NCCC-134
APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

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Suggested citation format:

Bekkerman, A., G. W. Brester, and T. McDonald. 2011. "A Quantile Regression Approach to Analyzing Quality-Differentiated Agricultural Markets." Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. St. Louis, MO. [http://www.farmdoc.illinois.edu/nccc134].

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Paper presented at the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management
St. Louis, Missouri, April 18-19, 2011

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Hedonic models are commonly used to quantify the value of characteristics implicit in a product's price. However, when products are heterogenous across quality levels, using traditional parametric methods for estimating characteristic values may provide poor inferences about quality effects. We propose using a quantile regression framework for estimating the value of characteristics in quality-differentiated products. Semi-parametric quantile regressions allow the data to flexibly identify and estimate quality effects across a conditional price distribution. Using purchase price data from a bull auction, we show complementary non-linear relationships exist between quality and bull carcass and growth traits. Improved precision in understanding consumer valuation of product characteristics across quality market segments can help producers tailor products for each segment.

KEYWORDS: bull sales, heritable traits, quality differentiation, quantile regression, RFI

Introduction

Agricultural products are often differentiated by quality and these differences affect consumer valuation of product characteristics and purchase decisions. Understanding how valuations differ with quality can help producers develop better production and marketing strategies by improving characteristics that are most valued in a targeted quality market segment. However, identifying and controlling for quality differences in empirical analyses is often complicated because it is difficult to explicitly quantify and characterize every quality consideration. For example, hedonic models are commonly used to estimate the value of characteristics implicit in a product's price, but these models are traditionally estimated using parametric regressions, which require quality considerations to be quantified and parameterized. Furthermore, even when quality measures (or appropriate proxies) are available, commonly used empirical methods only estimate *average* marginal effects of quality across all possible quality levels.

We propose using a semi-parametric quantile regression (QR) framework for estimating characteristic valuations when quality differentiation is substantial. The quantile regression method has been used extensively in the labor economics literature to study topics such as the heterogeneity of wage effects, returns to education, and school quality (Chamberlain 1994, Buchinsky 1997, Eide and Showalter 1998, Levin 2001). We show that quantile regressions can also be used to quantify the effects of quality on consumers' valuation of implicit product characteristics.

Our application focuses on estimating the marginal value of hedonic characteristics associated with purchases of cattle genetics. Genetic seedstock represents a classic example of a quality-differentiated agricultural product (Dhuyvetter et al. 1996). Cattle producers purchase bulls to produce calves, improve product quality, and upgrade the genetic composition of cow herds through heifer calf retention. Previous studies show that producers select seedstock based on production needs, environmental conditions, and marketing strategies (Chvosta, Rucker, and Watts 2001; Vanek, Watts, and Brester 2008). These results were generally based on bull purchase transactions occurring at relatively homogenous production sales, so hedonic price estimates that represent averages across bull qualities at such auctions may be relatively accurate.[1] Some bull auctions offer bulls that are substantially heterogeneous in quality; therefore, buyers may more heavily condition their valuations of implicit characteristics on bull quality.

We characterize "quality" as a set of measurable and unmeasurable considerations affecting consumers' valuation of product characteristics.[2] We seek to determine whether meaningful interactions exist between a product's revealed quality and its measurable product traits. Although quality perceptions may be difficult to directly identify and quantify by economists, it is likely that buyers consider quality aspects in their purchasing decisions and these considerations are implicitly manifest in a product's sale price. For example, if bull buyers more heavily value a marginal increase in the feed-to-gain trait in higher-quality bulls, then these buyers would be willing to pay more for marginal feed-to-gain increases in high-quality bulls than for identical increases in lower-quality bulls. Using quantile regressions, we seek to exploit heterogeneity in the conditional price distribution to determine whether buyers' valuations change across products of different perceived qualities.

In this study, we show that consumers' valuations of many bull characteristics are affected by quality considerations. After adjusting for non-normality in the price distribution and investigating the presence of non-linear relationships between a bull's sale price and its observable characteristics, quantile regressions reveal non-constant marginal effects of bull traits across the conditional sale price distribution. That is, the semi-parametric estimation method provides important inferences about consumers' quality considerations by identifying information implicit in the product's price. These results provide a better understanding of bull buyer behavior and may improve bull producers' ability to tailor products to specific quality market segments.

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[1] However, Vanek, Watts, and Brester (2008) provide some evidence that quality segmentation exists across seedstock markets.

[2] Alternatively, quality can be described as a product's perceived overall worth, which may not be fully characterized by observable and measurable product traits.

Modeling Quality in a Quantile Regression Framework

Conditional-mean functions are a foundation for a large number of modeling techniques, including linear regressions, weighted and non-linear least squares regression specifications, and maximum likelihood models. Under appropriate statistical conditions, estimation using these techniques is relatively simple and empirical results are easily interpreted. However, the conditional-mean estimation framework has several important limitations. First, conditional-mean functions are not easily generalizable to modeling data in non-central locations of the conditional dependent variable distribution, making analyses of the distribution tails difficult. Second, conditional-mean functions focus primarily on marginal effects of modeled regressors on the central tendency of the conditional distribution. When marginal effects vary across the conditional dependent variable distribution, marginal effects at the data's central tendency may substantially distort economic inferences. Quantile regressions can provide a more robust characterization of data relationships (Koenker and Bassett 1978).[3]

A linear quantile regression is similar to least squares models in that both minimize weighted sums of residuals, but differ in their specification of weighting mechanisms. For a model $y = \boldsymbol{x'\beta} + \varepsilon$, ordinary least squares estimate the conditional mean function $E[y|X = x] = \boldsymbol{x'\beta}$ by solving $\hat{\boldsymbol{\beta}} = \min_{\beta \in \boldsymbol{R}} \sum_i (y_i - \boldsymbol{x_i'\beta})^2$. Similarly, Koenker and Bassett (1978) show that for the ϕ^{th} regression quantile, the model $y = \boldsymbol{x'\beta(\phi)} + \varepsilon(\phi)$ can be estimated using a linear conditional quantile function, $Q(\phi)(y|X = x) = \boldsymbol{x'\beta(\phi)}$, solving:

$$\widehat{\boldsymbol{\beta}}(\boldsymbol{\phi}) = \min_{\beta \in \boldsymbol{R}^{\phi}} \left\{ \sum_{i \in (i : y_i \geq x_i' \beta(\phi))} \phi |y_i - \boldsymbol{x_i'\beta(\phi)}| + \sum_{i \in (i : y_i < x_i' \beta(\phi))} (1-\phi) |y_i - \boldsymbol{x_i'\beta(\phi)}| \right\}, \quad (1)$$

where $0 < \phi < 1$ is the proportion of y with outcomes below the ϕ^{th} sample quantile.[4] The estimation of a linear conditional quantile function is loosely analogous to least squares estimation, in which a Euclidian distance $\|y - \hat{y}\|$ is minimized over all $\hat{y}$ in the column span of X (Koenker 2005). A quantile regression minimizes a weighted sum of absolute errors, with weights ϕ and $(1-\phi)$ assigned to positive and negative residuals, respectively. A different set of weights is assigned and a different associated conditional quantile function is estimated for each ϕ^{th} sample quantile.[5]

[3] An overview of quantile regression methods and examples of applications in economics is provided by Hao and Naiman (2007). A more thorough theoretical model and derivation of statistical inferences associated with quantile regressions are shown in Koenker (2005).

[4] The term "quantile" should not be confused with "quartile" or "quintile." A quantile is a general term describing any value $\phi \in [0, 1]$ such that the probability of the response variable is less than or equal to ϕ. Quartiles and quintiles are special cases of a quantile, referring to four and five equally-spaced quantiles on the interval [0,1].

[5] A special case of QR is the least absolute deviation (LAD) estimator, which is equivalent to a quantile regression at the 50^{th} quantile (Greene 2003).

