Bayesian Analysis of a Comprehensive Model for Agricultural Futures

by

Adam Schmitz, Zhiguang Wang, and Jung-Han Kimn

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Abstract

Agricultural futures price features stochastic volatility, seasonal spot price volatility, and stochastic cost-of-carry. We propose a single comprehensive model that inludes all these features. We apply the proposed model to analyze the corn futures market from January 3rd, 1989, to December 31st, 2008. We conduct parameter estimation using Markov chain Monte Carlo (MCMC) with a novel dynamic tuning scheme. We also employ a parallel MCMC scheme for state variable estimation. Parameter estimates and model errors indicate the comprehensive model to be effective for modeling corn futures.

1 Introduction

Trading and interest in agricultural commodities has increased within the past decade leading to new highs in both price and trading volume for many tradable crops such as corn, soybeans, and wheat. The increase in trading activity can be attributable to a number of different factors, most notably an increase in global food prices, active pursuit of alternative energy, such as biofuels – ethanol and diesel, and financalization of commodities, such ad tradeable commodity indices. As the markets focus more attention on the agricultural commodities, practitioners and regulators both seek research defining and examining characteristic behaviors of these markets, futures markets in particular.

Previous studies on commodity futures have examined various characteristics of the price behavior, including mean reverting price, mean reverting stochastic volatility, seasonal spot price changes, and the term structure of futures price. A widely used form of stochastic volatility is Heston's (1993) mean reverting square root model that was proposed for financial options. Brooks and Prokopczuk (2013) model the log spot using a Heston-style

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stochastic volatility model with a seasonal price drift for energy, hard metals, and agricultural commodities as well as the S&P 500. Back et al. (2011) model natural gas futures and options using a stochastic volatility model with a seasonal long run mean. Geman and Nguyen (2005) use two-factor and three-factor models with mean reversion and seasonality in both the spot price and volatility using monthly soybean data from U.S., Argentina, and Brazil. Back et al. (2013) model the deseasonalized commodity spot as mean reverting in price with a seasonal volatility component for soybean and heating oil options. The log spot consists of the deseasonalized spot and a seasonal component. Trolle and Schwartz (2009) model crude oil spot using a two factor spanned mean reverting volatility model with term cost of carry. Sørensen (2002) and Richter and Sørensen (2003) study the effects of seasonal prices for agricultural commodities. Jin et al. (2010) generalize the model in Schwartz (1997) including spot price mean reversion and seasonality. Lence et al. (2013) construct the wheat futures curve using a two-factor mean-reverting model with spot seasonality.

The existing literature examines price characteristics of agricultural commodities separately. Specifically, Trolle and Schwartz (2009) use a model with stochastic volatility and convenience yield (indirectly modeling term structure of futures); Sørensen (2002) and Richter and Sørensen (2003) study the affects of seasonality on agricultural commodity prices; and Brooks and Prokopczuk (2013) model the log spot price with stochastic volatility and seasonality. We propose a comprehensive agricultural futures model that incorporates the characteristic behaviors of mean-reverting stochastic volatility, seasonalized spot volatility and stochastic convenience yield into a single model form.

We adopt Markov chain Monte Carlo (MCMC) method to estimate the proposed model. MCMC simulation has been widely used in parameter estimation for equity and commodities models, especially when there are a large number of parameters and state variables. Karali et al. (2011) and Schmitz et al. (2014) use a Bayesian procedure for parameter estimation of a stochastic volatility model for agricultural commodities futures and options, respectively. Equity model estimation via MCMC is used in Eraker (2004). Because of the number of state variables and super parameters in our comprehensive futures model, we find that MCMC is the most effective form of parameter estimation.

In addition to estimating parameters via MCMC, we employ a relatively new estimation procedure for (unobserverable) state variables. Previous studies that employ MCMC estimation often simulate state varaibles in a sequential fashion (see Erkaer (2004) for financial index options and Schmitz et al. (2014) for agricultural commodity options). The sequential estimation of state variables requires that a state variable could be only simulated once for any given iteration. When the number of days in the sample data increases, the total number of iterations to obtain a complete series of a state variable, which is equal to days multiplied by iterations, will increase linearly in a sequential estimation. Because the number of iterations is usually in hundreds of thousands, even adding one more day of state variable will result in a significant increase in simulation time. To expedite the simulation of state variables, we estimate them in parallel as proposed in Wilkinson (2005). The parallel simulation of state variables is implemented by splitting the daily time series into even and odd days, in which the even days are estimated first in a parallel fashion and then, using the immediately estimated even days, the odd days are estimated in the same parallel fashion. In addition, for both sequential and parallel estimation, we propose a dynamic tuning scheme in which tuning parameters are adjusted so that parameter acceptance is within the acceptable range of 15% to 30% as described in Roberts and Rosenthal (2001).

We apply the comprehensive model and estimation procedure to corn futures from January 3rd, 1989, to December 31st, 2008, and find model absolute relative errors are on average 7.35% with a standard deviation of 6.05%. Parameter estimates imply that long run mean variance is 0.3622 with a low mean reversion rate of 0.0010 (with a half time of about two years). There is very low but positive correlation between the volatility and spot processes. Furthermore, statistically significant spot volatility parameters imply a seasonal component to the volatility in the spot prices. These parameters indicate that the spot volatility tends to increase as harvest approaches and then decreases thereafter.

