Can Real Option Value Explain Why Producers Appear to Store Too Long?

by

Hyun Seok Kim, and B. Wade Brorsen

Suggested citation format:

Kim, H. S., and B. W. Brorsen. 2008. "Can Real Option Value Explain Why Producers Appear to Store Too Long?" Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. St. Louis, MO. [http://www.farmdoc.uiuc.edu/nccc134].

Can Real Option Value Explain Why Producers Appear to Store Too Long?

Hyun Seok Kim,

and

B. Wade Brorsen

Paper presented at NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management
St. Louis, Missouri, April 21-22, 2008

Kim is a Ph.D. candidate (*hyunseok.kim@okstate.edu*), and Brorsen is a regents professor and Jean & Patsy Neustadt Chair (*wade.brorsen@okstate.edu*), Department of Agricultural Economics, Oklahoma State University.

Can Real Option Value Explain Why Producers Appear to Store Too Long?

Practitioner's Abstract: *Previous studies suggest that producers tend to store crops longer than makes economic sense. Since decisions to sell are irreversible, there can be a real option value from waiting to sell grain. This real option value may explain why producers appear to store too long. A seasonal mean reversion model is estimated that allows prices to be a random walk within a season, but mean reverting across crop years. Unless prices are extremely low, it is optimal for producers to sell before the mean reversion begins. Thus, the real option value of waiting cannot explain why producers seem to store at a loss in the latter part of crop years.*

Keywords: real option value, seasonal mean reversion

Introduction

Some studies show that producers store longer than is profitable (Hagedorn, et al.; Anderson and Brorsen). One possibility is that producers may store crops longer than makes economic sense due to myopic loss aversion which means that producers get more disutility from a loss than they get utility from receiving an equally sized gain. Producers' decisions to sell grain are stock irreversible. This irreversibility creates a real option value from waiting to sell grain (Fackler and Livingston). This research will focus on answering the question, "Can real option values explain why producers appear to store too long?"

There are, recently, some studies that developed concerning the effects of irreversibility on optimal investment decision. McDonald and Siegel (1986) studied the optimal timing of investment in an irreversible project and concluded that it is optimal to wait until benefits are twice the investment costs. Brennan and Schwartz (1985) recognized that this dynamic aspect of the investment decision of constructing a mine is closely related to the problem of determining the optimal strategy for exercising an option on a share of common stock. Fackler and Livingston (2002) proposed that optimal storage decision rule follows a cutoff price[1] rule which is optimal to continue holding stocks when the market price is below the cutoff price and to sell all stocks when the market price exceeds the cutoff price and recognized that this rule is analogous to the decision to exercise an American call option. Their empirical result showed that including real option value increase the optimal length of storage.

To determine the real option value, first, we model and estimate price process. The model attempts to capture two important features of agricultural commodity prices, mean reversion and seasonality. This study models and estimates a seasonal mean reversion price process which allows price to be a random walk within a season, but mean reverting across crop years. Our model goes beyond considering mean reversion to a seasonal mean which is common in the electricity literature. The model allows the rate of convergence to the

[1] A cutoff price represents the price at which the producers are indifferent between selling and holding stocks.

mean to be seasonal. After estimation of price process, we employed universal lattice model to determine real option value. This study conducts simulations using cash price of crops to determine differences of net returns of optimal strategy under two different price processes, which are mean reversion price process and seasonal mean reversion price process. This empirical work determines whether real option value can explain why producers appear to store too long.

Theory of Dynamic Programming

Decision maker's optimization problem can be modeled as a discrete dynamic programming problem. The model has the following structure: in every period *t*, a decision maker observes the state of an economic system k_t, takes an action x_t and earns rewards $f(k_t, x_t)$. The state space *K* and the action space *X* are both finite. The probability distribution of the next period's state depends only on the current state and the decision maker's action which can be expressed as

(1) $$\Pr(k_{t+1} = k' \mid k_t = k,\ x_t = x) = P(k' \mid k, x)$$

The decision maker seeks a sequence of policies $\{x_t^*\}$ that prescribes the action $x_t = x_t^*(k_t)$ that should be taken in any given state and period so as to maximize the present value of current and expected future rewards over a time horizon *T*, discounted at a per-period factor r. The state of an economic system is governed by transition equation