Each conditional quantile estimation uses the entire sample data set. However, absolute positive and negative residuals are weighted differently depending on the sample quantile value ϕ. Consequently, the minimized value of this weighted sum also differs for each estimated conditional quantile function, and each estimation defined by ϕ yields a unique set of conditional-quantile parameter values.[6] Parenthetically, OLS estimates minimize the sum of equally weighted squared residuals around a conditional mean, resulting in a single set of estimated conditional-mean parameters. Each set of regression quantiles, $\widehat{\beta}(\phi)$, specifies a fitted line, with $\phi\%$ of observations lying below the line and $(1-\phi)\%$ lying above the line. Alternatively, $\phi\%$ of data have negative residuals and $(1-\phi)\%$ have positive residuals (Hao and Naiman 2007).

Quantile regression estimation is essentially a linear programming problem. Consequently, numerous methods exist for solving such problems. A simplex algorithm for median regressions (Barrodale and Roberts 1973) and its variations are popular, stable methods for analyzing relatively small data samples. Koenker and d'Orey (1994) generalize the simplex algorithm for quantile regression. For larger sample sizes, Karmarkar (1984) developed an interior point algorithm adapted for quantile regression by Portnoy and Koenker (1997). Additionally, Chen (2007) describes the use of smoothing algorithms, which approximate conditional quantile functions and use a Newton-Raphson procedure to iteratively determine a solution.[7] Chen and Wei (2005) present computational comparisons of various algorithms.

When random variables are assumed to be independent and identically distributed ($i.i.d.$), the covariance matrix for $\widehat{\boldsymbol{\beta}}(\boldsymbol{\phi})$ is $\sum_{\widehat{\beta}(\phi)} = \frac{\phi(1-\phi)}{n}\frac{1}{f_{\varepsilon(\phi)}(0)^2}(X'X)^{-1}$ (Hao and Naiman 2007). The term $f_{\varepsilon(\phi)}(0)^2$ is an assumed local probability density function of the error term $\varepsilon(\phi)$.[8] When $i.i.d.$ variables cannot be assumed, bootstrapping is an alternative method for obtaining standard errors. In addition, Koenker and Machado (1999) describe an analog to the familiar R^2 statistic used to measure goodness-of-fit in least squares estimations. A pseudo-R^2 for the ϕ^{th} quantile is calculated as $R^2(\phi) = 1 - \frac{S_1(\phi)}{S_0(\phi)}$, where $S_1(\phi)$ and $S_0(\phi)$ are the sum of weighted errors for fully specified and restricted models. Typically, the restricted model is estimated with only an intercept term. As with least squares models, a higher pseudo-R^2 indicates a better fit.

Note that regression quantiles cannot be obtained by performing least squares estimation on sub-samples of the data associated with the ϕ^{th} dependent variable quantile. Such sub-sampling truncates the dependent variable and results in biased and inefficient parameter estimates (Hausman and Wise 1977, Heckman 1979). Estimation methods for truncated data exist;

[6]Because the entire data sample is used for estimating each conditional quantile function, increasing the number of regression quantiles does not decrease degrees of freedom.

[7]Because the conditional quantile function minimizes the sum of absolute errors, non-differentiability exists when a residual is zero, $|y_i - x_i'\beta(\phi)| = 0$.

[8]For a complete description of estimating the term $\frac{1}{f_{\varepsilon(\phi)}(0)^2}$, see Koenker (2005).

however, using these methods on sub-samples can omit relevant information present in the excluded sample portions. Truncation and information loss does not occur with quantile regression estimation because each conditional quantile function is estimated using the entire sample.

QR Estimations for Quality-Differentiated Products

Following Rosen (1974) and Arias, Hallock, and Sosa-Escudero (2001), we specify a hedonic model with quality interaction effects as:

$$P_i = \sum_j X_{ij}\beta_j + g(X_{ij}, q_i)\gamma_{ij} + e_i \; . \tag{2}$$

The term P_i represents the price of product i, X_{ij} is the j^{th} measurable product characteristic, q_i is an unquantifiable measure of quality, e_i is a random disturbance term, and β_{ij} is the marginal price change associated with an additional unit of X_{ij}. An *a priori* unknown relationship between a product's quality and the trait X_{ij} is characterized by the function $g(X_{ij}, q_i)$, such that the term γ_{ij} represents the marginal contribution to price from changes in $g(X_{ij}, q_i)$. For example, γ_{ij} is expected to be positive if consumers value additional units of X_{ij} more in higher-quality products.

Parametric regression frameworks are widely used in agricultural economics research to estimate marginal values of characteristics in hedonic models. Estimating equation (2) using these frameworks requires that the quality interaction term, $g(X_{ij}, q_i)$, be parameterized. For example, one common parameterization is the multiplicative form: $g(X_{ij}, q_i) = X_{ij}q_i$. The parameterization requires that the term q_i be explicitly identified by a quantifiable measure describing a product's quality or by a closely correlated proxy. However, even if such measures exist, it may still be difficult to quantify quality effects across a spectrum of quality levels. For example, an ordinary least squares (OLS) estimation of equation (2) yields the following marginal effect of X_{ij}:

$$\frac{\partial E[P_i|X_{ij}]}{\partial X_{ij}} = \widehat{\beta}_{ij} + \gamma_{ij} \cdot \overline{q} \; . \tag{3}$$

This implies that the marginal valuation of X_{ij} is altered by average quality across all products. When product quality is relatively homogeneous, OLS estimates may accurately describe the marginal effect of quality on the conditional price distribution. However, average marginal effect estimates of product characteristics may be less conclusive when quality heterogeneity exists.[9]

When products are relatively heterogenous, consumers' valuations of product traits can differ across a spectrum of quality levels. It is frequently the case that quality metrics either do not exist or are unmeasurable. In such cases, buyers reveal their quality considerations through willingness-to-pay valuations. For example, in auctions, bidders actively reveal their preferences by contributing to the price determination process, and these contributions can differ based on the bidder's quality considerations. These differences are reflected in the conditional price distribution, because a product's perceived quality level can affect the premiums that consumers are willing to pay for marginal changes in a characteristic. Quantile regressions provide a flexible, semi-parametric estimation framework for quantifying marginal impacts of product quality across the conditional price distribution. For a sample quantile ϕ, the conditional quantile marginal effect (regression quantile) is characterized as follows:

$$\frac{\partial Q(\phi)[P_i|X_{ij}]}{\partial X_{ij}} \equiv \widehat{\beta}_{ij}(\phi) = \widehat{\beta}_{ij} + \gamma_{ij} \cdot \frac{\partial Q(\phi)[g(X_{ij}, q_i)|X_{ij}]}{\partial X_{ij}} \, . \tag{4}$$

Unlike conditional marginal effects implied by parametric specifications, the conditional quantile effect does not necessitate an explicit parameterization of the quality interaction term $g(X_{ij}, q_i)$. Rather, performing a set of quantile regressions across different values of ϕ allows the data (instead of the selected parameterization) to identify the impacts of quality effects across heterogeneous products. If quality considerations do not impact valuations of the characteristic X_{ij}, then $\frac{\partial Q(\phi)[g(X_{ij},q_i)|X_{ij}]}{\partial X_{ij}} = 0$. That is, consumers value marginal changes in X_{ij} equally, regardless of a product's quality.

It is critical to recognize that the quantile regression framework is not a simple correction for a misspecified linear conditional-mean function. That is, it is inappropriate to assume that marginal effects of modeled variables in different parts of the conditional price distribution can be replicated by simply adding interaction terms, transforming the data, or subsampling. For example, Ladd and Martin (1976) show that in modeling agricultural commodity prices, the marginal effect of changes in a particular product characteristic, x_{ij}, may be non-linearly related to price. That is, the effect of changes in characteristic x_{ij} is conditional on the level of the characteristic. The authors recommend modeling prices as $P_i = \beta_0 + \beta_1 x_{ij} + \beta_2 x_{ij}^2 + \varepsilon_i$, such that the marginal effect of x_{ij} is $\partial P_i / \partial x_{ij} = \hat{\beta}_1 + 2\hat{\beta}_2 x_{ij}$. However, this marginal effect describes only the average impact on price at different levels of x_{ij}. It does not reveal whether $\partial P_i / \partial x_{ij}$ is different across non-central portions of the conditional price distribution, such as

[9] Other parametric frameworks can be envisioned for estimating quality valuation across a spectrum of quality levels. However, these methods would still require a parameterization of the quality interaction term, $g(X_{ij}, q_i)$, as well as *a priori* knowledge of the quality spectrum.

the upper and lower tails. Therefore, altering the functional form using interaction terms does not substitute for inferences provided by quantile regressions. Alternatively, conditional quantile estimates indicating non-constant marginal effects of x_{ij} across the price distribution does not immediately imply that the same information can be obtained by simply altering the functional form of a conditional-mean model.