The remainder of this paper is as follows: Section 2 states the model and propositions for the calculation of the state variables; Section 3 outlines the data used in the study and the methodologies employed in parameter estimation; Section 4 analyzes the results of the estimation procedure and the model errors; Section 5 makes concluding remarks and discusses possible future research using the comprehensive model.

2 Models for Agricultural Futures

The dynamics for spot S_t, forward cost of carry y_t and volatility of spot price V_t are specified as follow:

$$\frac{dS_t}{S_t} = \delta(t)dt + \sigma_1\sqrt{V_t}dW_{1,t} \quad (1)$$

$$dy_t = \mu_y(t)dt + \sigma_2(t,T)\sqrt{V_t}dW_{2,t} \quad (2)$$

$$dV_t = \kappa(\overline{V} - V_t)dt + \sigma_3\sqrt{V_t}dW_{3,t} \quad (3)$$

with drift for the cost of carry

$$\mu_y(t) = \overline{\mu} + \eta\sin\left(2\pi(t+\zeta)\right)$$

and seasonal volatility

$$\sigma_1 = e^{\Psi} = e^{\Theta\sin(2\pi(t+\zeta))}$$

and correlations

$$dW_{1,t}dW_{3,t} = \rho_{13}dt.$$

The latent volatility follows a mean reverting process as in Heston (1993). The spot price has seasonal changes around a long-run mean. Instantaneous correlation between the spot price and volatility processes ρ_{13} captures the leverage effect.

Following Trolle and Schwartz (2009) we introduce the integrated forward cost of carry

$$Y(t,T) = \int_t^T y(t,u)du$$

and find

$$dY(t,T) \quad = \quad \left(-\delta(t) + \int_t^T \mu_y(t,u)du\right)dt + \sqrt{V_t}\int_t^T \sigma_2(t,u)du\, dW_{2,t}.$$

The time-t price of futures with expiration T can be written as

$$F(t,T) = S(t)e^{Y(t,T)}. \tag{4}$$

Using Itô's Lemma we find

$$\frac{dF(t,T)}{F(t,T)} = \frac{dS(t)}{S(t)} + dY(t,T) + \frac{1}{2}\left(dY(t,T)\right)^2 + \frac{dS(t)}{S(t)}dY(t,T).$$

Here we have

$$\left(dY(t,T)\right)^2 \quad = \quad V(t)\left(\int_t^T \sigma_2(t,u)du\right)^2 dt.$$

and

$$\frac{dS(t)}{S(t)}dY(t,T) \quad = \quad \left(V(t)e^{\Psi}\rho_{13}\int_t^T \sigma_2(t,u)du\right)dt.$$

Now we obtain the futures price dynamics:

$$\begin{aligned}\frac{dF(t,T)}{F(t,T)} \quad &= \quad \delta(t)dt + e^{\Psi}\sqrt{V_t}dW_{1,t} \\ &+ \left(-\delta(t) + \int_t^T \mu_y(t,u)du\right)dt + \sqrt{V(t)}\int_t^T \sigma_2(t,u)du\, dW_{2,t} \\ &+ \frac{1}{2}\left(\left(V(t)\left(\int_t^T \sigma_2(t,u)du\right)^2\right)dt + V(t)e^{\Psi}\rho_{13}\int_t^T \sigma_2(t,u)du\right)dt.\end{aligned}$$

We gather the drift terms (dt) and differentiate it with respect to T. Because the futures price should be diftless under the risk neutral measure, setting it to zero yields the following:

$$\frac{\partial}{\partial T}\left(\int_t^T \mu_y(t,u)du + \frac{1}{2}\left(\left(V(t)\left(\int_t^T \sigma_2(t,u)du\right)^2\right)dt + V(t)e^{\Psi}\rho_{13}\int_t^T \sigma_2(t,u)du\right)\right) = 0.$$

Further simplification shows

$$
\begin{aligned}
\mu_y(t,T) &= -\left(V(t)\sigma_2(t,T)\int_t^T \sigma_2(t,u)du + V(t)e^{\Psi}\rho_{13}\sigma_2(t,T)\right)\\
&= -\left(V(t)\sigma_2(t,T)\left(e^{\Psi}\rho_{13} + \int_t^T \sigma_2(t,u)du\right)\right.
\end{aligned}
$$

To incorporate the Samuelson Effect we model $\sigma_{yi}(t,T)$ as a determinstic process. We set the following:

$$
\sigma_2(t,T) = \alpha e^{-\gamma(T-t)}
$$

with the requirement of both α and γ to be greater than or equal to 0.

