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Measuring and Explaining Skewness in Pricing Distributions Implied from Livestock Options

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**Measuring and Explaining Skewness in Pricing Distributions
Implied from Livestock Options**

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Practitioner's Abstract

We characterize volatility skews implied by options on futures for hogs and cattle. Both markets have shown a persistent leftward skew. The skew is much more pronounced in live cattle. As a practical matter, the volatility skew is evidence that the cost of using options to insure against large price declines has been considerably more expensive than the cost of using options to insure against similarly large price increases. Out-of-the-money put options are expensive in livestock markets and this is especially the case for out-of-the-money put options on cattle futures. We also examine the relationship between the volatility skew and the *ex ante* physical returns distribution. We do this by measuring volatility skews just before releases of USDA reports and determine whether they can be empirically linked to the direction of the large price changes that often result. Some responses in live/lean hog futures prices could be explained by characteristics of the pre-report volatility skew. However, there was little evidence linking the volatility skew to post-report responses in live cattle futures.

Introduction

The relationship between implied volatility and option moneyness has drawn considerable attention in the finance literature. In particular, the observation that implied volatilities derived from S&P 500 index options are inversely related to strike prices has been of interest. One explanation is that this volatility skew reflects characteristics of the *ex ante* physical returns distribution. According to this argument, the skew shows the potential for low-probability price collapses. Explanations tracing skews in S&P 500 index options to the potential for low frequency but very large price swings are provided by Aït-Sahalia, Wang, and Yared (2001) and Bondarenko (2003). A second explanation is that this volatility skew reflects risk preferences of market participants and so may depart, in important ways, from the true *ex ante* physical returns distribution. Previous work has shown that when there are market frictions, differences in expectations or risk preferences among market participants can induce persistent skews in implied pricing densities. Bollen and Whaley (2004) argue that at some point, the marginal cost of writing additional options at a given strike becomes an increasing function of the number of contracts written. In support of this argument, they present empirical evidence showing that S&P 500 index options smirks are the result of hedging pressures. Specifically, they show that demand for out-of-the-money puts, used to hedge against large stock market declines, pushes up the implied volatilities on low strike options. Similarly, Buraschi, and Jiltsov (2006) illustrate

how heterogeneous beliefs among market traders can better account for smirks in S&P 500 index options than alternative volatility models. Their modeling approach is motivated by the idea that agents with a more pessimistic expectation of future returns demand state-contingent insurance protection from agents with a more optimistic outlook, which results in greater demand and relatively higher prices for out-of-the-money puts than would be predicted by the Black-Scholes model. Han (2008) finds statistically significant relationships between slopes of IV functions from S&P 500 index options and several proxies of bullish or bearish sentiment. He shows that the strength of relationships between sentiment and slope are affected by impediments to arbitrage.

While the volatility skew in S&P 500 index options has received considerable attention, there has been comparatively little focus on the relationship between implied volatilities and strike prices in agricultural commodities. In this paper we examine this relationship using options on two important livestock futures markets, live/lean hogs and live cattle. Specifically, we characterize the relationship between implied volatility and strike price in two different ways. One way is through plots of implied volatility smiles derived from Black's (1976) model and the other is based on a model-free measure of the implied skew. Regardless of the approach, we show a persistent leftward volatility skew in both live/lean hogs and live cattle. The skew is most pronounced in live cattle.

The general arguments used to explain volatility skews in the S&P index options are probably germane to these livestock markets as well. An argument could be made that the structure of cash markets for cattle and hogs would lead to greater demand for protection against price declines through out-of-the-money puts and so hedging pressures may be a key factor behind the volatility skew in these markets. Unfortunately, our data are not well suited to addressing the causal role of hedging pressures. We do, however, attempt to shed light on the relationship between the volatility skew and the *ex ante* physical returns distribution. We characterize volatility skews just before releases of USDA reports and determine whether they can be empirically linked to the direction of the large price changes that often result. Specifically, we examine responses in live/lean hogs futures prices following the release of quarterly *Hogs and Pigs* reports and responses in live cattle futures prices following the release of monthly *Cattle on Feed* reports.

Evidence of the Volatility Skew in Live/Lean Hogs and Live Cattle

To characterize volatility skews we first grouped options into the following five moneyness categories:

1. *Deep-out-of-the-money puts*. Put options with strike prices that were 12.5 to 7.5 percent below the underlying futures price.

2. *Out-of-the-money puts*. Put options with strike prices that were 7.5 to 2.5 percent below the underlying futures price.
3. *Near-the-money puts and calls*. Put or call options with strike prices that were 2.5 percent below or above the underlying futures price.
4. *Out-of-the-money calls*. Call options with strike prices that were 2.5 to 7.5 percent above the underlying futures price.
5. *Deep-out-of-the-money calls*. Call options with strike prices that were 7.5 to 12.5 percent above the underlying futures price.

Our source of historic data on futures prices and options premiums is Bridge CRB. The data are daily and cover futures and options contracts on live hogs with maturities through 1996, lean hogs with maturities from 1997 through 2008, and live cattle with maturities through 2008. The potential for stale prices is one problem with daily data. The use of the moneyness ranges defined above is one attempt to address this problem as near-the-money and out-of-the-money options generally have more liquidity than options that are well into the money. Also, we exclude puts and calls that are very deep out of the money – more than 12.5 percent out of the money – because trading in these options is generally light. Category 3 above does include options that are in the money. All in-the-money options were pre-screened to ensure that the premiums reported were no less than the intrinsic value of the option when evaluated at the daily futures settlement price. Very few options violated this condition, especially in comparison to the size of the dataset. An implied volatility measure was computed for each option falling into any of the five moneyness categories above. These volatilities were then averaged by category. Implied volatilities were imputed from Black's (1976) options pricing formula. The risk-free rate used in the formula reflected yields on 6-month treasury bills.

A second method of characterizing the volatility skew is based on the model-free approach developed by Bakshi, Kapadia, and Madan (2003). These authors show that the discounted risk-neutral expectation of a general payoff function can be (1) expressed as function of out-of-the-money put and call options, and (2) replicated by a position in market-traded assets. The idea behind Bakshi, Kapadia, and Madan's (2003) approach is to apply risk neutral valuations to payoff functions that can then be used to uncover the volatility and higher moments of the pricing density. We use the Bakshi, Kapadia, and Madan (2003) approach to compute a measure of the implied skew. Again, we use data on out-of-the-money puts and calls with strike prices that are no more than 12.5 percent away from the money. We compute a measure of skewness for each trading day on which we observed premiums for at least two out-of-the-money put options and two out-of-the-money call options within this range. Additional details on implementing the approach are provided in the appendix.

Figure 1 summarizes implied volatilities across the different moneyness ranges for futures contracts on live/lean hogs. Over time, the highest implied volatilities have typically been associated with deep-out-of-the-money put options suggesting that put options have traded at a premium relative to call options that were similarly out of the money. This volatility skew was

less pronounced in the late 1980s and through the 1990s when both out-of-the money puts and calls traded at a premium relative to options that were near the money. However, the volatility skew became much more pronounced in contracts with 1999 maturities and later. In fact, implied volatilities shown in Figure 1 indicate that out-of-the-money calls often traded at a substantial discount relative to near-the-money options or out-of-the-money puts after 1999.

Figure 2 shows the same relationship for live cattle futures contracts. Beginning in about 1988, a leftward volatility skew has been a consistent feature of options on cattle futures. Volatilities from deep-out-of-the-money puts are much higher than those recovered from put options classified as out-of-the-money, which in turn are markedly higher from those recovered from options with strike prices that are near the money. Volatilities from out-of-the-money calls and deep-out-of-the-money calls have generally been near if not below implied volatilities from near-the-money options. While both live cattle and live/lean hog futures have shown a volatility skew over time, the skew in live cattle has been much more pronounced. This is evident from the data presented in Figures 1 and 2, especially when one considers that the vertical axis scale in Figure 2 (live cattle) is much wider than that in Figure 1 (live/lean hogs). Figure 3, shows the long-term average volatility smile in both markets and clarifies the more pronounced volatility skew in live cattle relative to live/lean hogs.

Implied skews (Figure 4) based on the Bakshi, Kapadia, and Madan (2003) approach are consistent with volatility metrics discussed above. Implied skews for both live cattle and live/lean hogs have been consistently negative over time. As shown in Figure 4, there is an inverse – almost mirror image – relationship between the degree of skewness and price levels (more negative skewness measures correspond to higher price levels). Regardless, the cattle market has almost invariably shown a more pronounced skew than the hog market.