(2) $$k_{t+1} = g(k_t, x_t)$$

The optimization problem can be solved using dynamic programming methods developed by Richard Bellman (1957), which is based on the principle of optimality. The principle of optimality formally may be expressed in the form of a value function $V_t(k)$ and this function satisfies the Bellman equation:

(3) $$V_t(k) = \max_{x \in X(k)} \left\{ f(k, x) + r \sum_{k' \in K} P(k' \mid k, x) V_{t+1}(k') \right\}, \quad k \in K,\ t = 1, 2, \mathbf{K}, T$$

The Bellman equation captures the essential problem faced by a dynamic optimizing decision maker: the need to optimally balance an immediate reward $f(k_t, x_t)$ against expected future rewards $dE_t V_{t+1}(k_{t+1})$. In a finite horizon decision model, can be solved recursively by repeated application of the Bellman equation: having V_{T+1}, solve for $V_T(k)$ for all states *k*; having V_T, solve for $V_{T-1}(k)$ for all states *k*; having V_{T-1}, solve for $V_{T-2}(k)$ for all states *k*; and so on. The process continues until $V_1(k)$ is derived for all states *k*. For the continuous stochastic dynamic optimization problem, we can rewrite equation (2) and (3) as

(4) $$dk = g(k,x)dt + s\,(k)dz$$

and

(5) $$V(k,t) = \max_x \left\{ f(k,x)\Delta t + \frac{1}{1 + r\Delta t} E_t[V(k_{t+\Delta t}, t + \Delta t)] \right\}$$

where $s\,(k)$ indicates the instantaneous variance of the process, and z is a standard Wiener processes. Multiplying equation (5) by $(1 + r\Delta t)/\Delta t$ and rearranging:

(6) $$rV(k,t) = \max_x \left\{ f(k,x)(1 + r\Delta t) + \frac{E_t[V(k_{t+\Delta t}, t + \Delta t) - V(k,t)]}{\Delta t} \right\}$$

Taking the limits of this equation at $\Delta t \to 0$ yields the continuous time version of the Bellman equation:

(7) $$rV(k,t) = \max_x \left\{ f(k,x) + \frac{E_t dV(k,t)}{dt} \right\}$$

Therefore, the Bellman equation states that the rate of return on the asset, rV, must equal the current income flow, *f*, plus the expected rate of capital appreciation, EdV/dt.
By Ito's lemma

(8) $$dV = [V_t + g(k,x)V_k + \tfrac{1}{2}s\,(k)^2 V_{kk}]dt + s\,(k)V_k dz$$

where the subscripts denote partial derivatives. Taking expectations and dividing by *dt* the term EdV/dt is replaced and then equation (7) can be rewritten as

(10) $$rV(k,t) = \max_x \left\{ f(k,x) + V_t + g(k,x)V_k + \tfrac{1}{2}s\,(k)^2 V_{kk} \right\}$$

This study uses dynamic programming models to determine crop producer's post-harvest marketing decision problem. The two state variables are governed by transition equations. The cash price is assumed to follow an Ito diffusion process described by the stochastic differential equation:

(11) $$dp(t) = a(p(t),t)dt + b^2(p(t),t)dz$$

where $p(t)$ is the price at time *t*, a represents the instantaneous mean, b^2 indicates the instantaneous variance of the process, z is a standard Wiener process. The stock transition equation, which implies the rate of change in stocks is equal to the negative of the rate of sales, is

(12) $$ds(t) = -q(t)dt$$

where $s(t)$ is the stock level at time *t*, and $q(t)$ is the rate of sales at time *t*. The optimization problem can be defined by using equation (10), which is