Adapting the solution procedure in Trolle and Schwartz (2009) to our problem, we find the time t cost-of-carry:

$$
\begin{aligned}
y(t,T) &= y(0,T) + \int_0^1 \mu_y(u,T)du + \int_0^1 \sigma_2(u,T)\sqrt{V(u)}dW_2(t)\\
&= y(0,T) + \left(\alpha e^{-\gamma(T-t)}x(t) + \alpha e^{-2\gamma(T-t)}\phi(t)\right)
\end{aligned}
$$

with

$$
\begin{aligned}
x(t) &= -\int_0^t V(u)\left(\frac{\alpha}{\gamma} + \rho_{13}e^{\Psi}\right)e^{-\gamma(t-u)}du + \int_0^t e^{-\gamma(t-u)}\sqrt{V(u)}dW_2(u),\\
\phi(t) &= \int_0^t V(u)\frac{\alpha}{\gamma}e^{-2\gamma(t-u)}du.
\end{aligned}
$$

Now we apply Itô's Lemma to the above to get the dynamics of the state variables x and ϕ to get

$$
\begin{aligned}
dx(t) &= \left(-\gamma x(t) - \left(\frac{\alpha}{\gamma} + \rho_{13}e^{\Psi}\right)V(t)\right)dt + \sqrt{V(t)}dW_2(t),\\
d\phi(t) &= \left(-2\gamma\phi(t) + \frac{\alpha}{\gamma}V(t)\right)dt.
\end{aligned}
$$

We let $s(t) \equiv \log S(t)$ be log spot price and apply Itô's Lemma to obtain the following log spot price process:

$$
ds(t) = \left[y(0,t) + \alpha(x(t) + \phi_t) - \frac{1}{2}\sigma_1^2 V_t\right]dt + \sigma_1\sqrt{V_t}dW_1(t).
$$

For ease of reference, we summarize the state variables in the following:

$$ds(t) = \left[y(0,t) + \alpha(x_t + \phi_t) - \frac{1}{2}\sigma_1^2 V_t\right] dt + \sigma_1 \sqrt{V_t} dW_1(t) \quad (5)$$

$$dx(t) = \left(-\gamma x(t) - \left(\frac{\alpha}{\gamma} + \rho_{13} e^{\Psi}\right) V(t)\right) dt + \sqrt{V(t)} dW_2(t) \quad (6)$$

$$dV_t = \kappa(\overline{V} - V_t)dt + \sigma_3 \sqrt{V_t} dW_{2,t}. \quad (7)$$

Taking the logarithm of Equation 4, we obtain the log futures price at time t expiring at T as follows:

$$\log F(t,T) = \varphi + s(t) + \frac{\alpha}{2\gamma}(1 - e^{-2\gamma\tau})x(t) + \frac{\alpha}{2\gamma}(1 - e^{-2\gamma\tau}). \quad (8)$$

3 Data and Methodology

3.1 Data

Daily corn futures prices are obtained from the CME group. Futures with maturities less 5 days are removed from the sample due to the potential microstructural noise. The dataset ranges from January 3rd, 1989 to December 31st, 2008, which amounts to 5,037 trading days. On any given day, the nearest March, May, July, September and December contracts are employed for the subsequent analysis. Descriptive statistics are reported in Table 1.

Table 1 shows that corn futures prices exhibit a contango structure, higher prices for longer maturities. The excess kurtosis and non-zero skewness indicate non-normal distribution of futures prices. The inverse relationship between standard deviation and length to maturity increases indicates that futures contracts become more volatile the nearer they are to maturity. This effect is known as the "Samuelson effect" (Samuelson 1965).

Table 1: Descriptive statistics for the five nearest corn futures

Corn	Mean	Std. Dev.	Skewness	Kurtosis	Min	Max
Futures1	2.6810	0.8378	2.4980	10.2866	1.7450	7.5450
Futures2	2.7374	0.8274	2.6659	11.3973	1.8650	7.6800
Futures3	2.7811	0.8191	2.9062	12.9918	1.9600	7.8800
Futures4	2.8106	0.8096	3.1436	14.5454	2.0700	8.0500
Futures5	2.8336	0.8073	3.2975	15.4398	2.1425	8.1300

3.2 Methodology for Parameter Estimation

The futures model consists of eight super parameters, κ, $\overline{V}$, σ_3, found in Equation 3, Θ, ζ, found in Equation 1, α, found in Equations 6, and 5, γ, found in Equation 6, and ρ_{13}, and the three state variables of cost-of-carry process, χ_t, Equation 6, spot price process, s_t, Equation

5, and the volatility process, V_t, Equation 3. These parameters and state variables are all simulated using the Metropolis-Hastings Markov Chain Monte Carlo (MCMC) algorithm. The first step in the algorithm is to choose a candidate value randomly from a distribution and then test that value against the current parameter value. Testing involves measuring the ratio of probabilities generated from the current value to the proposed value. A ratio greater than one makes the proposed parameter value the current parameter value. If the ratio is less than one, a random number g between 0 and 1 is drawn. If the proposed value is greater than g then the proposed becomes the current. Otherwise, the current remains the parameter value. One point worth to note is that there is one spot price s_t but with five futures prices. We first estimate contract-specific spot price and then average the five estimates to obtain the model-implied spot price for every day.

State variable estimation is conducted in both a sequential and a parallel emulated fashion.[1] All state variables utilize the days prior to and after the currently estimated day. We use V_t as an example to explain the two schemes. Let V_t^i represent the estimated volatility on day t in ith iteration. First of all, the estimation of V_t utilizes both V_{t-1} and V_{t+1}. For sequential estimation, the simulation of $V_t^{(i)}$ utilizes both the values of $V_{t-1}^{(i)}$ and $V_{t+1}^{(i-1)}$. More specifically, the current iterative value of day t, V_t^i, is based on the value of day $t-1$, V_{t-1}^i and the previous iterative value of day $t+1$, V_{t+1}^{i-1}.