Is there Information in the Volatility Skew?

Previous research has investigated whether *Hogs and Pigs* and *Cattle on Feed*, two governmental reports containing information on market fundamentals, impact livestock markets. These studies have addressed whether the reports contain unanticipated information and if livestock futures markets react efficiently to unanticipated information that is contained in the reports (Koontz, Hudson and Purcell 1984; Colling and Irwin 1990; Schroeder, Blair, and Mintert 1990; Schaefer, Myers, and Koontz 1990; Grunewald, McNulty and Biere 1993; Carter and Galopin 1993; Mann and Downen 1996; Mann and Downen 1997; Isengildina, Irwin, and Good 2006). Two general conclusions of these studies are that statistically large futures price movements are often observed following the report release dates and futures markets appear to be efficient at impounding the new information. The market efficiency conclusion stems from the fact that even though futures prices tend to react to report releases, a systematic strategy set up prior to the report would result in trading profits that are not statistically different from zero. In other words,

there is no systematic directional movement in futures prices following report releases. Finally, taken together, these studies suggest that while live cattle futures do respond to *Cattle on Feed* report releases and live/lean hog futures prices respond to *Hogs and Pigs* report releases, live/lean hogs futures are comparatively much more responsive.

The fact that livestock futures tend to respond to report information provides a natural setting within which to examine the linkage between volatility skews and the *ex ante* physical price distribution. The key research question is whether the volatility skew can be empirically linked to the direction of post-report price changes. Following Doran, Peterson, and Tarrant (2007), we examine this question using binomial response models in which the probability of a post-report price change above or below a given threshold is modeled as a function of the magnitude of the pre-report volatility skew.

To estimate the models, we first gathered information on report release dates and computed corresponding post-report changes in the four nearest to maturity live/lean hogs and live cattle futures prices. Table 1 provides the correspondence between report release months and the four nearest livestock futures contract months (deferral classes) used in our analysis. Options on futures expire some days before futures contracts mature and so in many instances, options on the first deferred futures contract had expired when USDA reports were released. As shown in Table 1, this is always the case for *Cattle on Feed* reports released in even months. Both *Hogs and Pigs* and *Cattle on Feed* are released at 3:00 pm, after the market close. For this reason, post report price changes were computed as logged differences between the settlement price on the first non-limit trading day after report release and the settlement price on the release day. Post-report limit moves were relatively infrequent among the four nearest-to-maturity live cattle contracts but occurred regularly (nearly a third of the time) among the four nearest-to-maturity live/lean hog contracts.

Our sample of quarterly *Hogs and Pigs* releases included all reports from September 20, 1984 through March 28, 2008. Our sample of *Cattle on Feed* releases included all reports from January 24, 1986 through May 16, 2008. In both cattle and hogs, there was at least one instance of a limit move on the report release day itself. Since, it is impossible to impute meaningful pre-release values for implied volatilities or the implied skew, any observation with a limit move on the release date was excluded from the analysis. Finally, due to concerns about data quality, any observation among the first contract deferral class that involved fewer than 10 trading days between the release of a report and the expiration date of options contracts was excluded.

Empirical distributions of the post-report price changes across all four contract deferral classes are presented in Figure 5. We use these distributions to define relatively large price response thresholds. While somewhat arbitrary, we define large price declines as those below the 10th, 15th, and 20th percentiles and large price increases as those above the 80th, 85th, and 90th percentiles. Table 2 shows the total number of report releases included in our final samples by

contract deferral class along with the number of price responses exceeding these percentile thresholds.

Based on earlier specifications used by Doran, Peterson, and Tarrant (2007), we estimate the following three binomial response models. Each model involves a slightly different characterization of the volatility skew. In models 1 and 2, the degree of skewness is defined in terms of the slope of the volatility smile (see Figure 3) whereas in model 3 the degree of skewness is measured by Bakshi, Kapadia, and Madan's (2003) implied skew.

$$(1) \text{ Prob}(\Delta P \leq (\geq) X) = F \left(\alpha_0 + \alpha_1 V_n + \alpha_2 \frac{V_d^p - V_o^p}{V_n} + \alpha_3 \frac{V_o^p - V_n}{V_n} + \alpha_4 \frac{V_o^c - V_n}{V_n} + \alpha_5 \frac{V_d^c - V_o^c}{V_n} + \alpha_6 \text{Days} \right)$$

$$(2) \text{ Prob}(\Delta P \leq (\geq) X) = F \left(\beta_0 + \beta_1 V_n + \beta_2 \frac{V_d^p - V_n}{V_n} + \beta_3 \frac{V_o^c - V_n}{V_n} + \beta_4 \text{Days} \right)$$

$$(3) \text{ Prob}(\Delta P \leq (\geq) X) = F(\gamma_0 + \gamma_1 V_n + \gamma_2 \text{Skew} + \gamma_3 \text{Days})$$

In models 1 – 3, ΔP is the change in futures price from the day of the report to the first non-limit trading day following the report, X is a threshold value, and $F()$ refers to the probability distribution. V represents implied volatility, $Skew$ is the implied skew, and $Days$ is the number of trading days from the release of the report until options expiration. Subscripts n , o , and d correspond to near-the-money, out-of-the-money, and deep out-of-the-money, respectively. Superscripts p and c indicate that implied volatility is taken from puts and calls, respectively.

Descriptive statistics for explanatory variables in these models are presented by contract deferral class in Table 3. Our hypotheses are that α_1 , β_1 , and γ_1 will be positive, as higher levels of volatility should imply a higher likelihood of a large price change in either direction. In cases where we are modeling the probability of a price decline (the probability that $\Delta P \leq X$), we would anticipate positive values for α_2 , α_3 , and β_2 and non-positive values for α_4 , α_5 , and β_3 . The logic here is that high implied volatilities on out-of-the-money put (call) options relative to those near the money are consistent with a leftward (rightward) skew in the *ex ante* price distribution and so should correspond to the likelihood of a large price decrease (increase). Of course, the opposite signs on these coefficients are to be expected when modeling the probability of a price increase. In model 1, the magnitude and statistical significance of α_2 relative to α_3 and of α_5 relative to α_4 are expected to depend on the value of X . For example, when modeling a price decline one would expect smaller (more negative) values of X to be associated with larger (more positive) estimates of α_2 relative to α_3 . This is because high volatilities from deep-out-of-the-money put options would more closely correspond to large price declines than would high volatilities from puts that are nearer-the-money. We expect γ_2 to be negative when modeling the probability of a price decline. This inverse relationship is expected because the lower (more negative) the implied skew the greater the mass in the left tail of *ex ante* price distribution. By the same

reasoning, we expect γ_2 to be positive when modeling the probability of a price increase. Finally, we have no specific hypotheses regarding the signs of α_6 , β_4 , and γ_3 . However, the number of days remaining until option expiration is a potentially important control variable as both *Hogs and Pigs* and *Cattle on Feed* reports contain information that will be of importance to prices in the short term (i.e., statistics on livestock numbers marketed) as well as information that is of importance to prices several months into the future (i.e., statistics on breeding inventories or numbers of animals placed on feed). Table 3 shows that there is a relatively wide range in the number of days to options expiration within any of the four deferral classes.

The binomial response models were estimated using SAS's *genmod* procedure. Because there are relatively low response frequencies, especially when modeling price changes below the 10th percentile or above the 90th percentile, both probit and logit specifications are used. As shown in Table 2, there was only one price change that exceeded the 90th percentile among nearest to maturity contracts in the live/lean hogs datasets and so binomial response models were not estimated for this scenario.

Tables 4 and 5 present selected model estimates for price responses in live/lean hog futures following the release of *Hogs and Pigs* reports. Table 4 shows results for price declines while Table 5 shows results for price increases. Across the different deferral classes and price change thresholds, most coefficients on implied volatility measurements are insignificant (models 1 and 2). However, where they are significant, the signs generally conform to the hypothesized values outlined above. Also, the results are relatively robust to different characterizations of the slope of the volatility smile that are inherent in models 1 and 2 and to the price change thresholds, especially for models involving price increases in the 3rd contract deferral class. No estimates on implied volatility variables are significant in the results for the price changes below the 10th percentile (not shown). The findings presented in Tables 4 and 5 are also robust in the sense that there are no noteworthy differences in signs or significance levels obtained from logit model estimates (not reported). The implied skew (model 3) is significant in only three scenarios reported in Tables 4 and 5 and in each case its sign is opposite that of the hypothesized value. Again, logit model estimates (not reported) are consistent with this finding.