(13) $$rV(s,p,t) = \max_q qp - sc + V_t(s,p,t) - qV_s + a(p,t)V_p(s,p,t) + \frac{b^2(p,t)}{2}V_{pp}(s,p,t)$$

where *c* is a per period, per unit storage cost rate. The optimization problem is subject to the constraints that $q \geq 0$ and $s \geq 0$, and the only feasible condition when $s = 0$ is $q = 0$ since the study assumes irreversibility. The first order condition for optimization is

$$V_s - p \geq 0,\ q \geq 0,$$
$$q(V_s - p) = 0, \text{ for } s > 0$$

This condition implies that if the market price is less than the shadow price of stocks it is optimal to hold all of the stocks. On the other hand, if the market price is higher than the shadow price, then it would be optimal to sell all instantly.
To determine the cutoff price at which a decision maker is indifferent between holding and selling, we solve the Bellman equation for low prices where $q = 0$ along with boundary conditions. Then we can rewrite the equation (13) as

(14) $$rV = -sc + V_t(s,p,t) + a(p,t)V_p(s,p,t) + \frac{b^2(p,t)}{2}V_{pp}(s,p,t)$$

This equation is satisfied by the function $sv(p,t)$, where $v(p,t)$ satisfies

(15) $$rv(p,t) = -c + v_t(p,t) + a(p,t)v_p(p,t) + \frac{b^2(p,t)}{2}v_{pp}(p,t)$$

This is a direct result of the linearity of the problem in the stock level. This implies that the optimal decision rule can be determined independently of the stock level by solving (15) subject to boundary conditions. The solution of optimization problem is characterized by the value function $v(p,t)$ and the optimal cutoff price $c(t)$ that solve (15) for $0 \leq p(t) \leq c(t)$ with terminal condition

(16) $$v(p,T) = p$$

and the boundary conditions

(17) $$v(c(t),t) = c(t)$$

and

(18) $$v_p(c(t),t) = 1.$$

When selling stocks is viewed as irreversible, the producer is not just holding stocks but is holding stocks and a put option to sell the stocks on or before time *T*. Holding stocks and a put is like holding a synthetic call option to buy the good on or before time *T* with an exercise price of $x = 0$. Thus, the optimal storage problem is equivalent to the determining the optimal time to exercise an American call option on a commodity that expires at time *T* with exercise price zero.

Data

The chosen agricultural commodities are corn, soybeans and wheat. This study obtained Thursday cash prices of South Central Illinois corn and soybean data from the National Agricultural Statistic Services (NASS) of the United States Department of Agriculture (USDA) website. Thursday cash price of wheat data at Medford, Oklahoma, are obtained from the Oklahoma Market Reports of USDA. The sample period extends from October 1976 through September 2007 for corn and soybean, and from June 1988 through May 2007 for wheat.

However, these primary data have some missing values for Thanksgiving and Christmas season. For these missing data, this study uses data of a week or day before days which have missing value. For example, we uses the data of a week before Thanksgiving and Christmas week for corn and soybean and uses Wednesday data of Thanksgiving and Christmas week for wheat.

Annual average price data are also obtained from the NASS of USDA website. To estimate the price processes, 5 year moving averages of annual average prices for each crop are used as mean prices.

To conduct simulation, corn and soybean storage costs from 1995 through 2004 are from Farmdoc, University of Illinois, AgMas report "The Pricing Performance of Market Advisory Services". We calculate previous 20 year storage costs using consumer price index and assume that storage costs of 2005 and 2006 equal to the cost of 2004. Storage costs of wheat from 1975 to 2006 are obtained from Oklahoma Cooperative Extension Service at Oklahoma State University. The interest cost is calculated at the prime rate for that year plus 2%. The prime rate is the prime charged by banks in June for that year, quoted from the Kansas City Federal Reserve Bank (2008).

Procedures

This paper has three main procedures – estimation of price process parameters, determining real option value and simulation. To determine the real option value, this study employs a universal lattice model (Chen and Yang) as a continuous stochastic dynamic programming.

Estimation of Price Process Parameters

This study attempts to capture two important features of agricultural commodity prices, mean reversion and seasonality. A number of studies documented mean reversion in commodity cash prices (Fama and French; Dixit and Pindyck; Bessembinder et al.; Brennan). Also, some other studies have found that futures prices follow a near random walk within a contract month (Yoon and Brorsen; Bessler and Covey), but are mean reverting when prices across multiple contract months are used (Schroeder and Goodwin). Seasonality in the mean level of price and price variability has been also well documented in commodity (Anderson and Danthine; Anderson; Streeter and Tomek; Kenyon et al.) and electricity markets (Cartea and Figueroa). For example, prices of seasonally produced goods tend to rise during the marketing season to cover the cost of storage and be more volatile during the growing season. A model which represents these features is the Ito process described by