For parallel state estimation the current day's state is estimated using the estimates based on the last iteration, or $V_t^{(i)}$ utilizes $V_{t-1}^{(i-1)}$ and $V_{t+1}^{(i-1)}$. Wilkinson (2005) suggests that a vector of even days be split evenly among the available processors and estimated. After the even days have been estimated, the vector of odd days should be estimated in the same way but with the updated even values. We emulate this procedure by first estimating the even days in a sequential manner and then estimating the odd days. Errors and parameter estimates will be compared between the sequential estimation and the parallel emulation.

For all super and state parameters we employ a dynamic tuning scheme. The rate of proposed parameter acceptance using the Metropolis-Hastings algorithm should neither be too high nor too low. Statisticians suggest an acceptance rate in the range of 15% to 30%. The rate of acceptance is controlled by the parameter's tuning factor. For the "burn-in" period, depending on the acceptance rate, we allow the tuning factors to increase or decrease by 10% for every 1,000 grand iterations. An acceptance rate that is too high (low) indicates that the tuning factor should be increased (decreased). After the burn-in period, tuning factors are frozen for the remainder of the iterations.

4 Results

In this section we first present the super parameter and state variable estimates generated from the sequential and parallel MCMC methods with analysis. We then analyze the er-

[1] Although we do not actually estimate the states in a parallel fashion, we employ the same technique that would be used with parallelization. In fact, our technique is the parallel technique but with the employment of a single processor instead of multiple.

rors generated from the estimated super parameters and state variables for both estimation methods.

4.1 Sequential Parameter Estimates and Analysis

Estimates for the eight super parameters are shown in Table 2 for the sample period beginning in 1989 and ending in 2008. We ran a total of 100,000 iterations and calculated the mean of the last 90,000 for the parameter estimate. This left the first 10,000 as the "burn-in" and was discarded. All parameters are statistically significant at the 5% level. The parameters for stochastic volatility are κ, $\overline{V}$, and σ_3. We find that the volatility process is persistent with the mean reversion speed κ being low. The half time of volatility ($\ln(2)/\kappa$) is approximately 700 days indicating a substantial amount of time for volatility dispersion. The long-run average of deseasonalized variance, $\overline{V}$, is estimated to be 0.3622 while volatility of volatility, σ_3, is estimated to be 0.0428. The correlation parameter ρ_{13} is slightly positive at 0.00031 but statistically significant indicating that spot prices and volatility are positively correlated. Spot volatility parameters Θ and ζ indicate a seasonal trend to the spot price volatility. Figure 1 shows that the seasonal trend to spot volatility increases as harvest approaches in September and October and then decreases once the harvest is finished. This is consistent with previous research which also shows increasing volatility with an approaching harvest.

Latent volatility is shown in Figure 2. The mean and standard deviation of this process is 1.3108 and 0.0022 respectively. We find in this figure that the V_t process is fairly constant over the time frame indicating little movement during estimation. Although there is little movement in estimation on an absolute scale, the trends in the estimated volatility process generally follow the empirical price volatility trends. In addition, the magnitudes of the volatility plot roughly match the volatility seen in the corn markets. Figures 4.1 and 4 show the estimated spot and corn futures prices over the time period under study, respectively. As the corn prices start to increase in the middle 1990's, the volatility plot also increases indicating that the estimation process is capturing this change. Also, as corn prices start to decrease from the late 1990's to the middle 2000's, the volatility plot decreases. Finally, the volatility of the most recent phenomenon of dramatic price increases in the late 2000's is also captured by the estimated volatility process as there is also a dramatic increase in the volatility over this period.

4.2 Sequential Pricing Error Analysis

We analyze the pricing performance of the model by calculating the absolute dollar and absolute relative pricing errors generated from the parameter estimates. We compute the absolute dollar errors as

$$|P_{Mod,t} - P_{Mkt,t}|$$

Table 2: Parameter estimates from the MCMC method

Parameter	Mean	Std.Dev.	95% C.I.
κ	0.0010	0.0011	[0.0001, 0.0001]
$\overline{V}$	0.3622	0.4948	[0.3588, 0.3656]
Θ	0.0067	0.1014	[0.0060, 0.0074]
ζ	0.0013	0.1164	[0.0005, 0.0021]
σ_3	0.0428	0.1079	[0.0421, 0.0435]
ρ_{13}	3.1e-4	0.0190	[1.9e-4, 4.3e-4]
α	1.5994	1.6021	[1.5883, 1.6105]
γ	7.9392	4.3012	[7.9094, 7.9690]