Tables 6 and 7 present selected estimates for models of the probability of price responses in live cattle futures following the release of *Cattle on Feed* reports. As hypothesized, there is evidence that higher pre-report values of near-the-money implied volatility increase the likelihood of a price response, especially in the case of price increases (Table 7). However, most estimates on measurements designed to characterize the slope of the volatility smile are insignificant. The only exceptions are in the first deferred contract for price responses below the 20th percentile (Table 6) and price responses above the 80th percentile (Table 7). However, these are significant only in model 1. Results from model 2, with an alternative characterization of the smile, are insignificant. Coefficient estimates for the implied skew (model 3) are never

statistically significant. Logit model estimates (not reported) are consistent with these general findings. Estimates for price responses below the 10th or above the 90th percentiles (not shown) are also consistent with these general findings.

Discussion

In sum, futures price movements following report releases provide some evidence linking the volatility skew to the *ex ante* physical price distribution, however the evidence is not very strong. Some price responses in live/lean hog futures could be explained by characteristics of the pre-report volatility smile. However, it was important to explicitly model the shape of the smile, price responses were not explained by the overall degree of skewness. Price responses in live cattle could only be linked to pre-report levels of implied volatility and characteristics of pre-report volatility smiles or overall degree of skewness did not seem to matter.

That said, the persistent leftward volatility skew implied by livestock futures options is an interesting feature of these livestock markets. The bottom line is that the cost of using options to insure against large price declines has been considerably more expensive than the cost of using options to insure against similarly large price increases. Further research is needed to explain why out-of-the-money put options are so expensive. While the evidence we find linking the volatility skew to characteristics of the *ex ante* physical price distribution is tenuous, our results are by no means conclusive and there is a need to examine contexts other than the release of USDA reports. Additionally the volatility skew likely reflects the risk preferences of the market. The selling side of these livestock markets consists of cattle feedlots and hog operations that may depend heavily on options to manage price risk and this could explain, in part, the premium observed for deep out-of-the-money puts. Further research is needed to address the role of demand for options at different strikes and whether the persistent volatility skew can be explained by hedging pressures.

Appendix

Implementing the Bakshi, Kapadia, and Madan (2003) to Measure Implied Skewness

Bakshi, Kapadia and Madan (2003) define payoff functions in terms the underlying security price (s) in the current period (t) and at some terminal period ($T > t$) as $\text{Log}[s_T/s_t]^2$, $\text{Log}[s_T/s_t]^3$, and $\text{Log}[s_T/s_t]^4$, which they term volatility, cubic, and quartic contracts, respectively. They show that the discounted risk-neutral expectations of these payoff functions over the period $\tau = (T - t)$ can be valued in terms of out-of-the-money put and call options as follows:

$$(A1) \quad V[\tau] = \int_{s_t}^{\infty} \frac{2(1-\ln[k/s_t])}{k^2} C[\tau, k] dk + \int_0^{s_t} \frac{2(1+\ln[s_t/k])}{k^2} P[\tau, k] dk$$

for the volatility contract,

$$(A2) \quad W[\tau] = \int_{s_t}^{\infty} \frac{6\ln\left[\frac{k}{s_t}\right] - 3\left(\ln\left[\frac{k}{s_t}\right]\right)^2}{k^2} C[\tau, k] dk - \int_0^{s_t} \frac{6\ln[s_t/k] + 3(\ln[s_t/k])^2}{k^2} P[\tau, k] dk,$$

for the cubic contract, and

$$(A3) \quad X[\tau] = \int_{s_t}^{\infty} \frac{12\left(\ln\left[\frac{k}{s_t}\right]\right)^2 - 4\left(\ln\left[\frac{k}{s_t}\right]\right)^3}{k^2} C[\tau, k] dk + \int_0^{s_t} \frac{12(\ln[s_t/k])^2 + 4(\ln[s_t/k])^3}{k^2} P[\tau, k] dk$$

for the quartic contract.

In equations A1-A3, k is the strike price and $P[\tau, k]$ and $C[\tau, k]$ are put and call option premiums, respectively. The measure of skewness is computed in terms of these values as follows:

$$(A4) \quad \text{Skew}[\tau] = \frac{e^{r\tau}W[\tau] - 3e^{r\tau}\mu[\tau]V[\tau] + 2\mu[\tau]^3}{(e^{r\tau}V[\tau] - \mu[\tau]^2)^{3/2}},$$

$$\text{where } \mu[\tau] = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V[\tau] - \frac{e^{r\tau}}{6}W[\tau] - \frac{e^{r\tau}}{24}X[\tau],$$

To implement this approach we approximate equations (A1) through (A3) by numerically integrating over observed options premia. Specifically, let $i = 1, 2, \dots, m$ index the strikes on out-of-the-money put options, $i = m+1, m+2, \dots, N$ index the strikes on out-of-the-money call options, and let the observations be ordered by strike price so that $k_1 < k_2 < \dots < k_N$. We weighted each observed premium by the second derivative of the payoff under each contract. This provides $V_i[\tau]$, $W_i[\tau]$, and $X_i[\tau]$ as follows:

$$(A5a) \quad V_i[\tau] = \frac{2\left(1+\ln\left[\frac{s_t}{k_i}\right]\right)}{k_i^2} P[\tau, k_i] \quad \text{for } i = 0, \dots, m$$

$$(A5b) \quad V_i[\tau] = \frac{2\left(1-\ln\left[\frac{k_i}{s_t}\right]\right)}{k_i^2} C[\tau, k_i] \quad \text{for } i = m + 1, \dots, N$$

$$(A6a) \quad W_i[\tau] = -\frac{6\ln\left[\frac{s_t}{k_i}\right] + 3\left(\ln\left[\frac{s_t}{k_i}\right]\right)^2}{k_i^2} P[\tau, k_i] \quad \text{for } i = 1, \dots, m$$

$$(A6b) \quad W_i[\tau] = \frac{6\ln\left[\frac{k_i}{s_t}\right] - 3\left(\ln\left[\frac{k_i}{s_t}\right]\right)^2}{k_i^2} C[\tau, k_i] \quad \text{for } i = m + 1, \dots, N$$

$$(A7a) \quad X_i[\tau] = \frac{12\left(\ln\left[\frac{s_t}{k_i}\right]\right)^2 + 4\left(\ln\left[\frac{s_t}{k_i}\right]\right)^3}{k_i^2} P[\tau, k_i] \quad \text{for } i = 1, \dots, m$$

$$(A7b) \quad X_i[\tau] = \frac{12\left(\ln\left[\frac{k_i}{s_t}\right]\right)^2 - 4\left(\ln\left[\frac{k_i}{s_t}\right]\right)^3}{k_i^2} C[\tau, k_i] \quad \text{for } i = m + 1, \dots, N$$

Using the trapezoidal rule for numeric integration, the price of each contract is then computed as:

$$(A8) \quad V[\tau] = \frac{1}{2} \sum_{i=2}^N (V_i[\tau] + V_{i-1}[\tau])(k_i - k_{i-1})$$

$$(A9) \quad W[\tau] = \frac{1}{2} \sum_{i=2}^N (W_i[\tau] + W_{i-1}[\tau])(k_i - k_{i-1})$$

$$(A10) \quad X[\tau] = \frac{1}{2} \sum_{i=2}^N (X_i[\tau] + X_{i-1}[\tau])(k_i - k_{i-1})$$

Values resulting from A8-A10 are used to compute the implied skewness of the pricing density given in A4.