$$(19) \qquad d\ln p = a\,(a(t) - \ln p)dt + b(t)dz$$

where $a(t)$ and $b(t)$ are seasonal functions. This study allows prices to follow a random walk within a season, but be mean reverting across crop years. Such a price process can be rewritten as

$$(20) \qquad d\ln p = \begin{cases} a(t) + b(t)dz & \text{if } t < t_0 \\ a\,(a(t) - \ln p)dt + b(t)dz & \text{if } t \geq t_0 \end{cases}$$

The model, in this study, also goes beyond considering mean reversion to a seasonal mean and allows the rate of convergence to the mean to be seasonal. Then, the price process can be described as

$$(21) \qquad \ln p_{t+1} - \ln p_t = \begin{cases} f(t,1) + b\,(\ln \overline{p}_t - \ln p_t) + e_{t+1} & \text{if } t < t_0, \\ g(t,1) + (a + b)(\ln \overline{p}_t - \ln p_t) + e_{t+1} & \text{if } t \geq t_0, \end{cases}$$

where

$$e_{t+1} \sim N(0, s_t^2),$$

p indicates the cash price, $\bar{p}$ is an estimated mean price, t represents number of days after harvest, $f(t, 1)$ and $g(t, 1)$ are functions that reflect seasonality, and a and b are parameters to be estimated. This study adopts a polynomial functional form for the seasonal function $l(t)$, which is

$$\text{(22)} \qquad l(t) = \sum_{i=0}^{m} g_i t^i$$

where the g s are the parameters to be estimated. Then, $f(t, 1)$ and $g(t, 1)$ can be written as

$$\text{(23)} \qquad \begin{aligned} f(t, 1) &= l(t)\{1 + b(t-26)\} \\ g(t, 1) &= l(t)\{1 + (a+b)(t-26)\} \end{aligned}$$

If we impose a continuity restriction on the seasonal function $l(t)$ then the rate of change at harvest in the current year is equivalent to the rate of change at harvest next year. Since this study uses weekly cash price data, we can impose a continuity condition as $l(0) = l(52)$. Using (26) this can be rearranged as

$$\text{(24)} \qquad g_1 = \frac{-\sum_{i=2}^{m} g_i (52)^i}{52}$$

Then, g_1 can be obtained by other estimated parameters.

This study estimated (21) using cash prices of three crops – wheat, corn, and soybean and found that there is no evidence that coefficient b is significantly different from zero. Therefore, we allows to b equals zero and then (21) can be rewritten as

$$\text{(25)} \qquad \ln p_{t+1} - \ln p_t = \begin{cases} f(t, 1) + e_{t+1} & \text{if } t < t_0, \\ g(t, 1) + a(\ln \bar{p}_t - \ln p_t) + e_{t+1} & \text{if } t \geq t_0, \end{cases}$$

The study also estimated (21) but we considered two cases, that is, homoskedasticity or heteroskedasticity. Log-Likelihood value for homoskedasticity model was slightly higher than alternative one. Therefore, we assume homoskedasticity to estimate price process. A nonparmetric bootstrapping is also used to estimate price process. We resampled ten thousands samples of size 1664.

A Universal Lattice Model

Hull and White (1990, 1993) suggest a trinomial lattice model. In this trinomial lattice structure, the branches are up, flat, and down by an increment of change in underlying value Δx. That is,

$$x_{3,i,t} = x_{i,t} + \Delta x \quad (26)$$
$$x_{2,i,t} = x_{i,t}$$
$$x_{1,i,t} = x_{i,t} - \Delta x$$