The quantile regression method can reveal heterogeneous marginal quality effects by describing both location and scale shifts of the conditional price distribution. For example, when quality differences are considered to be non-existent or constant (as shown in equation (3)), a marginal effect of a covariate is characterized by a pure location shift of the conditional distribution. This shift is shown in figure 1(a), in which a one-unit change in covariate X_{ij} simply increases the mean of the distribution (shifts the distribution from the solid to the dashed line). That is, observations in the lower tail of the distribution are shifted a distance of ρ, from ϕ_L^0 to ϕ_L^1, and observations in the upper tail of the distribution are shifted a distance of ρ, from ϕ_U^0 to ϕ_U^1. These identical shifts imply equal marginal impacts of a covariate across all values in the response variable distribution.

Scale adjustments reveal how changes in a covariate affect observations in different segments of the conditional response distribution (CRD). That is, if a change in a covariate affects the scale of the CRD, then observations in one segment of the distribution may be affected differently than observations in a different segment. Figure 1(b) shows the effect of a change in X_{ij} on both the location and scale of the CRD. Not only does increasing X_{ij} cause the value of the central tendency of P_i to increase, but it also causes the conditional response distribution to have a larger spread around the new value. As a result, observations in different parts of the CRD are affected differently by changes in X_{ij}. For example, the marginal effect of X_{ij} on observations in the lower tail of the distribution is the distance ρ_L, from ϕ_L^0 to ϕ_L^1. However, observations in the upper tail are shifted a distance of ρ_U, from ϕ_U^0 to ϕ_U^1, such that $\rho_U > \rho_L$. Thus, increasing X_{ij} causes values of P_i in the upper tail to increase by more than values of y in the lower tail. Because quality differences are implicit in a product's price, recognizing conditional distribution shape shifts – both location and scale – is important for quality-differentiated product research.

Quality Differentiation at Bull Auctions

Bull auctions are one example of an agricultural market containing quality-differentiated products. When a heterogeneous set of producers sells bulls to a heterogeneous set of buyers, those bulls likely represent a wide range of quality. Bull purchasers use measurable information contained in sale catalogs along with visual inspections of bulls' physical characteristics, knowledge of sellers' reputations, a bull's expected length of service, and a bull's genealogical and heritability histories to make bidding decisions. Conditional on quality considerations, buyers implicitly value each characteristic to determine the price they are willing to pay for a bundle of these characteristics in a specific bull. It seems likely that the value placed on a particular trait may vary significantly with a bull's unquantifiable perceived quality.

Most genetic improvements in the beef industry occur through bull selection rather than cow selection with higher value placed on bulls with better expected progeny differences (EPDs) and simple performance measures (SPMs). EPDs are quantitative predictions of a bull's heritable traits based on genealogical histories. Vanek, Watts, and Brester (2008) found that buyers pay more for bulls with higher EPDs. Simple performance measures (SPMs) refer to observable bull measurements occurring during 70-100 day performance testing. Chvosta, Rucker, and Watts (2001) and Dhuyvetter et al. (1996) show that both EPDs and SPMs affect bull sale prices at auctions.

A third bull quality measure is residual feed intake (RFI). RFI is a measure of the difference between an animal's actual feed intake and its expected feed intake, and is uncorrelated with average daily weight gain (Koch et al. 1963).[10] A bull with a better RFI trait is able to add weight while consuming less feed relative to a bull possessing poorer RFI. Therefore, RFI may provide a means for selecting bulls with higher feed efficiency characteristics without negatively impacting cattle growth and carcass traits. Given that RFI is relatively heritable, it seems reasonable that bull purchasers would value this characteristic. Recent technological advances, such as the GrowSafe® system, have led to more accurate actual feed intake measurements. Midland Bull Test (Columbus, MT) is one of the first and largest facilities in North America to use the GrowSafe® system for measuring bull RFI.[11]

Along with RFI, the Midland Bull Test (MBT) facility measures other feed efficiency, weight gain, carcass quality, and fertility characteristics. More than 100 bull producers annually contract with MBT to conduct bull performance tests.[12] At the conclusion of the testing period, MBT publishes test results in sale catalogs and facilitates an open out-cry sale. Using sale catalogs and visual inspection, buyers evaluate bull characteristics and offer bids during a live auction. During the 2009 MBT bull auction, 202 producers sold bulls to 292 buyers.

We implement a semi-log linear hedonic price model to quantify bull characteristic effects on bull sale prices, because the price distribution is positively skewed. The price model is as follows:

[10]Expected feed intake is the amount of feed necessary for an animal to meet its maintenance and production requirements. Although the idea of using residual feed intake was introduced in the early 1960s, RFI has not been widely used in valuing bulls by buyers until recently.

[11]For a detailed description of RFI measurement and calculation, see McDonald et al. (2010).

[12]MBT is among a few venues that facilitate the testing and sale of bulls that are not owned by MBT. Most bull auctions focus on a single (or only a few) producer's bull offerings.

$$\ln p_i(\phi) = \beta_{i0}(\phi) + \sum_j \theta_{ij}(\phi) \cdot SPM_{ij} + \sum_k \gamma_{ik}(\phi) \cdot EPD_{ik} + \varepsilon_i(\phi) \;, \tag{5}$$

where $\ln p_i(\phi)$ is the logged purchase price of bull i, SPM_{ij} is the j^{th} simple performance measure, and EPD_{ik} is the k^{th} expected progeny difference measure. SPM measures include actual birth weight (pounds), weaning weight, 365-day weight, age (in days) at sale, age-squared, average daily gain (pounds per day), intramuscular-fat (percentage of fat in rib-eye area), and rib-eye area (square inches). Rib-eye area and intramuscular fat (marbling) are bull carcass characteristics that improve perceived product quality and are therefore valued by end-users. Expected progeny differences include birth weight EPD, birth-to-yearling gain EPD, rib-eye area EPD, intramuscular fat EPD, and milk EPD. Birth-to-yearling gain EPD is calculated as the difference between birth weight and yearling weight EPDs. High correlation between the birth weight and yearling weight EPD measures prevents directly including both measures in the model (Vanek, Watts, and Brester 2008). Finally, $\varepsilon_i(\phi)$ is an error term and $\beta_{i0}(\phi)$, $\theta_{ij}(\phi)$, and $\gamma_{ij}(\phi)$ are estimable parameters for the ϕ^{th} sample quantile.[13]

It is important to note that a correlation matrix among the independent variables reveals little co-movement relationships.[14] Only six of seventy-eight correlation statistics were greater than 0.40: weaning weight and 365-day weight (0.47); 365-day weight and average daily gain (0.65); 365-day weight and birth-to-yearling weight EPD (0.41); rib-eye area and rib-eye area EPD (0.51); birth weight EPD and birth-to-yearling weight EPD (0.42); and birth-to-yearling weight EPD and milk EPD (0.42). One important implication of the low characteristic correlations is that producers seeking to improve bull traits that are expected to yield higher sale prices will not incidentally alter other traits that may adversely affect the bull's price. For example, producers wishing to increase the 365-day weight may be concerned that they will concurrently increase birth weights, which are expected to negatively affect sales prices. However, low correlation values between birth weight and all other bull characteristics indicate that this type of adverse co-movement is unlikely. In general, the low correlation among the majority of bull traits suggests that producers may be able to improve specific traits that are highly valued by a particular market segment.

Least Squares and Quantile Regression Results

Information on bulls offered for sale in MBT's 2009 auction was published in a catalog and

[13] The order of bulls sold in auctions is frequently included in bull auction studies. Because MBT uses several bull test measures to determine its auction order, multicollinearity exists between auction order and several explanatory variables in the hedonic price model. We therefore exclude sale order from the specification.