References

- Aït-Sahalia, Y., Y. Wang, and F. Yared. 2001. Do Option Markets Correctly Price the Probabilities of Movement of the Underlying Asset? *Journal of Econometrics* 102:67-110.
- Bakshi, G., N. Kapadia, and D. Madan. 2003. Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options. *The Review of Financial Studies* 16:101-145.
- Black, F. 1976. The Pricing of Commodity Contracts. *The Journal of Financial Economics* 3:167-79.
- Bollen, N.P. and R.E. Whaley. 2004. Does net Buying Pressure Affect the Shape of Implied Volatility Functions? *The Journal of Finance* 59:711-753.
- Bondarenko, O. 2003. Why are Put Options so Expensive? Working paper.
- Buraschi, A. and L. Jiltsov. 2006. Model Uncertainty and Option Markets with Heterogeneous Beliefs. *The Journal of Finance* 59:2841-2897.
- Carter, C.A., and C.A. Galopin. 1993. Informational Content of Government Hogs and Pigs Reports. *American Journal of Agricultural Economics* 75:711-18.
- Colling, P.R., and S.H. Irwin. 1990. The Reaction of Live Hog Futures Prices to USDA Hogs and Pigs Reports. *American Journal of Agricultural Economics* 72:84-94.
- Doran, J.S., D.R. Peterson, and B.C. Tarrant. 2007. Is There Information in the Volatility Skew? *Journal of Futures Markets* 27:921-959.
- Han, B. 2008. Investor Sentiment and Options Prices. *The Review of Financial Studies* 21:387-414.
- Grunewald, O., M.S. McNulty, and A.W. Biere. 1993. Live Cattle Futures Response to Cattle on Feed Reports. *American Journal of Agricultural Economics* 75:131-37.
- Isengildina, O., S.H. Irwin, and D.L. Good. 2006. The Value of USDA Situation and Outlook Information in Hog and Cattle Markets. *Journal of Agricultural and Resource Economics* 31:226-82.

- Koontz, S.R., M.A. Hudson, and W.D. Purcell. 1984. The Impacts of Hog and Pig Reports on Live Hog Futures Prices: An Event Study of Market Efficiency. Staff Paper SP-84-11, Department of Agricultural Economics, Virginia Polytechnic Institute and State University.
- Mann, T.L., and R.J. Downen. 1996. Are Hog and Pig Reports Informative? *The Journal of Futures Markets* 16:273-87.
- Mann, T.L., and R.J. Downen. 1997. The Impact of Proprietary-Public Information on Pork Futures. *The Journal of Futures Markets* 17:417-32.
- Schaefer, M.P., R.J. Myers, and S.R. Koontz. 2004. Rational Expectation and Market Efficiency in the U.S. Live Cattle Futures Market: The Role of Proprietary Information. *The Journal of Futures Markets* 24:429-451
- Schroeder, T., J. Blair, and J. Mintert. 1990. Abnormal Returns in Livestock Futures Prices Around USDA Inventory Report Releases. *North Central Journal of Agricultural Economics* 12:293-304.

Table 1. Contract Deferral Periods Used in Binomial Response Models

Report Month	1 st Deferred Contract ¹	2 nd Deferred Contract	3 rd Deferred Contract	4 th Deferred Contract
Live/Lean Hog Contracts Used for Quarterly <i>Hogs and Pigs</i> Reports				
March	April	June	July	August
June	July	August	October	December
September	October	December	February	April
December	February	April	June	July
Live Cattle Contracts Used for <i>Cattle on Feed</i> Reports				
January	February	April	June	August
February	(NA)	April	June	August
March	April	June	August	October
April	(NA)	June	August	October
May	June	August	October	December
June	(NA)	August	October	December
July	August	October	December	February
August	(NA)	October	December	February
September	October	December	February	April
October	(NA)	December	February	April
November	December	February	April	June
December	(NA)	February	April	June

¹ For inclusion in the study, we required at least 10 calendar days between the report release and expirations date of options contracts. In even months, options on the nearby live cattle futures contracts expire before *Cattle on Feed* report releases.

Table 2. Number of Sample Report Releases with Prices Changes Exceeding Given Percentiles by Contract Deferral Class¹

Contract Deferral Class	Live/Lean Hogs After <i>Hogs and Pigs</i> Releases				Live Cattle After <i>Cattle on Feed</i> Releases			
	1 st	2 nd	3 rd	4 th	1 st	2 nd	3 rd	4 th
Total	58	92	94	94	98	269	269	269
$\Delta P \leq 10^{\text{th}}$ percentile	4	10	10	9	15	33	27	16
$\Delta P \leq 15^{\text{th}}$ percentile	5	15	15	15	20	46	40	32
$\Delta P \leq 20^{\text{th}}$ percentile	6	22	21	19	28	62	52	40
$\Delta P \geq 80^{\text{th}}$ percentile	8	18	21	22	17	64	55	44
$\Delta P \geq 85^{\text{th}}$ percentile	6	15	14	16	14	51	36	32
$\Delta P \geq 90^{\text{th}}$ percentile	1	8	13	12	9	37	25	19

¹ ΔP is computed as the difference in the settlement price on the report day and the next non-limit settlement price. See Figure 5 for price raw changes corresponding to percentiles.

Table 3. Descriptive Statistics of Explanatory Variables in Binomial Response Models¹

Explanatory Variable	Live/Lean Hogs on <i>Hogs and Pigs</i> Release Days				Live Cattle on <i>Cattle on Feed</i> Release Days			
	Mean	Std Dev	Min	Max	Mean	Std Dev	Min	Max
	Deferral Class 1 (N = 58)				Deferral Class 1 (N = 98)			
V_n	0.251	0.080	0.156	0.657	0.149	0.042	0.091	0.323
$\frac{V_d^p - V_o^p}{V_n}$	0.139	0.143	-0.092	0.582	0.319	0.162	-0.008	0.656
$\frac{V_o^p - V_n}{V_n}$	0.086	0.075	-0.056	0.370	0.220	0.125	-0.093	0.609
$\frac{V_n}{V_o^c - V_n}$	0.013	0.089	-0.149	0.377	0.047	0.117	-0.313	0.374
$\frac{V_d^c - V_o^c}{V_n}$	0.076	0.104	-0.090	0.344	0.168	0.110	-0.060	0.602
$\frac{V_d^p - V_n}{V_n}$	0.223	0.206	-0.128	0.952	0.554	0.246	0.137	1.142
$\frac{V_d^c - V_n}{V_n}$	0.090	0.176	-0.144	0.611	0.266	0.192	-0.164	0.807
Skew	-0.298	0.314	-0.975	0.844	-0.729	0.430	-2.011	0.671
Days to option expiration	27.810	13.078	12.000	51.000	15.949	3.154	11.000	22.000
	Deferral Class 2 (N = 92)				Deferral Class 2 (N = 269)			
V_n	0.249	0.063	0.147	0.538	0.151	0.043	0.081	0.307
$\frac{V_d^p - V_o^p}{V_n}$	0.043	0.056	-0.199	0.159	0.155	0.091	-0.064	0.471
$\frac{V_o^p - V_n}{V_n}$	0.033	0.077	-0.525	0.215	0.110	0.072	-0.012	0.343
$\frac{V_n}{V_o^c - V_n}$	-0.017	0.059	-0.139	0.197	-0.056	0.063	-0.471	0.087
$\frac{V_d^c - V_o^c}{V_n}$	0.001	0.048	-0.133	0.156	0.036	0.098	-0.282	0.826
$\frac{V_d^p - V_n}{V_n}$	0.076	0.091	-0.383	0.289	0.265	0.148	-0.050	0.676
$\frac{V_d^c - V_n}{V_n}$	-0.016	0.077	-0.184	0.146	-0.011	0.095	-0.260	0.355
Skew	-0.297	0.255	-0.777	0.337	-0.894	0.449	-2.541	-0.014
Days to option expiration	70.446	23.380	22.000	112.000	58.963	16.115	29.000	84.000

¹ Subscripts *n*, *o*, and *d* correspond to near-the-money, out-of-the-money, and deep out-of-the-money, respectively. Superscripts *p* and *c* indicate that implied volatility (*V*) is taken from puts and calls, respectively.