Figure 1 shows an example of a trinomial lattice structure. The branches are down, flat, and up with the risk neutral probabilities P_1, P_2, and P_3, respectively, which satisfy the following three equation

$$P_{1,i,t}x_{1,i,t} + P_{2,i,t}x_{2,i,t} + P_{3,i,t}x_{3,i,t} = x_{i,t} + m \quad (27)$$
$$P_{1,i,t}(x_{1,i,t})^2 + P_{2,i,t}(x_{2,i,t})^2 + P_{3,i,t}(x_{3,i,t})^2 - (x_{i,t} + m_{i,t})^2 = s_{i,t}^2$$
$$P_{1,i,t} + P_{2,i,t} + P_{3,i,t} = 1$$

where $x_{i,t}$ is the ***i***-th node of *x* at time *t*, $x_{n,i,t}$ is the ***n***-th lowest possible node at time $t + \Delta t$, and $m_{i,t}$ and $s_{i,t}^2$ are the expected change and the variance of $x_{i,t}$ during the next time interval Δt, respectively. However, in the trinomial lattice structure, the risk neutral probabilities of all nodes in the lattice could be negative. To solve this problem, Hull and White (1990, 1993) propose four alternative branching schemes. These alternatives include the branches of the lattice to go three ups, two ups, and one up; two ups, one up, and flat; flat, one down, and two downs; and one down, two downs, and three downs. However, Chen and Yang (1999) argue that, in alternative trinomial lattice, there seems to be no consistent way to construct the lattice in which all probabilities are guaranteed to be positive. Thus, they extend Hull and White's model and propose a general form of alternative branching schemes. With Chen and Yang's new lattice model, the three branches can be written as

$$x_{3,i,t} = x_{i,t} + (j + k)\Delta x \quad (28)$$
$$x_{2,i,t} = x_{i,t} + (j)\Delta x$$
$$x_{1,i,t} = x_{i,t} + (j - k)\Delta x$$

where the variable *j* and *k* provide flexibilities for the branches to yield non-negative probabilities with any level of mean and variance, respectively. With this branching method, the risk neutral probabilities can be obtained solving (27) then the results are

$$(29) \qquad P_{1,i,t} = \frac{(j\Delta x - m_{i,t})((j+k)\Delta x - m_{i,t}) + s_{i,t}^{2}}{2k^{2}\Delta x^{2}}$$

$$P_{2,i,t} = 1 - \frac{(m_{i,t} - j\Delta x)^{2} + s_{i,t}^{2}}{k^{2}\Delta x^{2}}$$

$$P_{3,i,t} = 1 - pr_{1,i,t} - pr_{2,i,t}$$

To guarantee the convergence of the model, the constraints of $0 \le P_{n,i,t} \le 1$ translate into the following two sets of sufficient conditions:

$$(30) \qquad \frac{s_{i,t}}{\Delta p} \le k \le \frac{2s_{i,t}}{\Delta p}$$

and

$$\frac{m_{i,t} - \sqrt{k^{2}\Delta x^{2} - s_{i,t}^{2}}}{\Delta x} \le j \le \frac{m_{i,t} + \sqrt{k^{2}\Delta x^{2} - s_{i,t}^{2}}}{\Delta x}$$

and

$$(31) \qquad k > \frac{2s_{i,t}}{\Delta p}$$

and

$$\frac{m_{i,t}}{\Delta x} - \frac{\sqrt{k^{2}\Delta x^{2} - s_{i,t}^{2}}}{\Delta x} \le j \le \frac{-k}{2} + \frac{m_{i,t}}{\Delta x} - \frac{\sqrt{k^{2}\Delta x^{2} - s_{i,t}^{2}}}{\Delta x} \qquad \text{or}$$

$$\frac{-k}{2} + \frac{m_{i,t}}{\Delta x} + \frac{\sqrt{k^{2}\Delta x^{2} - 4s_{i,t}^{2}}}{\Delta x} \le j \le \frac{k}{2} + \frac{m_{i,t}}{\Delta x} - \frac{\sqrt{k^{2}\Delta x^{2} - 4s_{i,t}^{2}}}{\Delta x} \quad \text{or}$$

$$\frac{k}{2} + \frac{m_{i,t}}{\Delta x} + \frac{\sqrt{k^{2}\Delta x^{2} - 4s_{i,t}^{2}}}{\Delta x} \le j \le \frac{m_{i,t}}{\Delta x} + \frac{\sqrt{k^{2}\Delta x^{2} - s_{i,t}^{2}}}{\Delta x}.$$

Since this study assumes the constant volatility, which means $k = 1$, the risk neutral probabilities and the sets of sufficient conditions for constraints of $0 \le P_{n,i,t} \le 1$ can be rewritten as