[14] To conserve space, we do not present the correlation matrix. It is available from the authors on request.

distributed in advance of the sale. Although eleven different bull breeds were offered at this auction, only Black Angus and Red Angus bulls were sold in sufficient quantities to allow for meaningful inferences. An indicator variable is included to account for the Red Angus bull type. All Angus bulls were sold on the same day and the MBT facility excluded bulls from the sale that ranked in the bottom 30% of all test measures. Specific seller and buyer information was not available.[15] If bulls offered for sale did not meet a consignor's reservation price or did not sell, then these bulls did not receive a sale price and associated data were also unavailable. In some cases, 67% and 75% fractional interests in a bull were auctioned. Because we are unable to observe whether fractional interest sales indicate a seller's semen retention rights or some other factor, we assign indicator variables for fractional interest bull sales.

Table 1 presents summary statistics and shows that the standardized median absolute deviation (MAD) of logged bull prices (0.39) is lower than its standard deviation (0.49). Because MAD is a robust measure of location and scale, the discrepancy is indicative of a long upper tail (Huber 1981).[16] The histogram and fitted kernel density of the logged price presented in Figure 2 provide visual evidence that the logged sale price distribution is skewed. These reveal distinguishing properties of a quality-differentiated product. Because each buyer seeks to purchase bulls for breeding purposes, the observed substantial price variation is likely indicative of quality differences. That is, buyers signal their valuation of bull quality by paying prices that are not equally proportionate to changes in observable bull traits. Because there is no clear *a priori* knowledge about how the interaction between quantifiable bull characteristics and quality considerations affects consumers' valuations, quantile regressions are used to semi-parametrically estimate this relationship.[17] Moreover, it is not that case that the highest-priced bulls possessed the highest levels of all SPM and EPD measures. Thus, when quality differences are expected to be substantial, estimated marginal quantile effects across the conditional price distribution may appropriately characterize bull quality levels.

For comparison purposes, equation (5) is estimated using ordinary least squares (OLS)

[15] Information about the sellers and buyers can allow explicit control for potential reputation effects, which could represent one of the factors contributing to quality valuation. Although these data contain anonymous seller and buyer identification, assigning indicator variables for 202 unique sellers and 260 unique buyers as controls for 480 total transactions would substantially limit the estimation power.

[16] The statistical measures for the explanatory variables indicate that leverage points (outliers; see Mahalanobis (1936)) in the covariate space are not of concern. While quantile regression is robust to outliers in the distribution of the response variable, it is not robust to extreme values (leverage points) of covariates.

[17] Quantile regression estimation is semi-parametric and inferences do not depend on distributional assumptions of the error structure (Hao and Naiman 2007). Highly skewed or other unconventional data can, therefore, be appropriately analyzed using the QR method. Although M and MM estimators (see Huber 1973; Yohai 1987) can also be used for robustly estimating non-normal data, the estimation results do not reveal heterogeneous marginal effects across multiple parts of the conditional dependent variable distribution.

regression and quantile regression (QR) with five quantiles.[18] Table 2 presents parameter estimates, R^2 and pseudo-R^2, and tests for joint significance of parameters. Covariates were centered around their means such that the intercept represents a "typical" bull.[19] OLS estimates of bull traits represent average marginal effects, which appear to be reasonable relative to previous research and *a priori* expectations (e.g., Vanek, Watts, and Brester 2008; Chvosta, Rucker, and Watts 2001). Birth weight, weaning weight, average daily gain, rib-eye area, residual feed intake, birth weight EPD, birth-to-yearling EPD, and the Red Angus bull breed indicator are all statistically significant at the 1% level.

Lower birth weight and birth weight EPD indicate that a bull's progeny are on average expected to have lower birth weights. This is a positive genetic attribute because lower birth weights reduce manual labor and animal mortality during the birth process (parturition). A one pound decrease in a bull's birth weight and a one unit decrease in birth weight EPD increases a conditional bull's price by approximately 0.9% and 6.8%, respectively. A one pound increase in weaning weight increases conditional bull prices by 0.1% because higher bull weaning weights are an indicator of higher weaning weights for its progeny. Bulls that have higher average daily gains are more highly valued. A one pound increase in average daily gain increases bull prices by 54.7%.[20] In addition, bulls that higher birth-to-yearling EPDs are expected to increase conditional prices by 1.1%.

Intramuscular fat, rib-eye area, and rib-eye area EPD are end-user carcass quality characteristics. A one-square inch increase in rib-eye area is on average expected to increase bull prices by 6.6%. However, neither intramuscular fat nor rib-eye EPD are statistically different from zero. RFI is a measure of feed efficiency after accounting for animal size. For two bulls with the same growth and carcass characteristics, the bull with a lower RFI is able to attain those characteristics while consuming less feed. Hence, a negative parameter estimate indicates that RFI is positively valued. OLS estimates indicate that a one pound per day gain in RFI increases the conditional value of a bull by an average of 5.7%. Finally, bulls offered at fractional interest were statistically different from others, and Red Angus bulls increased the price of an average bull by 15.5%. The latter suggests that unquantifiable considerations are controlled for by the bull breed indicator variable.

[18]Standard errors are estimated using a 200-resample Markov chain marginal bootstrap procedure (He and Hu 2002). Without bootstrapping, observations are assumed to be $i.i.d.$, which implies that covariates do not cause scale shifts of the dependent variable's conditional distribution (Hao and Naiman 2007). Bootstrapping provides appropriate standard error estimates.

[19]In an OLS regression, a "typical" bull represents the price of an average bull. In a quantile regression, a "typical" bull describes the price of a bull at the ϕ^{th} sample quantile.

[20]A one point increase in average daily gain represents nearly a three standard deviation change. Attaining such increases is difficult and highly valued by bull buyers, as indicated by the estimated coefficient. A one standard deviation increase in average daily gain yields a 14.78% average change in a bull's price.

To evaluate potential non-linear marginal effects of explanatory variables, we modeled logged bull sale prices as a function of squared explanatory variables. However, including the squared variables lead to substantial multicollinearity with the associated level terms, indicated by high variance inflation factors (VIFs).[21] Consequently, the adverse effects of multicollinearity become evident, including substantial changes in parameter estimates and reductions in t statistic values relative to an OLS specification without squared terms. These results provide statistical evidence that non-linearities cannot be explicitly controlled for and little new information can be gained by including squared terms in the exogenous variable space.

Different trait valuations across perceived bull quality levels can be observed in the conditional quantile regression estimates, presented in both tabular (table 2) and graphical forms (figures 3 and 4). Because a tabular presentation of conditional quantile estimates across the entire spectrum can be excessive, we only show estimates for the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} conditional quantiles. These estimates provide inferences about marginal effects of bull characteristics in the tails and in the central parts of the conditional sale price distribution. Estimated coefficients in each quantile are presented in table 2 and represent, *ceteris paribus*, the expected location shift in that particular quantile of the conditional bull price distribution.

An interpretation of results in table 2 is best described using an example. Focusing on the birth weight characteristic, we find that a one-pound reduction in a bull's birth weight (improvement) is expected to increase bull prices by 0.6% at the 50^{th} quantile (median). This conditional-median coefficient estimate is lower than the OLS conditional-mean location shift estimate of 0.9%. This suggests that the skewed conditional price distribution may inflate expected location shifts implied by the OLS model. Additionally, parameter estimates across conditional-quantiles disclose scale shifts in the conditional response distribution caused by changes in a covariate, and reveal the effect of the quality interaction term, $g(Birth\ Weight_i, q_i)$. For birth weight, a one-pound decrease is not expected to change the selling price of bulls perceived to be of lower quality (those at the 10^{th} quantile of the conditional price distribution) and a 1.5% increase for high quality bulls (those at the 90^{th} quantile of the conditional price distribution). This indicates a complementary effect of birth weight and quality on conditional bull prices. Buyers in the lower tail of the conditional price distribution (those purchasing lower-quality bulls as revealed by actual winning prices) are willing to pay less for improvements in birth weight than buyers of higher-quality bulls. In general, the results indicate that birth weight is valued heterogeneously across the bull quality distribution.