Table 3. Descriptive Statistics of Explanatory Variables in Binomial Response Models (continued)¹

Explanatory Variable	Live/Lean Hogs on <i>Hogs and Pigs</i> Release Days				Live Cattle on <i>Cattle on Feed</i> Release Days			
	Mean	Std Dev	Min	Max	Mean	Std Dev	Min	Max
	Deferral Class 3 (N = 94)				Deferral Class 3 (N = 269)			
V_n	0.232	0.054	0.134	0.444	0.141	0.038	0.081	0.272
$\frac{V_d^p - V_o^p}{V_n}$	0.030	0.041	-0.090	0.135	0.113	0.070	-0.027	0.347
$\frac{V_o^p - V_n}{V_n}$	0.029	0.088	-0.679	0.211	0.096	0.071	-0.025	0.555
$\frac{V_o^c - V_n}{V_n}$	-0.041	0.087	-0.690	0.148	-0.072	0.064	-0.414	0.332
$\frac{V_d^c - V_o^c}{V_n}$	0.002	0.081	-0.085	0.654	0.005	0.059	-0.186	0.343
$\frac{V_d^p - V_n}{V_n}$	0.062	0.087	-0.567	0.221	0.209	0.125	-0.009	0.694
$\frac{V_d^c - V_n}{V_n}$	-0.042	0.075	-0.263	0.143	-0.060	0.075	-0.255	0.345
Skew	-0.437	0.255	-1.027	0.106	-1.097	0.562	-3.011	0.062
Days to option expiration	124.330	26.966	81.000	173.000	119.926	16.085	92.000	147.000
	Deferral Class 4 (N = 94)				Deferral Class 4 (N = 269)			
V_n	0.221	0.047	0.135	0.368	0.132	0.034	0.030	0.239
$\frac{V_d^p - V_o^p}{V_n}$	0.034	0.038	-0.068	0.120	0.094	0.205	-0.238	3.234
$\frac{V_o^p - V_n}{V_n}$	0.027	0.043	-0.062	0.144	0.094	0.076	-0.039	0.889
$\frac{V_o^c - V_n}{V_n}$	-0.037	0.091	-0.734	0.083	-0.080	0.085	-0.407	0.785
$\frac{V_d^c - V_o^c}{V_n}$	0.001	0.083	-0.124	0.612	0.002	0.180	-0.475	2.421
$\frac{V_d^p - V_n}{V_n}$	0.058	0.060	-0.090	0.195	0.188	0.265	-0.231	4.123
$\frac{V_d^c - V_n}{V_n}$	-0.040	0.075	-0.265	0.103	-0.071	0.238	-0.494	3.206
Skew	-0.528	0.340	-1.711	0.331	-1.404	0.909	-8.374	-0.029
Days to option expiration	168.904	26.922	113.000	206.000	180.888	16.066	148.000	204.000

¹ Subscripts *n*, *o*, and *d* correspond to near-the-money, out-of-the-money, and deep out-of-the-money, respectively. Superscripts *p* and *c* indicate that implied volatility (*V*) is taken from puts and calls, respectively.

Table 4. Selected Probit Model Estimates for Large Price Declines Following Quarterly Hogs and Pigs Report Releases.

Contract	Magnitude of price response (percentile)	Model	Explanatory Variables ¹								Days to option expiration
			V_n	$\frac{V_d^p - V_o^p}{V_n}$	$\frac{V_o^p - V_n}{V_n}$	$\frac{V_o^c - V_n}{V_n}$	$\frac{V_d^c - V_o^c}{V_n}$	$\frac{V_d^p - V_n}{V_n}$	$\frac{V_d^c - V_n}{V_n}$	Skew	
1 st deferred	$\leq 15^{\text{th}}$	1	0.99	-4.51	6.88	7.70	-10.84 *				0.02
		2	2.23					-0.53	-2.09		0.01
		3	2.72							-0.16	0.02
2 nd deferred	$\leq 15^{\text{th}}$	1	3.86	7.25 *	1.13	1.00	-7.66 *				-0.01
		2	4.09					1.88	-2.29		-0.01
		3	0.92							1.15	-0.01
3 rd deferred	$\leq 15^{\text{th}}$	1	1.82	-4.70	5.95	2.63	-2.57				0.00
		2	0.56					1.56	-1.23		-0.01
		3	-2.60							1.29 *	-0.01
4 th deferred	$\leq 15^{\text{th}}$	1	5.84	1.77	1.26	4.34	9.75				0.00
		2	5.76					-2.16	4.49		0.00
		3	3.14							-0.29	0.00
1 st deferred	$\leq 20^{\text{th}}$	1	0.75	-2.75	6.25	4.25	-9.24 *				0.01
		2	1.91					0.00	-2.67		0.01
		3	3.09							-0.55	0.02
2 nd deferred	$\leq 20^{\text{th}}$	1	2.42	7.62 **	2.93	2.74	-3.68				0.00
		2	2.85					3.56	-0.08		0.00
		3	-0.61							0.88	0.00
3 rd deferred	$\leq 20^{\text{th}}$	1	1.62	-2.63	0.04	-2.86	0.85				-0.01
		2	1.38					-2.41	-2.14		-0.01
		3	0.20							0.78	-0.01
4 th deferred	$\leq 20^{\text{th}}$	1	5.53	3.71	-0.36	1.60	6.28				0.01
		2	4.67					-0.82	2.86		0.01
		3	2.53							0.22	0.01

¹ Subscripts n , o , and d correspond to near-the-money, out-of-the-money, and deep out-of-the-money, respectively. Superscripts p and c indicate that implied volatility (V) is taken from puts and calls, respectively. Asterisks indicate significance: * at the 10 percent level and ** at the 5 percent level.

Table 5. Selected Probit Model Estimates for Large Price Increases Following Quarterly *Hogs and Pigs* Report Releases.

Contract	Magnitude of price response (percentile)	Model	Explanatory Variables ¹								Skew	Days to option expiration
			V_n	$\frac{V_d^p - V_o^p}{V_n}$	$\frac{V_o^p - V_n}{V_n}$	$\frac{V_o^c - V_n}{V_n}$	$\frac{V_d^c - V_o^c}{V_n}$	$\frac{V_d^p - V_n}{V_n}$	$\frac{V_d^c - V_n}{V_n}$			
1 st deferred	≥ 80 th	1	-0.39	-6.43 *	5.87	5.62	-1.97					0.00
		2	0.59					-1.91	1.78			-0.01
		3	0.55							0.32		0.00
2 nd deferred	≥ 80 th	1	1.50	-0.73	2.18	4.81	2.10					0.00
		2	1.67					0.62	3.57			0.00
		3	1.11							0.01		0.00
3 rd deferred	≥ 80 th	1	8.99 **	2.63	3.38	7.68 **	8.47 *					0.02 **
		2	8.99 **					3.34	7.93 **			0.02 **
		3	6.55 **							-0.66		0.01
4 th deferred	≥ 80 th	1	7.00	6.45	-9.73 *	-0.25	7.21					0.00
		2	3.64					-0.86	4.08			0.00
		3	5.84							-1.15 **		0.00
1 st deferred	≥ 85 th	1	3.29	-9.74 **	10.79	3.05	3.32					-0.03
		2	2.85					-3.18	3.92			-0.05
		3	1.34							0.25		-0.03
2 nd deferred	≥ 85 th	1	1.26	0.90	2.91	3.92	1.62					0.00
		2	1.37					1.62	2.88			0.00
		3	1.29							-0.20		0.00
3 rd deferred	≥ 85 th	1	4.51	-4.09	3.64	6.89 *	10.58 **					0.02 *
		2	4.60					0.95	8.19 **			0.01 *
		3	3.48							-0.92		0.00
4 th deferred	≥ 85 th	1	-5.56	-6.23	-5.55	2.44	-1.51					0.00
		2	-5.15					-4.41	2.64			0.00
		3	-2.17							-1.29 **		0.00

¹ Subscripts *n*, *o*, and *d* correspond to near-the-money, out-of-the-money, and deep out-of-the-money, respectively. Superscripts *p* and *c* indicate that implied volatility (*V*) is taken from puts and calls, respectively. Asterisks indicate significance: * at the 10 percent level and ** at the 5 percent level.

Table 6. Selected Probit Model Estimates for Large Price Declines Following *Cattle on Feed* Report Releases.