(32) $$P_{1,i,t} = \frac{(j\Delta x - m_{i,t})((j+1)\Delta x - m_{i,t}) + s_{i,t}^{2}}{2\Delta x^2}$$

$$P_{2,i,t} = 1 - \frac{(m_{i,t} - j\Delta x)^2 + s_{i,t}^{2}}{\Delta x^2}$$

$$P_{3,i,t} = 1 - pr_{1,i,t} - pr_{2,i,t}$$

and

(33) $$\frac{s_{i,t}}{\Delta p} \le k \le \frac{2s_{i,t}}{\Delta p}$$

and

$$\frac{m_{i,t} - \sqrt{\Delta x^2 - s_{i,t}^{2}}}{\Delta x} \le j \le \frac{m_{i,t} + \sqrt{\Delta x^2 - s_{i,t}^{2}}}{\Delta x}.$$

Using (28) and (32), option value can be determined. If $P_{1,i,t}x_{1,i,t} + P_{2,i,t}x_{2,i,t} + P_{3,i,t}x_{3,i,t}$ is less than $x_{i,t}$ then the option is exercised.

Simulation

This study conducts simulations to determine differences of net returns of optimal strategy under two different price process models which can be defined as

M1: The price of agricultural commodity follows mean reversion process.
M2: The price of agricultural commodity follows seasonal mean reversion process.

The simulations are conducted based on four different scenarios. These scenarios depend on the level of storage and interest costs which can be designed as

S1: The models include storage and interest costs.
S2: The models do not include interest cost but storage cost.
S3: The models include half of storage cost and interest cost.
S4: The models do not include interest cost but half of storage cost.

Using a universal lattice model, the simulations are conducted with weekly cash price data for corn, soybean, and wheat. We also conduct a paired-difference test for M1 and M2 under the null hypothesis:

$$H_0 : p_{M1} = p_{M2}$$
$$H_0 : p_{M1} \neq p_{M2}$$

where p_{M1} and p_{M2} indicate that net returns of M1 and M2.

Results

This study uses a fifth power polynomial functional form for seasonality to estimate price process. The results of the estimation of price process parameters are presented in table 1. The estimated a coefficients are all significant at the 5% level. The estimated t_0 is 42, 41, and 39 for corn, soybean and wheat, respectively. These results imply that, for corn, mean reversion process appears from 41 weeks after harvest with 2.2% rate of mean reversion process per week. For soybean, mean reversion process appears from 41 weeks after harvest with 3.4% rate per week. Mean reversion process appears from 39 weeks after harvest for wheat and its rate is 1.8% per week. That is, total percentages of mean reversion each year are 21.8% for corn, 37.3% for soybean, and 22.8% for wheat.

Table 2 shows results of a nonparametric bootstrapping. Mean reversion process appears from 42 weeks, 39 weeks, and 38 weeks after harvest and rates of mean reversion are 3.8%, 3.7%, and 2.5% for corn, soybean, and wheat, respectively. Total mean reversion rates for a marketing year are 38.2% for corn, 48.4% for soybean, and 34.4% for wheat.

Figures 2 through 4 show the shapes of seasonality for corn, soybean, and wheat, respectively. For corn and soybean, seasonal price change is negative about two weeks from harvest but increases very rapidly until beginning of December. Then seasonal price change is positive but slowly decreases and is negative again on early June for corn and beginning of July for soybean. For wheat, seasonal price change is also negative about a month from harvest but increases rapidly until early September. Then seasonal price change is positive but slowly decreases and is negative again on mid January.

Optimal cutoff prices are illustrated in figures 5 through 10. To determine cutoff price, we use 1975 cash prices at harvest as values of $x_{i,t}$ on (28) for each crop and assume Δx on (28) are 12 cents for corn, 29 cents for soybean, and 16 cents for wheat. The shapes of graphs of M1s are very different from those of M2s. This implies that, with seasonal mean reversion, much of the real option value will disappear. Thus, the finding of Fackler and Livingston (2002) of a large real option value that can explain why producers store too long is not supported.