and the absolute relative errors as

$$\frac{|P_{Mod,t} - P_{Mkt,t}|}{P_{Mkt,t}}$$

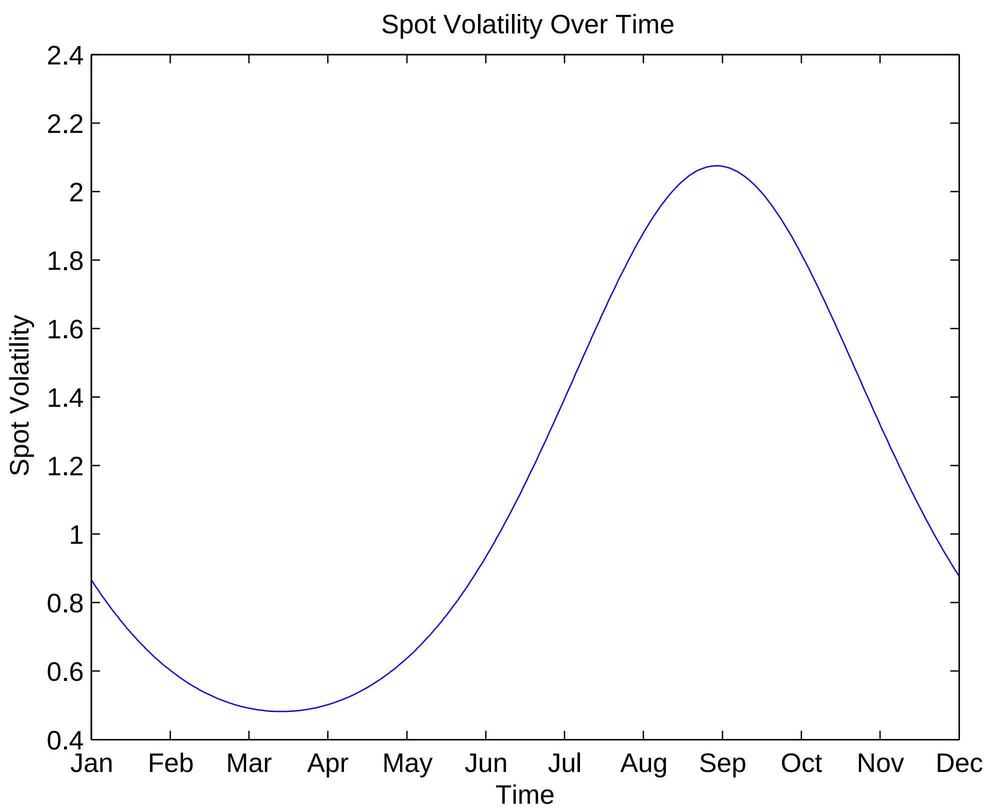

Figure 1: Seasonality of Volatility

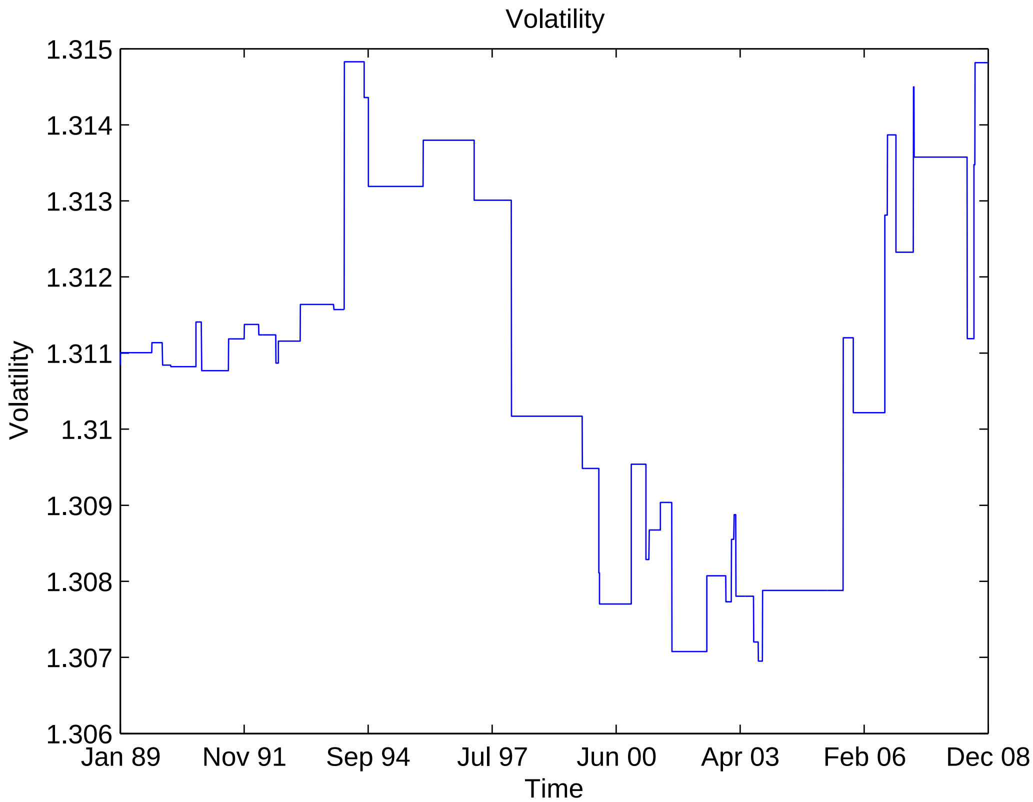

Figure 2: Latent Stochastic Volatility

with "Mod" being our model price and "Mkt" being the market price. The mean and standard deviation for the sequential absolute dollar errors are \$0.076 and \$0.067 respectively. The mean and standard deviation for the sequential relative dollar errors are 7.35% and 6.05% respectively. Figure 5 shows the absolute relative errors over time.