Contract	Magnitude of price response (percentile)	Model	Explanatory Variables ¹								
			V_n	$\frac{V_d^p - V_o^p}{V_n}$	$\frac{V_o^p - V_n}{V_n}$	$\frac{V_o^c - V_n}{V_n}$	$\frac{V_d^c - V_o^c}{V_n}$	$\frac{V_d^p - V_n}{V_n}$	$\frac{V_d^c - V_n}{V_n}$	Skew	Days to option expiration
1 st deferred	$\leq 15^{\text{th}}$	1	-10.35	5.41	-11.05	-9.48	2.34				0.08
		2	0.23					-0.97	1.32		0.15
		3	3.05							0.54	0.07
2 nd deferred	$\leq 15^{\text{th}}$	1	2.60	-2.05	1.66	2.12	1.34				0.01
		2	2.37					-0.69	1.31		0.01
		3	0.36							0.35	0.00
3 rd deferred	$\leq 15^{\text{th}}$	1	0.56	1.79	0.16	3.10	0.35				0.01
		2	-0.69					0.08	1.35		0.00
		3	2.49							-0.23	0.01
4 th deferred	$\leq 15^{\text{th}}$	1	2.35	-2.42	1.22	2.37	1.21				0.01
		2	2.33					-1.15	1.10		0.00
		3	7.34 *							-0.08	0.00
1 st deferred	$\leq 20^{\text{th}}$	1	-8.48	3.08	-9.43	-16.22 **	15.01 **				-0.01
		2	-5.25					-2.12	1.63		0.01
		3	2.50							0.38	0.03
2 nd deferred	$\leq 20^{\text{th}}$	1	-1.07	-2.06	-0.19	1.48	0.68				0.01
		2	-1.27					-1.49	0.73		0.01
		3	0.26							0.10	0.00
3 rd deferred	$\leq 20^{\text{th}}$	1	-2.02	0.39	-0.45	2.76	1.82				0.00
		2	-3.23					-0.69	1.82		0.00
		3	0.30							-0.10	0.00
4 th deferred	$\leq 20^{\text{th}}$	1	0.93	-2.06	0.80	1.33	1.35				0.00
		2	0.69					-0.98	0.84		0.00
		3	6.00							-0.14	0.00

¹ Subscripts n , o , and d correspond to near-the-money, out-of-the-money, and deep out-of-the-money, respectively. Superscripts p and c indicate that implied volatility (V) is taken from puts and calls, respectively. Asterisks indicate significance: * at the 10 percent level and ** at the 5 percent level.

Table 7. Selected Probit Model Estimates for Large Price Increases Following *Cattle on Feed* Report Releases.

Contract	Magnitude of price response (percentile)	Model	Explanatory Variables ¹								Days to option expiration	
			V_n	$\frac{V_d^p - V_o^p}{V_n}$	$\frac{V_o^p - V_n}{V_n}$	$\frac{V_o^c - V_n}{V_n}$	$\frac{V_d^c - V_o^c}{V_n}$	$\frac{V_d^p - V_n}{V_n}$	$\frac{V_d^c - V_n}{V_n}$	Skew		
1 st deferred	$\geq 80^{\text{th}}$	1	-0.42	-3.32	-1.18	7.61 *	-4.20					-0.04
		2	1.30					-0.72	0.73			0.00
		3	9.04 **								-0.08	0.00
2 nd deferred	$\geq 80^{\text{th}}$	1	8.65 **	1.30	1.31	1.23	-0.50					0.00
		2	8.48 **					1.00	-0.15			-0.01
		3	6.42 **								-0.13	-0.01
3 rd deferred	$\geq 80^{\text{th}}$	1	7.62 **	-0.62	1.39	-0.92	0.02					-0.01
		2	7.88 **					0.53	-0.67			-0.01
		3	7.90 **								-0.07	-0.01
4 th deferred	$\geq 80^{\text{th}}$	1	4.44	-1.93	1.81	2.00	-1.52					-0.01
		2	5.95					-1.24	-0.49			0.00
		3	10.20 **								-0.08	0.00
1 st deferred	$\geq 85^{\text{th}}$	1	-2.99	-1.37	-1.86	5.35	-6.21					-0.02
		2	1.31					0.00	-0.40			0.04
		3	9.32 **								-0.39	0.00
2 nd deferred	$\geq 85^{\text{th}}$	1	8.89 **	1.50	0.67	0.50	-0.42					-0.01
		2	8.85 **					1.02	-0.20			-0.01
		3	7.49 **								-0.15	-0.01
3 rd deferred	$\geq 85^{\text{th}}$	1	4.81	-0.98	0.67	1.04	-3.12					-0.01
		2	5.75					-0.60	-1.22			-0.01
		3	4.19								0.37	-0.01 *
4 th deferred	$\geq 85^{\text{th}}$	1	3.89	-2.23	2.54	2.23	-1.73					-0.01
		2	5.75					-1.19	-0.63			-0.01
		3	6.42								0.15	-0.01

¹ Subscripts n , o , and d correspond to near-the-money, out-of-the-money, and deep out-of-the-money, respectively. Superscripts p and c indicate that implied volatility (V) is taken from puts and calls, respectively. Asterisks indicate significance: * at the 10 percent level and ** at the 5 percent level.

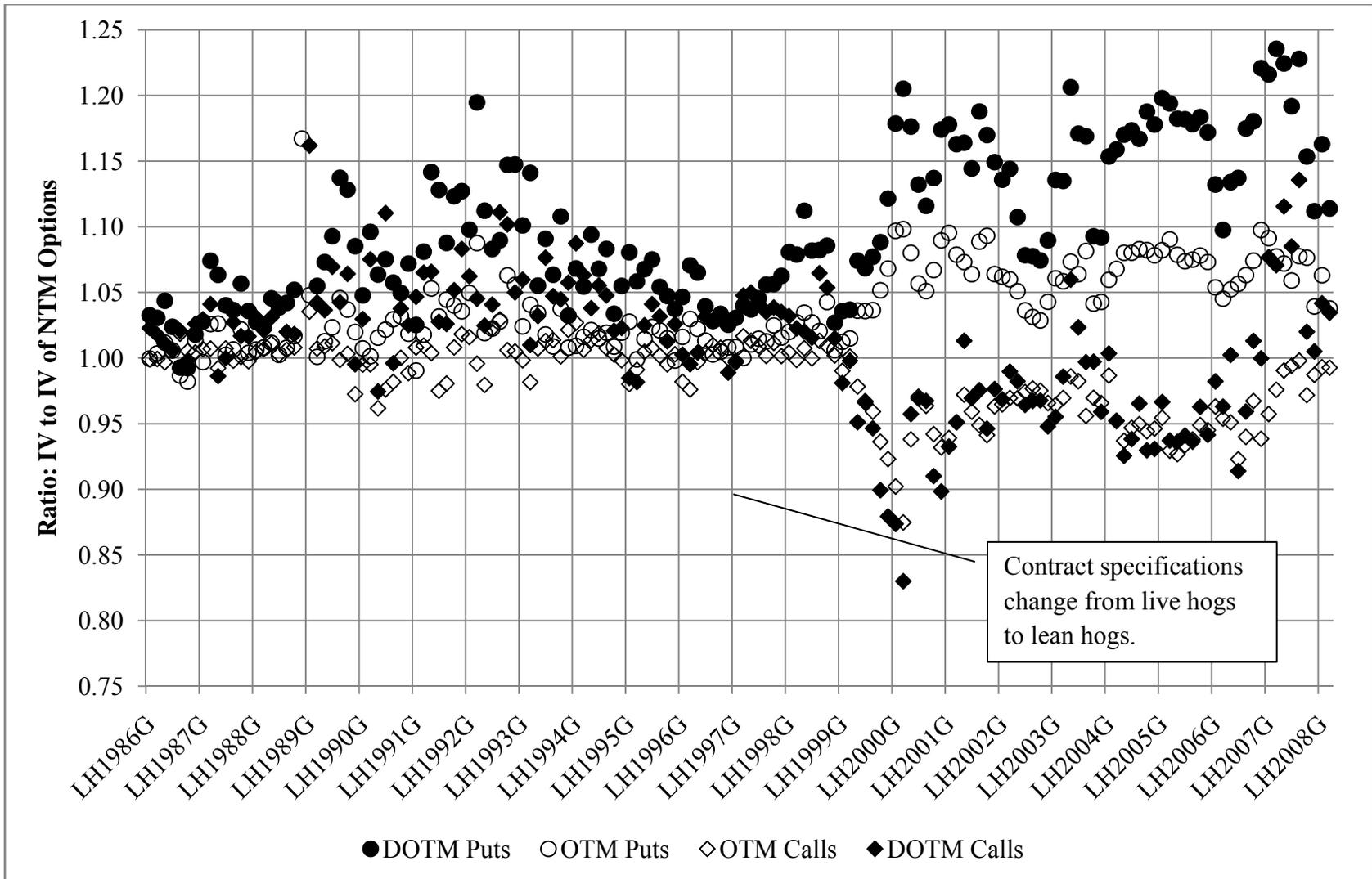


Figure 1. Ratios of out-of-the-money (OTM) and deep-out-of-the-money (DOTM) implied volatility (IV) to IV computed from near-the-money (NTM) puts and calls for live/lean hog contracts 1986 through April 2008. Values presented are averages by contract and are based on the period consisting of 18 weeks to 2 weeks prior to expiration of options contracts.