The results of simulations for corn, soybean, and wheat are presented in table 3, 4 and 5 respectively. For corn, the average net returns over 32 years of M2s are higher than those of M1s except S4. The result of soybean shows that the 32 year average net returns of M1s for S2, S3, and S4 are greater than those of M2s. In case of wheat, M2s have the higher average net returns over 32 years than M1 for all scenarios, S1 through S4.

Table 6 represents results of paired difference test for M1 and M2 for scenario 1 through 4. All *t*-values on table 6 are not significant at 5% level. Therefore, we can conclude that there is little evidence that, for all scenarios, the net returns over 32 years for M1 and M2 for all crops are different.

Table 1. Parameter Estimates of Seasonal Mean Reversion Price Process (1975-2006)

	Variable	Coefficient	Standard Error	*t*-statistic
Corn	a	0.0218	0.0073	2.98*
	g_0	-0.0053	0.0028	-1.89
	g_2	-4.49E-04	1.5303	-2.94*
	g_3	2.02E-05	7.7179	2.62*
	g_4	-4.08E-07	16.5270	-2.47*
	g_5	3.00E-09	12.6148	2.38*
	s^2	0.0013	4.5E-05	28.62*
	t_0	42	0	
Soybean	a	0.0339	0.0086	3.95*
	g_0	-0.0037	0.0024	-1.57
	g_2	-2.24E-04	1.3595	-1.65
	g_3	9.93E-06	6.8506	1.45
	g_4	-1.99E-07	14.6054	-1.37
	g_5	1.46E-09	11.0867	1.31
	s^2	0.0011	3.8E-05	28.6
	t_0	41	0	
Wheat	a	0.0175	0.0055	3.18*
	g_0	-0.0072	0.0027	-2.67*
	g_2	-9.92E-05	1.4314	-0.69
	g_3	4.93E-07	7.1605	0.07
	g_4	4.30E-08	15.2377	0.28
	g_5	-5.91E-10	11.5789	-0.51
	s^2	0.0012	4.0E-05	29.47*
	t_0	39	0	

Note: Asterisk (*) denotes that estimated coefficient is significant at 5% level.

Table 2. Parameter Estimates of Seasonal Mean Reversion Price Process by Nonparametric Bootstrapping (1975-2006)

	Corn		Soybean		Wheat	
	Coefficient	Standard Deviation	Coefficient	Standard Deviation	Coefficient	Standard Deviation
a	0.0382	0.0129	0.0372	0.0143	0.0246	0.0116
g_0	-0.0051	0.0027	-0.0037	0.0029	-0.0067	0.0031
g_2	-4.4E-04	1.2893	-2.3E-04	1.2803	-8.4E-05	1.4085
g_3	2.0E-05	6.5257	1.1E-05	6.2714	-3.3E-08	6.8482
g_4	-4.4E-07	14.1054	-2.2E-07	13.2849	5.0E-08	14.4405
g_5	2.9E-09	11.0942	1.6E-09	10.1231	-6.2E-10	10.9854
s^2	0.0013	8.3E-05	0.0011	6.0E-05	0.0012	6.6E-05
t_0	42	10.9709	39	5.9332	38	10.5421