4.3 Parallel Estimation Results

As with the sequential estimation, parallel estimation burn-in is for the first 10,000 iterations. During these iterations all state variables are estimated in a sequential manner with the parallel emulation scheme starting after the burn-in period. The mean and standard deviation of the absolute relative errors are 14.38% and 10.19% respectively. Figure 6 displays the errors for the parallel MCMC estimation. As a comparison, the errors generated from the sequential estimation procedure are lower and in a tighter range.

Table 3 displays the parameter estimates and standard deviations from both the sequential and parallel estimations. With the exceptions of α and γ, parameter estimates for the sequential and parallel methods are not inline with some being orders of magnitude off. An explanation for this may have to do with the loss of information inherent in the parallel method. When the sequential method estimates day i's state variable, it is using new infor-

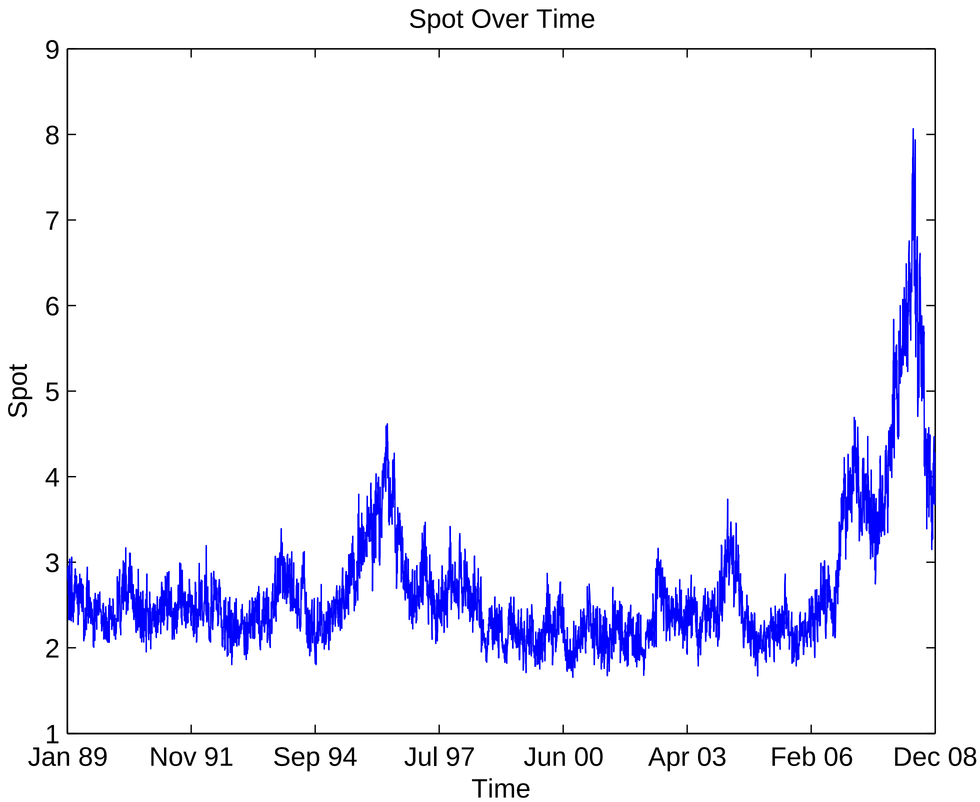

Figure 3: Spot prices over time

mation in the other parameters that were estimated on the previous grand iteration. In the parallel method, the information in the state estimation is from an even earlier iteration, namely the two iterations preceding the current one. Given this loss of information, the parallel scheme may require more iterations than the sequential. The increase in time for requiring more iterations can be more than offset by parallelizing the estimation process over multiple processors.

Implementing the parallel scheme in a lower level language such as C may make the results more reliable because of the control the user has over the pseudo random number generator. When using random number generators over separate processors it is crucial those processors are generating streams of numbers that are statistically independent from each other. Using a Scalable Parallel Random Number Generator (SPRNG) programs each processor so that a unique and independent stream of numbers is generated on that processor.[2]

[2] Find more information on SPRNGs at http://www.sprng.org/.