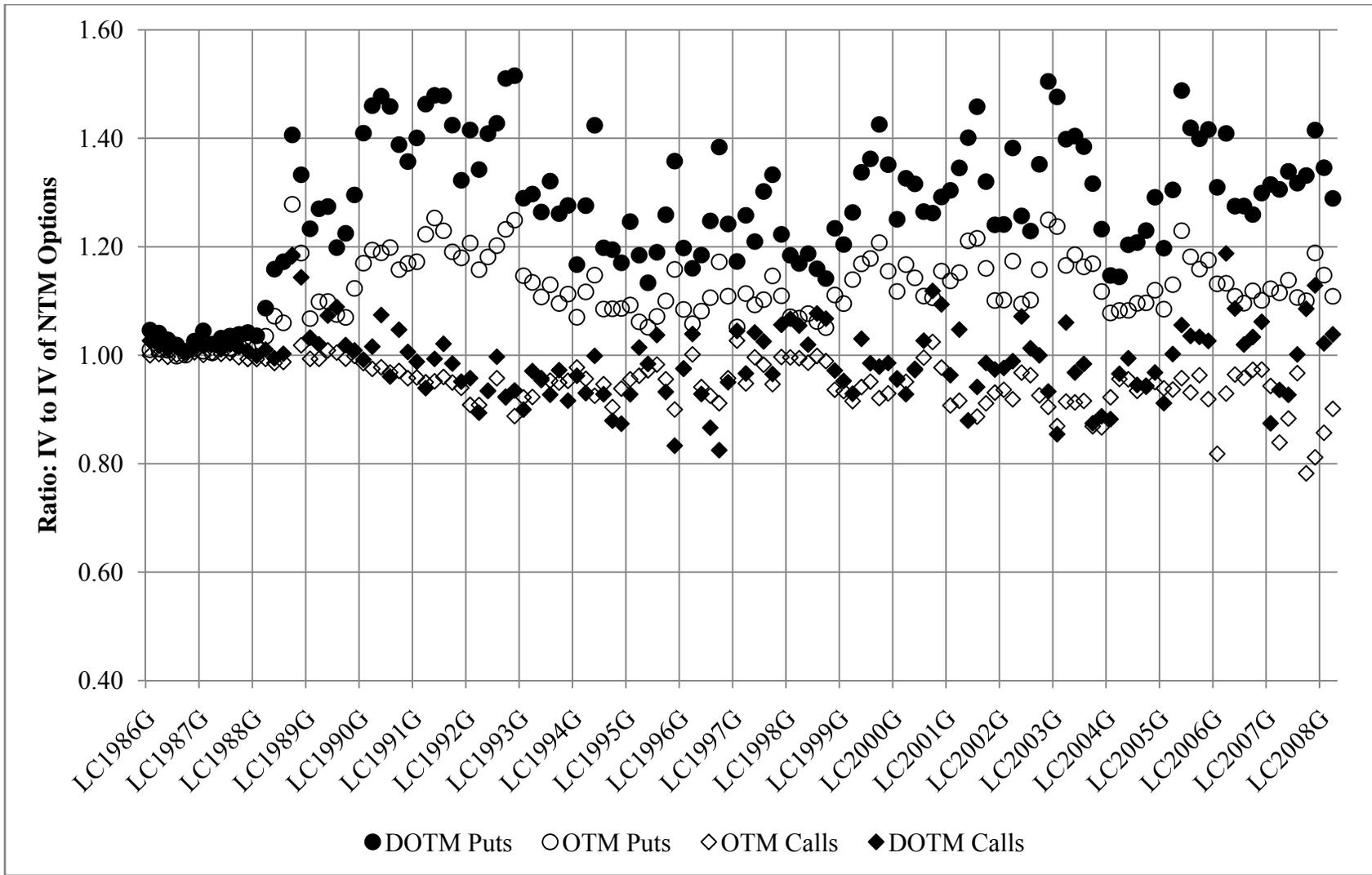


Figure 2. Ratios of out-of-the-money (OTM) and deep-out-of-the-money (DOTM) implied volatility (IV) to IV computed from near-the-money (NTM) puts and calls for live cattle contracts 1986 through April 2008. Values presented are averages by contract and are based on the period consisting of 18 weeks to 2 weeks prior to expiration of options contracts.

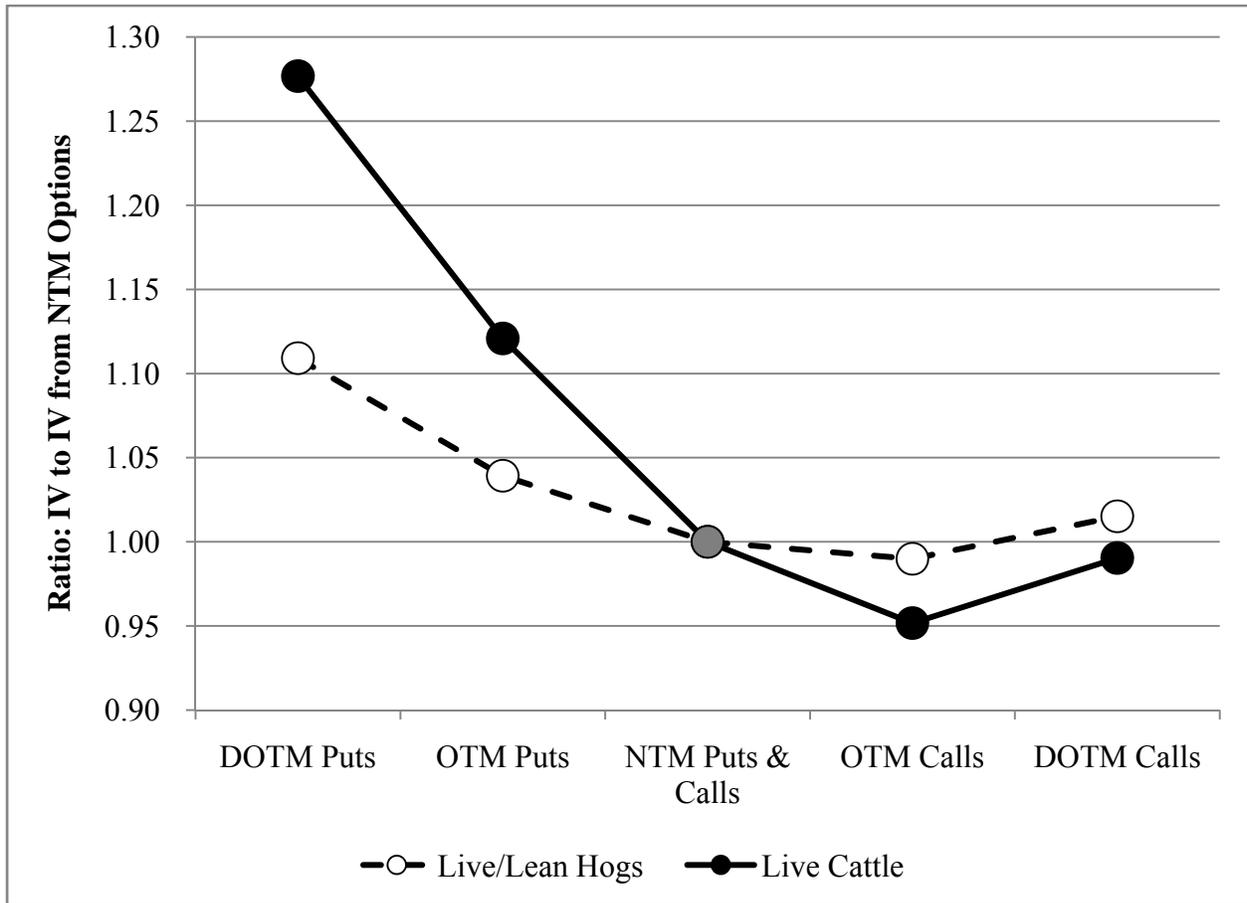


Figure 3. Typical shape of the volatility smile. Values in the figure are averages recovered from options on the February 1986 through April 2008 contracts. A volatility skew is present in both markets as higher volatilities are associated with deep-out-of-the-money (DOTM) and out-of-the-money (OTM) put options. The skew is very pronounced in live cattle.