Location and scale effects obtained from quantile regression models can be summarized with

[21] These results are omitted to conserve space; however, many VIFs were 100 to 160 times higher when squared terms are included in the specification. Variance inflation factors indicate the amount by which an independent variable inflates the model's variance relative to a hypothetical situation when no multicollinearity exists. For a detailed discussion of variance inflation factors, see Belsley, Kuh, and Welsh (1980).

process plots. These plots also help reveal the shape of quality impacts. Figure 3 illustrates regression quantiles for the birth weight covariate.[22] The conditional quantile effect of birth weight is represented by the solid line and a 95% bootstrapped confidence interval is represented by the shaded region around the solid line. The solid line describes the change in conditional-price quantiles resulting from a one-pound change in birth weight, holding all other covariates fixed. Birth weight has a statistically significant effect on conditional bull prices in all but the lowest regression quantiles, because the confidence interval region does not include zero (shown by the thick black reference line). Moreover, the marginal effect of birth weight steadily increases across the conditional logged price distribution, suggesting that buyers of higher quality bulls are willing to pay a higher premium for improvements in birth weight. These estimated birth weight conditional quantile effects indicate that quality considerations non-linearly affect both location and scale of the conditional response distribution. For comparison, a pure location shift is depicted by the superimposed horizontal dashed and dotted lines, which represent the OLS estimate and a 95% confidence interval of birth weight. The zero-sloped conditional-mean estimate line implies that buyers place the same value on birth weight across all bull quality levels.

Process plots for other bull characteristics are shown in figures 4a and 4b. Because covariates are centered around their mean, the intercept plot represents the estimated conditional-quantile function of the sale price distribution for a "typical" bull. The slope of the weaning weight covariate is relatively constant for bulls below the 60^{th} conditional quantile, but increases rapidly for higher-quality bulls. This indicates that quality considerations become important only for higher-quality bulls. A one-unit decrease in weaning weight is valued at 0.1% for bulls at the 25^{th} and 50^{th} conditional quantile and at 0.2% for bulls at the 75^{th} and 90^{th} conditional quantiles. Conditional quantile estimates of the 365-day weight and bull age provide important inferences that were not shown in the OLS parameter estimates. Although OLS results suggest that 365-day weight is not statistically significant in affecting conditional bull sale prices, quantile regressions indicate that these OLS inferences are not robust across the entire conditional price distribution. Marginal changes in 365-day weight affect prices of only lower-quality bulls, increasing the bull's value by 0.1% for one-pound increases in yearling weight.

Average daily gain is statistically significant at all conditional quantiles with non-linear quality consideration effects for bulls above the 75^{th} regression quantile. For lower-quality bulls, a one-pound increase in average daily gain causes bull prices to increase between 43.7% and 50.1%, but marginal increases of this trait are valued at 80.7% for high-quality bulls. Carcass quality characteristics indicate that buyers are willing to pay a premium for improvements in higher-quality bulls relative to lower-quality bulls. Estimated parameter estimates for intramuscular fat are statistically significant only for bulls above the 75^{th} conditional quantiles (8.8% for

[22] To better represent covariate effects on the shape of the conditional price distribution, process plots represent 33 equally spaced conditional-quantile estimates, ranging from the 10^{th} to the 90^{th} quantile.

one-percent improvements for bulls at the 90th quantile); this result is not suggested by OLS estimates, which imply that buyers do not place statistically significant value on intramuscular fat traits. Marginal increases in rib-eye area are also non-linearly affected by quality for higher-quality animals.

Finally, valuation of improvements in residual feed intake are relatively constant across the entire conditional logged price distribution, implying that the OLS estimate provides a relatively accurate representation of bull buyers' valuation across all quality levels. Changes in birth weight EPD lead to relatively similar, statistically significant changes in bull prices except for highest quality bulls, for which changes in EPD values do not affect buyers' valuations. Birth-to-yearling EPD is statistically significant for all conditional quantiles, but changes in the EPD are more highly valued for bulls that are perceived to be of higher quality. Conditional quantile coefficients for the rib-eye EPD, the milk EPD, and bulls sold at fractional interests are statistically insignificant across all regression quantiles. Finally, Red Angus bulls are values relatively similarly from the 25th to the 75th conditional quantiles, and valued 14% more at the 90th regression quantile.

Conclusions and Implications

This study uses quantile regression estimates of a hedonic model to evaluate marginal implicit values of a quality-differentiated agricultural product. Because consumers may value product characteristics differently depending on a set of non-quantifiable quality considerations, traditional parametric estimation methods may be unable to reveal quality effects on consumer valuations. Furthermore, parameterizing quality may yield an inaccurate representation of quality considerations by consumer behavior. Quantile regressions provide a semi-parametric framework that allows data to flexibly identify and estimate quality effects across a conditional price distribution.

We use a quantile regression framework to investigate how quality considerations affect consumers' marginal valuation of bull growth and carcass traits. Auctions that facilitate sales for a heterogeneous set of bull producers offer buyers an opportunity to evaluate and bid on bulls of varying quality. Although not explicitly observable, consumers' quality considerations may be affected by seller's reputation, knowledge of bull trait heritability, and visual evaluations. Buyers seeking to purchase lower-quality bulls may value specific bull traits differently than buyers interested in higher-quality animals. Although some quality attributes may be measurable, non-quantifiable traits are revealed by prices bidders actually pay for bulls. A hedonic model of bull sale prices obtained from the 2009 Midland Bull Test auction is estimated. Regression quantiles show that substantial differences exist among buyer preferences and valuations of bull characteristics. For most bull growth and carcass traits, there is a complementary non-linear relationship between traits and quality.

Quantile regression analyses can improve marketing and pricing strategies for agricultural producers who target different market segments. Understanding how consumers value specific characteristics across a spectrum of quality segments is important for making effective production decisions that are conditional on a product's expected quality. That is, because producers typically know the expected quality of their products – reputation effects and other quality signals are usually common knowledge (although often unquantifiable) – they can predict with relative certainty the quality segment to which the product is marketed. More precise knowledge of consumers' product characteristic valuations within a particular quality market segment can help producers tailor products that maximize returns. For example, improving products by focusing on more highly-valued characteristics within a market segment may help reduce production costs. Cost efficiency may be most important to producers of lower-quality products, because simply imitating higher-quality producers and improving all traits may substantially reduce profits.

Quality-differentiated products exist in many agricultural markets for which data do not explicitly reveal quality considerations; potential examples include alfalfa hay, fruit, flour, wine, farm land and source-verified products. In each case, only limited information exists that clearly distinguishes values of these characteristics across the quality spectrum (e.g., historic land productivity, overall quality of products from the originating region). However, additional unobserved quality differences may explain price variations in these markets. For quality-differentiated products, quantile regression methods may provide more informative analyses of consumer valuations than traditional parametric estimation techniques.

References

Arias, O., K. F. Hallock, and W. Sosa-Escudero. 2001. Individual heterogeneity in the returns to schooling: instrumental variables quantile regression using twins data. *Empirical Economics* 26: 7–40.

Barrodale, I., and F. Roberts. 1973. An improved algorithm for discrete l_1 linear approximation. *SIAM Journal of Numerical Analysis* 10: 839–848.

Belsley, D., E. Kuh, and R. Welsh. 1980. *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity*: New York: John Wiley and Sons.

Buchinsky, M. 1997. The dynamics of changes in the female wage distribution in the USA: a quantile regression approach. *Journal of Applied Econometrics* 13: 1–30.

Chamberlain, G. 1994. *"Quantile regression, censoring and the structure of wages"*, 171–209. in *Advanced in Econometrics*: New York: Elsevier, C. Sims, ed.

Chen, C. 2007. A finite smoothing algorithm for quantile regression. *Journal of Computational and Graphical Statistics* 16: 136–164.

Chen, C., and Y. Wei. 2005. Computational issues for quantile regression. *Sankhya: The Indian Journal of Statistics* 67: 399–417.

Chvosta, J., R. Rucker, and M. Watts. 2001. Transaction cost and cattle marketing: the information content of seller-provided presale data at bull auctions. *American Journal of Agricultural Economics* 83: 286–301.