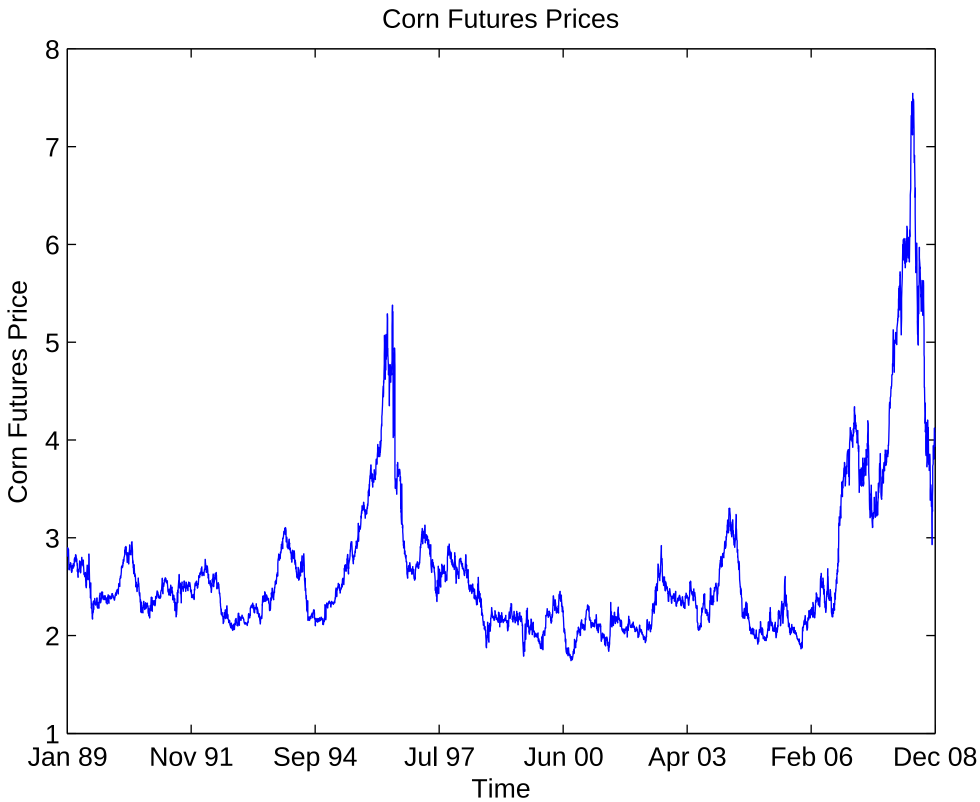

Figure 4: Corn Futures Prices

5 Conclusion

Stochastic volatility, spot seasonality, cost-of-carry, and seasonal spot volatility are all accepted characteristic behaviors of agricultural futures. These behaviors have been treated separately before in Back, et al. (2011), Sørensen (2002), and Brooks and Prokopczuk (2013). We propose to model these behaviors in an all-inclusive, comprehensive model and apply this model to the corn futures market.

Given the number of super parameters and state variables, we implement the MCMC procedure for parameter estimation. These techniques have been previously used in Eraker (2004) for financial derivatives, and Karali et al. (2011) and Schmitz et al. (2014) for commodity derivatives. We propose a novel technique for adjusting the tuning parameters so that parameter estimation remains viable throughout the run of the program. In addition to dynamic tuning, we implement a parallel estimation scheme proposed in Wilkinson (2005). This scheme separates the vector of daily state observations into an even and odd group in which both are estimated separately.

The parameter estimates from the sequential run of the MCMC procedure are all statistically significant. The estimates imply that mean reversion of volatility is quite slow meaning that

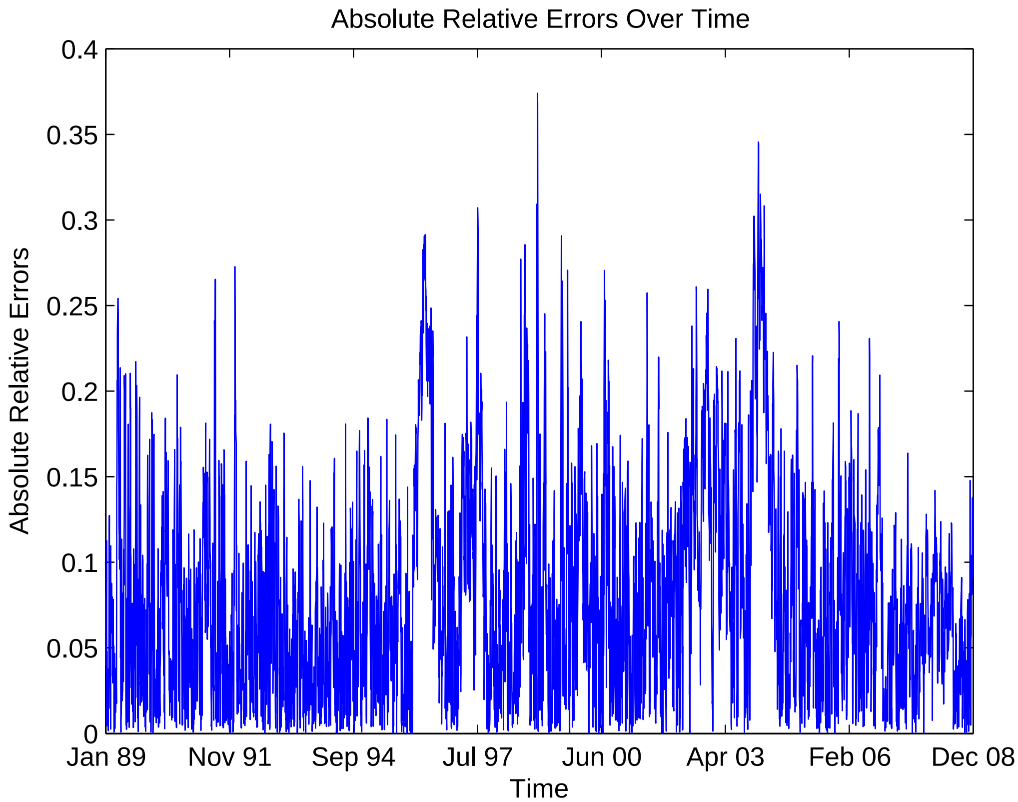

Figure 5: Absolute relative errors over time

volatility disruptions dissipate at a slow rate. The spot volatility indicates that volatility in the spot price is highest when the harvest is near and decreases afterwards. This finding is inline with previous studies of spot price seasonality. There is little correlation between volatility and spot. However, the correlation is positive and statistically significant. Errors generated from the sequential MCMC parameter estimation are reasonable with the overall relative mean of approximately 7%.