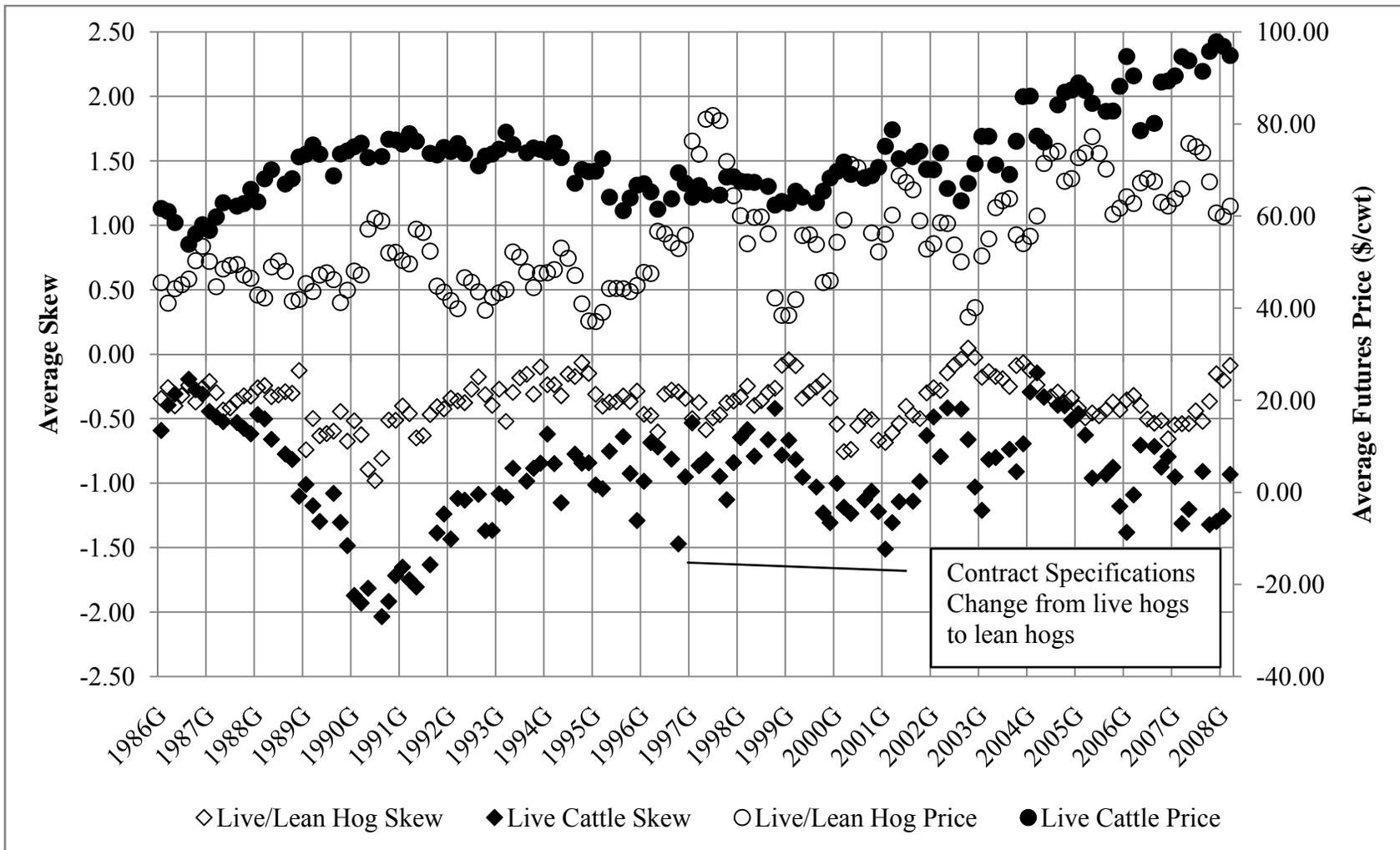


Figure 4. Average implied skew and futures settlement prices for live cattle and live/lean hogs 1986 through April 2008. Values presented are averages by contract and are based on the period consisting of 18 weeks to 2 weeks prior to expiration of options contracts.

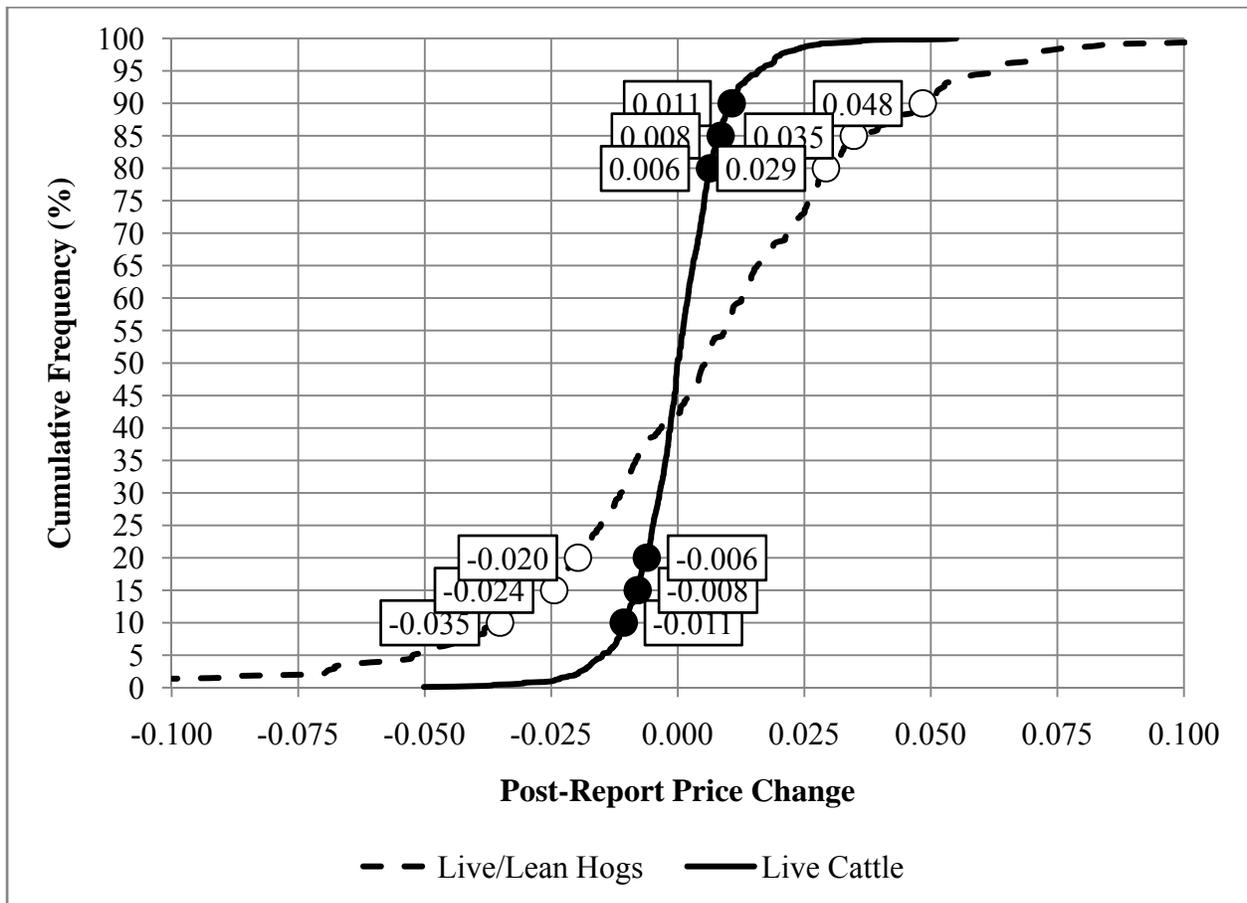


Figure 5. Price changes and percentile cutoffs used in binomial response models. Price changes are computed as the logged price difference between the settlement price on the first non-limit trading day following a report release and the settlement price on the report release day. The sequence for live/lean hogs is based on quarterly *Hogs and Pigs* releases. The sequence for live cattle is based on *Cattle on Feed* releases. Empirical distributions shown include price changes across contracts in all deferral classes.