Dhuyvetter, K., T. Schroeder, D. Simms, R. Bolze, and J. Geske. 1996. Determinants of purebred beef bull price differentials. *Journal of Agricultural and Resource Economics* 21: 396–410.

Eide, E., and M. Showalter. 1998. The effect of school quality on student performance: a quantile regression approach. *Economic Letters* 58: 345–350.

Greene, W., editor 2003. *Econometric Analysis*, 750–751, 5: Pearson Education, Inc.

Hao, L., and D. Naiman., editors 2007. *Quantile Regression*. Quantitative Applications in the Social Sciences: Sage, California.

Hausman, J., and D. Wise. 1977. Social experimentation, truncated distributions, and efficient estimation. *Econometrica* 45: 319–339.

He, X., and F. Hu. 2002. Markov chain marginal bootstrap. *Journal of the American Statistical Association* 97: 783–795.

Heckman, J. 1979. Sample selection bias as a specification error. *Econometrica* 47: 153–161.

Huber, P. 1973. Robust Regression: Asymptotics, Conjectures and Monte Carlo. *Annals of Statistics* 1: 799–821.

Huber, P. 1981. *Robust Statistics*: New York: John Wiley & Sons.

Karmarkar, N. 1984. A new polynomial-time algorithm for linear programming. *Combinatorica* 4: 373–395.

Koch, R., L. Swiger, D. Chambers, and K. Gregory. 1963. Efficiency of feed use in beef cattle. *Journal of Animal Science* 22: 486–494.

Koenker, R., editor 2005. *Quantile Regression*: Cambridge University Press, New York, NY.

Koenker, R., and G. Bassett. 1978. Regression Quantiles. *Econometrica* 46: 33–49.

Koenker, R., and V. d'Orey. 1994. Remark AS R92: A remark on algorithm AS 229: Computing dual regression quantiles and regression rank scores. *Applied Statistics* 43: 410–414.

Koenker, R., and A. Machado. 1999. Goodness of fit and related processes for quantile regression. *Journal of American Statistical Association* 94: 1296–1310.

Ladd, G., and M. Martin. 1976. Priced and demands for input characteristics. *American Journal of Agricultural Economics* 58: 21–30.

Levin, J. 2001. For whom the reductions count: a quantile regression analysis of class size on scholastic achievement. *Empirical Economics* 26: 221–246.

Mahalanobis, P. C. 1936. On the generalised distance in statistics. In *Proceedings National Institute of Science, India*, volume 2, 49–55.

McDonald, T., G. Brester, A. Bekkerman, and J. Patterson. 2010. Case Study: Searching for the Ultimate Cow: The Economic Value of Residual Feed. *Forthcoming* in Professional Animal Scientist.

Portnoy, S., and R. Koenker. 1997. The Guassian hare and the Laplacian tortoise: Computation of squared-error vs absolute-error estimators. *Statistical Science* 12: 279–300.

Rosen, S. 1974. Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition. *The Journal of Political Economy* 82: 34–55.

Vanek, J., M. Watts, and G. Brester. 2008. Carcass quality and genetic selection in the beef industry. *Journal of Agricultural and Resource Economics* 33: 349–363.

Yohai, V. 1987. High Breakdown Point and High Efficiency Robust Estimates for Regression. *Annals of Statistics* 15: 642–656.

Table 1: Summary Statistics

Variable (units)	Obs.	Mean	Median	Std. Dev.	MAD[a]	Max.	Min.
Logged bull sale price	480	7.80	7.82	0.49	0.39	9.85	6.91
Birth weight (lbs.)	480	82.08	82	8.06	7.41	111	50
Weaning weight (lbs.)	480	723.50	722	72.56	69.68	969	500
365-day weight (lbs.)	480	1267.20	1260.5	86.94	81.54	1639	1074
Bull age (days)	480	433.20	435	23.26	23.72	482	359
Average daily gain (lbs./day)	480	3.41	3.38	0.37	0.38	4.71	2.31
Intramuscular fat (percent)	480	3.58	3.47	0.72	0.67	6.4	1.99
Rib-eye area (squared inches)	480	12.08	12.20	1.21	1.33	15.3	8.9
Feed to gain ratio	480	6.47	6.36	1.04	0.95	12.25	4.06
Residual feed intake (lbs./day)	480	-0.02	0.16	1.95	1.67	7.12	-6.48
Birth weight EPD (lbs.)	480	1.67	1.80	1.68	1.48	6.2	-6.7
Birth-to-yearling gain EPD (lbs.)	480	84.20	86.80	15.40	14.60	119.7	34.4
Rib-eye area EPD (squared in.)	480	0.14	0.14	0.16	0.16	0.75	-0.3
Milk EPD (lbs.)	480	22.74	23	4.90	4.45	39	5
67% fractional sales	480	0.34	0	0.47	0	1	0
75% fractional sales	480	0.06	0	0.24	0	1	0
Bull breed indicator	480	0.21	0	0.41	0	1	0

[a] MAD is standardized median absolute deviation.

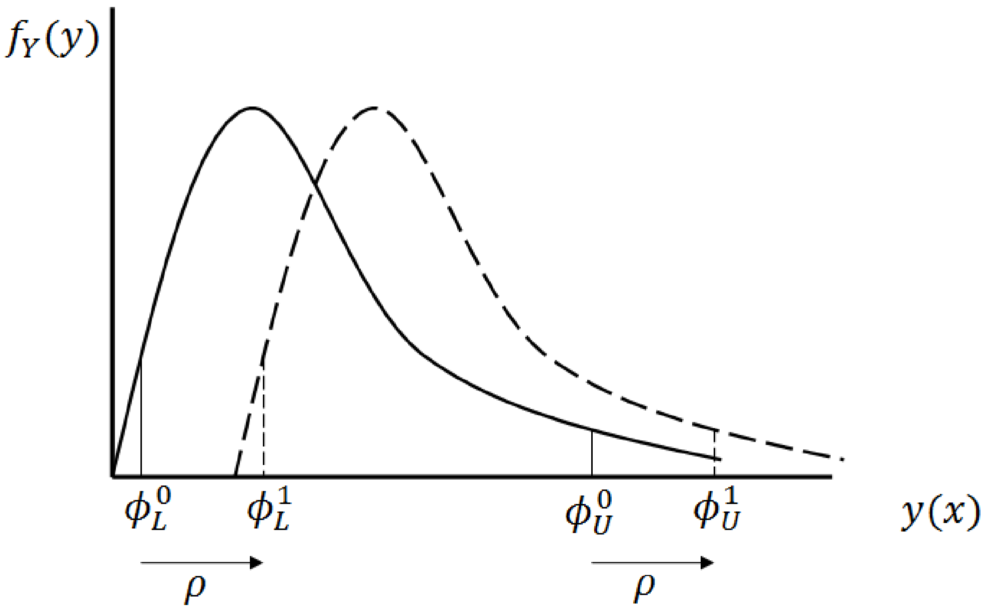
Table 2: Results for OLS and QR Estimation of Bull Sales Prices