Parallel parameter estimates are different from and model errors are inferior to the corresponding sequential results when the same number of iterations is simulated for both MCMC methods. Inherent in the parallel scheme is the loss of information due to the fact that the parallel method requires the use of $V_{t-1}^{(i-1)}$ and $V_{t+1}^{(i-1)}$ which were themselves estimated without using the most recent parameter estimates. This loss of information may require the increase in the number of iterations needed for the parallel method. However, the benefit of the parallel method can still be achieved by dividing the parallel tasks among a large number of CPUs or GPUs, thereby offsetting the disadvantage of information loss in the paralle scheme.

Future studies using the comprehensive agricultural futures model would likely include enhancements to the underlying model. These enhancements could include seasonal volatility

Table 3: Parameter estimates from sequential and parallel MCMC methods

Parameter	Mean (Seq.)	Std.Dev. (Seq.)	Mean (Par.)	Std.Dev. (Par.)
κ	0.0010	0.0011	0.0151	0.0110
$\overline{V}$	0.3622	0.4948	0.1132	0.0648
Θ	0.0067	0.1014	-0.0693	0.0804
ζ	0.0013	0.1164	0.0474	0.1037
σ_3	0.0428	0.1079	1.5847	0.1716
ρ_{13}	3.1e-4	0.0190	-0.0248	0.1022
α	1.5994	1.6021	1.4432	0.1019
γ	7.9392	4.3012	7.5304	4.3304

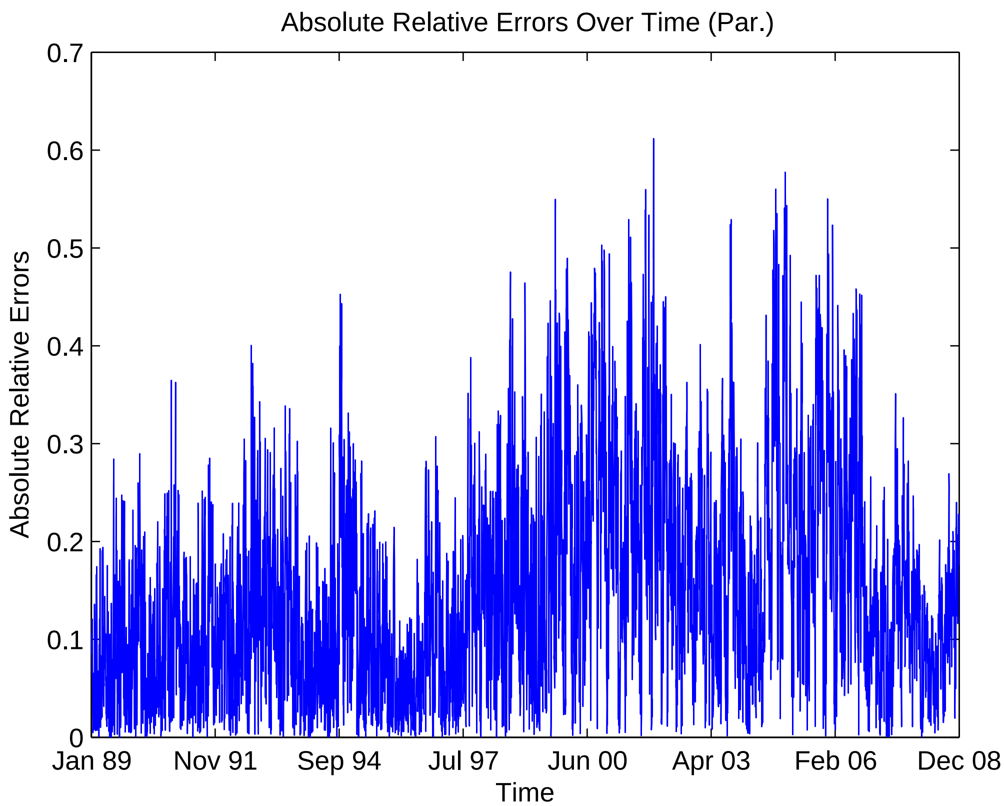

Figure 6: Absolute relative errors over time for the parallel MCMC procedure

of volatility. It is reasonable that as the spot price increases in volatility, the latent volatility process may also increase. The inclusion of spanned stochastic volatility as found in Trolle and Schwartz (2009) would indicate the degree of unhedgable volatility. Spot and volatiltiy jumps using a Poisson process may enhance the ability of the model to deal with extreme changes in price and volatility. In our study, we have found that stochastic volatility, spot seasonality, and cost-of-carry are necessary behaviors for inclusion in an agricultural futures model.

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