		Quantile Regression: Estimated Conditional Quantiles				
Variable	**OLS**	**10%**	**25%**	**50%**	**75%**	**90%**
Intercept	3.617***	7.388***	7.527***	7.713***	7.915***	8.131***
	(7.19)	(232.41)	(332.82)	(263.54)	(235.95)	(171.82)
Birth weight	-0.009***	-0.002	-0.006**	-0.006*	-0.010***	-0.015***
	(-3.55)	(-0.70)	(-2.55)	(-1.99)	(-2.92)	(-3.15)
Weaning weight	0.001***	0.001	0.001***	0.001***	0.002***	0.002***
	(4.79)	(1.14)	(3.66)	(3.29)	(2.88)	(3.14)
365-day weight	6E-4	0.001*	0.001	0.001	0.001	-0.001
	(1.09)	(1.97)	(1.86)	(1.53)	(1.15)	-(0.70)
Bull age	-3E-4	-3E-4	4E-4	1E-4	-0.001	-0.004**
	(-1.21)	(-0.28)	(0.54)	(-0.07)	(-0.75)	(-2.03)
Average daily gain	0.547***	0.501***	0.479***	0.470***	0.437***	0.807***
	(6.88)	(6.16)	(6.94)	(4.37)	(3.08)	(4.90)
Intramuscular fat	0.023	0.008	-0.035	0.007	0.029	0.088**
	(0.98)	(0.24)	(-1.35)	(0.23)	(0.89)	(2.38)
Rib-eye area	0.066***	0.046**	0.056***	0.060***	0.057**	0.108***
	(4.38)	(2.13)	(3.80)	(3.55)	(2.30)	(2.97)
Feed to gain ratio	0.021	0.023	0.020	-0.005	0.009	0.042
	(0.86)	(0.84)	(1.06)	(-0.14)	(0.23)	(0.72)
Residual feed intake	-0.057***	-0.053***	-0.056***	-0.049***	-0.050***	-0.074***
	(-4.84)	(-3.50)	(-5.97)	(-3.04)	(-2.75)	(-3.45)
Birth weight EPD	-0.068***	-0.074***	-0.069***	-0.056***	-0.060***	-0.033
	(-5.51)	(-4.80)	(-4.76)	(-3.93)	(-3.33)	(-0.99)
Birth-to-yearling gain EPD	0.011***	0.007***	0.010***	0.008***	0.013***	0.016***
	(6.24)	(2.74)	(5.78)	(3.56)	(4.98)	(4.57)
Rib-eye area EPD	0.029	0.037	0.081	0.201	0.049	-0.468
	(0.24)	(0.26)	(0.61)	(1.54)	(0.26)	-(1.82)
Milk EPD	0.002	-0.003	-0.002	0.003	0.005	0.003
	(0.61)	(-0.53)	(-0.49)	(0.66)	(0.99)	(0.55)
67% fractional sale	0.065	0.071	0.058	0.053	0.087	0.105
	(1.93)	(1.52)	(1.71)	(1.23)	(1.73)	(1.31)
75% fractional sale	0.070	0.002	0.067	0.047	0.084	0.153
	(1.11)	(0.02)	(0.81)	(0.62)	(0.71)	(0.85)
Bull breed indicator	0.155***	0.043	0.120**	0.127	0.219**	0.360***
	(2.77)	(0.51)	(2.05)	(1.76)	(2.43)	(2.88)
R^2	0.517	0.615	0.527	0.516	0.461	0.463
Tests of joint significance						
F-test	33.08***	–	–	–	–	–
Wald test	–	255.193***	625.302***	362.696***	244.374***	270.549***
Likelihood ratio test	–	158.168***	349.934***	287.415***	220.523***	171.141***

Pseudo-R^2 is used for quantile regression.
*, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels; t-values are in parentheses.

Figure 1: Illustration of Location and Scale Shape Shifts of a Conditional Distribution

(a) Location Shift



(b) Location and Scale Shifts

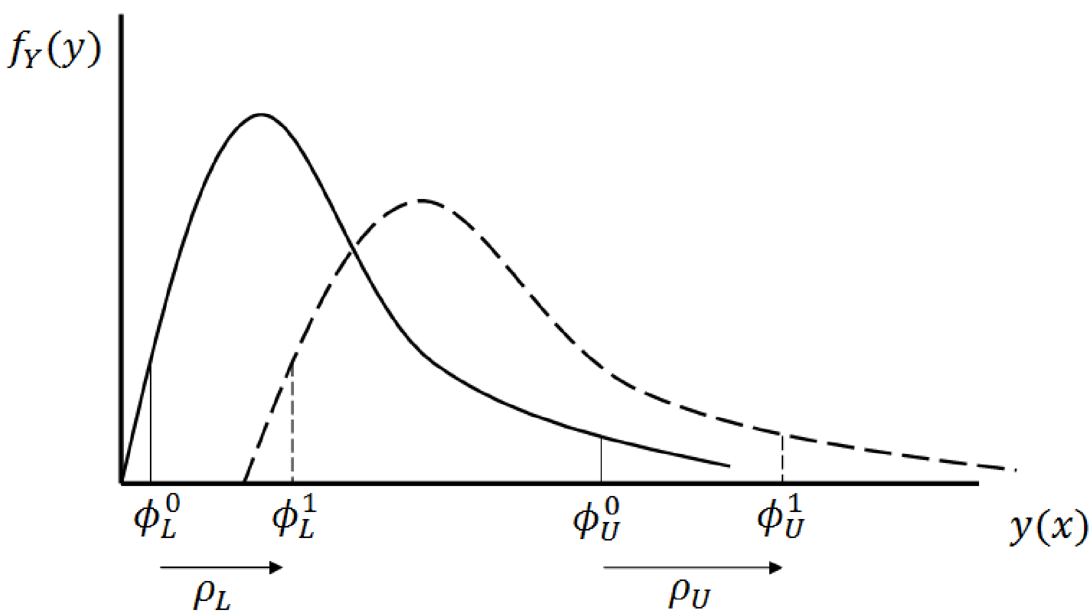


Figure 2: Histogram and Fitted Kernel Density of Logged Bull Sale Prices

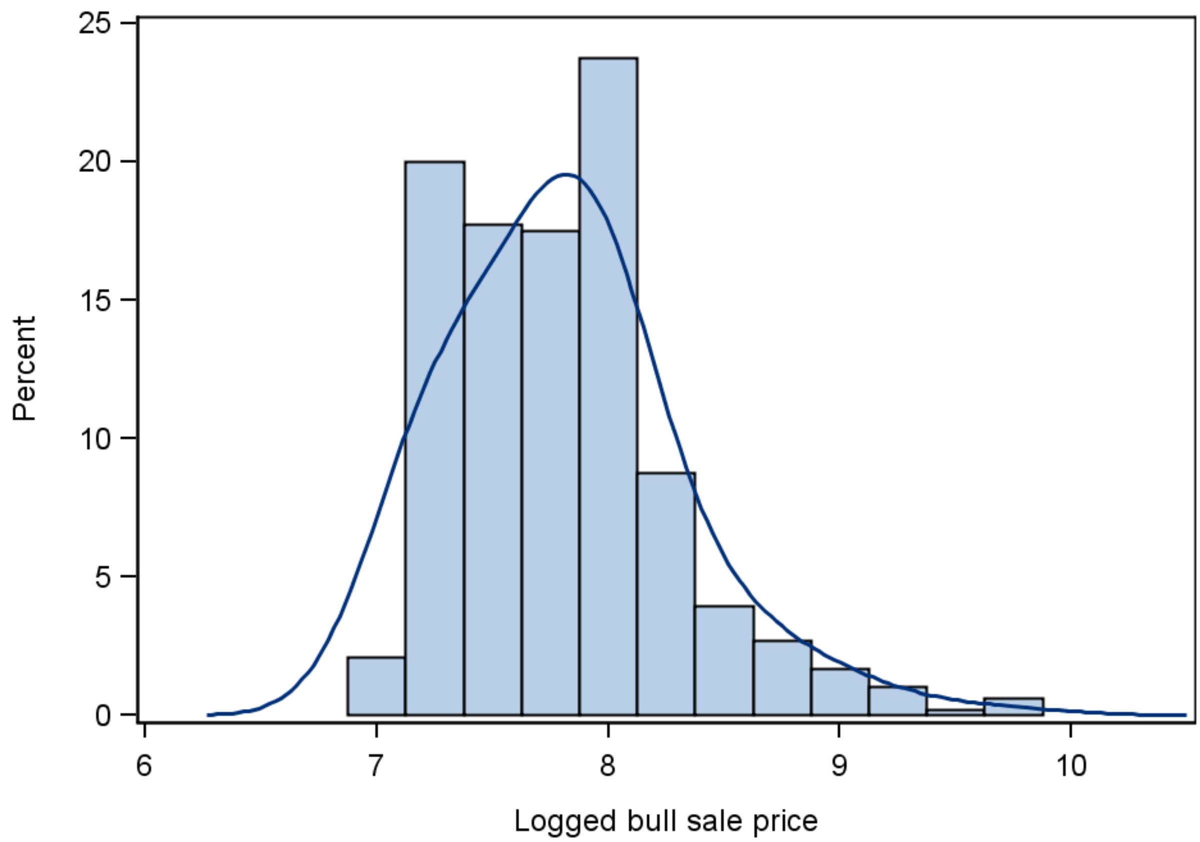


Figure 3: Birth Weight Estimated Parameters in QR and OLS Logged Bull Price Models

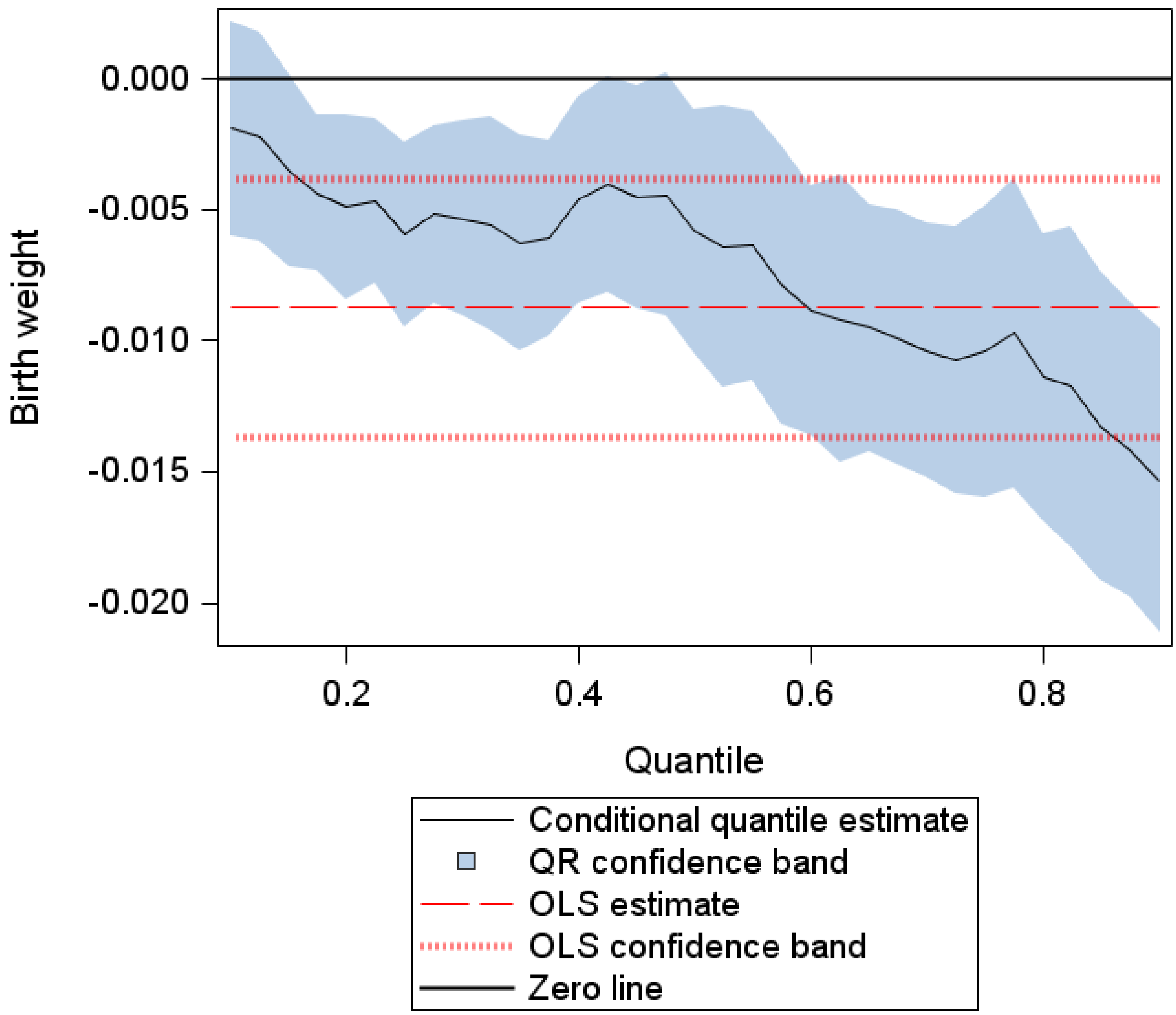


Figure 4a: Marginal Effects for Estimated QR and OLS Logged Bull Price Models

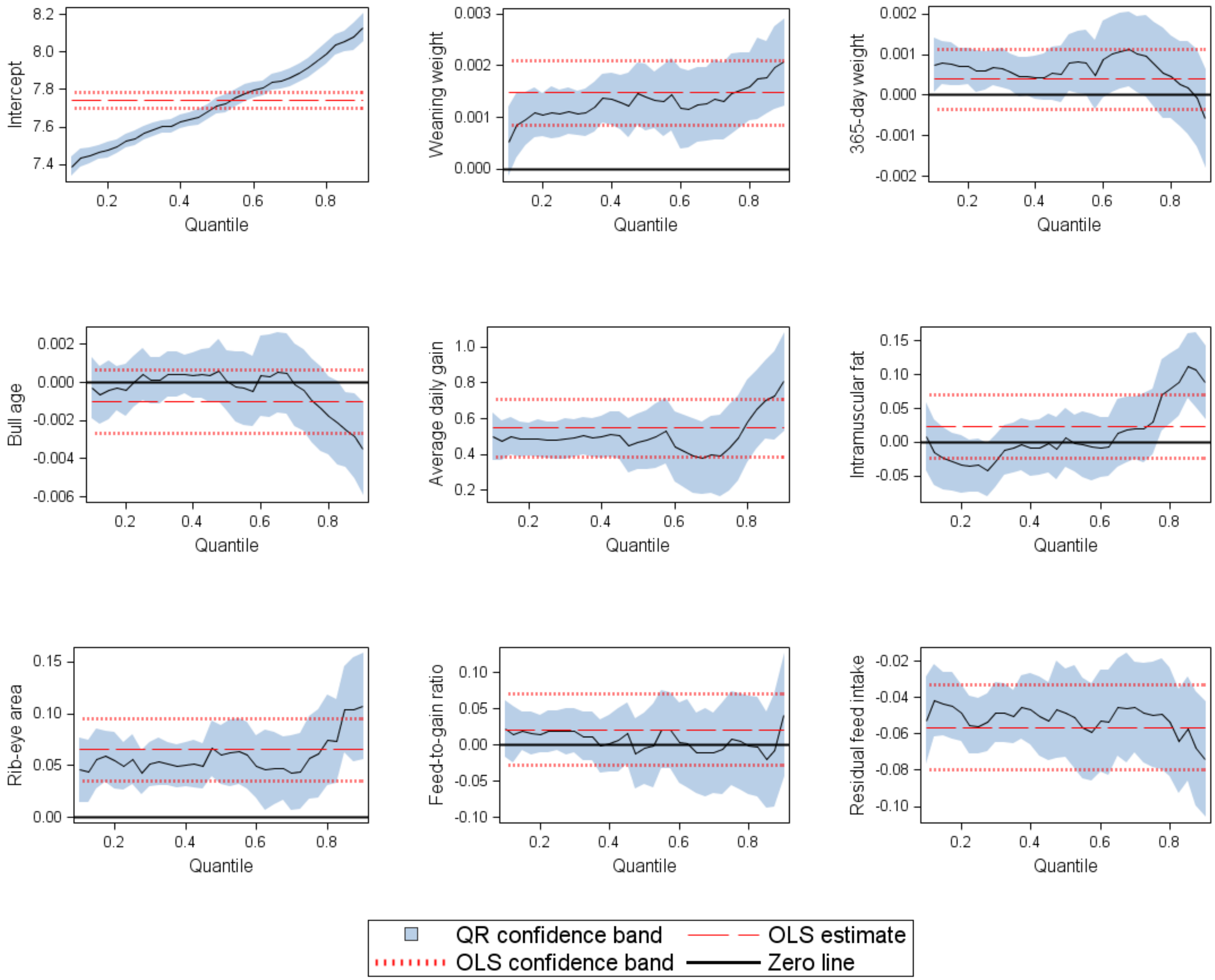


Figure 4b: Marginal Effects for Estimated QR and OLS Logged Bull Price Models