Table 3. Detailed Comparisons of Sales Dates and Net Returns for Corn

Year	Sale Dates (Weeks from Harvest)								Per Bushel Net Returns ($/bu)							
	S1		S2		S3		S4		S1		S2		S3		S4	
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
1975	19	0	20	0	24	0	35	22	2.354	2.605	2.444	2.605	2.402	2.605	2.663	2.530
1976	19	17	19	32	24	21	35	35	2.190	2.171	2.156	2.085	2.206	2.240	2.153	2.153
1977	19	24	19	33	24	32	34	36	1.891	2.015	2.123	2.185	2.076	2.115	2.275	2.231
1978	17	23	19	33	21	31	34	36	1.977	2.016	2.142	2.202	2.060	2.182	2.310	2.357
1979	0	0	18	31	19	0	34	35	2.615	2.615	2.218	2.280	2.277	2.615	2.317	2.333
1980	0	0	24	0	18	0	33	15	3.050	3.050	2.968	3.050	3.046	3.050	3.025	3.342
1981	0	0	18	22	0	0	33	33	2.380	2.380	2.234	2.263	2.380	2.380	2.480	2.480
1982	0	0	17	20	0	17	32	26	1.995	1.995	2.390	2.459	1.995	2.255	2.795	2.881
1983	0	0	16	0	18	0	32	21	3.405	3.405	2.940	3.405	2.989	3.405	3.202	3.040
1984	0	0	16	23	17	18	31	35	2.675	2.675	2.441	2.452	2.458	2.432	2.521	2.466
1985	0	17	0	32	18	32	31	37	2.100	2.108	2.172	2.103	2.171	2.108	2.239	2.292
1986	0	34	0	37	25	37	51	40	1.420	1.408	1.332	1.481	1.377	1.504	1.338	1.463
1987	0	19	0	33	18	33	30	36	1.625	1.689	1.743	1.719	1.756	1.750	1.792	2.180
1988	0	0	0	0	18	0	30	22	2.680	2.680	2.391	2.680	2.412	2.680	2.415	2.539
1989	0	0	0	0	16	0	29	25	2.265	2.265	2.120	2.265	2.137	2.265	2.510	2.339
1990	0	0	0	0	17	0	27	22	2.190	2.190	2.158	2.190	2.189	2.190	2.360	2.308
1991	0	0	0	0	17	0	26	18	2.415	2.415	2.415	2.415	2.343	2.415	2.364	2.442
1992	0	0	0	19	18	24	25	34	2.055	2.055	2.055	1.818	1.851	1.945	2.040	1.954
1993	0	0	0	0	18	13	24	17	2.225	2.225	2.225	2.225	2.599	2.743	2.573	2.718
1994	0	0	0	16	17	19	24	30	1.950	1.950	1.950	2.028	2.066	2.065	2.186	2.214
1995	0	0	0	0	16	0	23	0	2.940	2.940	2.940	2.940	3.172	2.940	3.614	2.940
1996	0	0	0	0	16	0	23	21	2.900	2.900	2.900	2.900	2.441	2.900	2.655	2.658
1997	0	0	0	0	16	1	23	35	2.445	2.445	2.445	2.445	2.434	2.706	2.496	2.042
1998	0	16	0	23	16	34	23	37	1.810	1.796	1.810	1.812	1.890	1.682	1.956	1.779
1999	0	16	0	22	17	31	23	38	1.770	1.756	1.770	1.755	1.867	1.905	1.905	1.592
2000	0	13	0	22	16	35	23	38	1.625	1.882	1.625	1.759	1.829	1.451	1.844	1.530
2001	0	0	0	18	17	30	23	34	1.845	1.845	1.845	1.696	1.765	1.612	1.798	1.815
2002	0	0	0	0	18	0	23	19	2.455	2.455	2.455	2.455	2.173	2.455	2.184	2.226
2003	0	0	0	0	18	15	23	16	2.030	2.030	2.030	2.030	2.450	2.350	2.645	2.476
2004	0	0	0	18	18	23	23	34	1.760	1.760	1.760	1.625	1.687	1.805	1.854	1.833
2005	0	0	0	17	17	19	23	27	1.665	1.665	1.665	1.811	1.863	1.929	1.928	2.051
2006	0	0	0	0	17	0	21	9	2.405	2.405	2.405	2.405	3.519	2.405	3.862	3.606
32 year average									2.222	2.243	2.196	2.236	2.246	2.284	2.384	2.338

Table 4. Detailed Comparisons of Sales Dates and Net Returns for Soybean

Year	Sale Dates (Weeks from Harvest)								Per Bushel Net Returns ($/bu)							
	S1		S2		S3		S4		S1		S2		S3		S4	
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
1975	0	0	36	0	33	0	39	36	5.150	5.150	5.758	5.150	4.666	5.150	6.367	5.994
1976	0	0	29	0	29	0	29	15	5.950	5.950	9.532	5.950	9.368	5.950	9.861	6.883
1977	0	0	35	32	33	25	38	35	5.045	5.045	6.447	6.344	6.290	6.373	6.212	6.738
1978	0	0	34	19	0	0	38	36	6.170	6.170	6.475	6.675	6.170	6.170	7.660	6.931
1979	0	0	35	0	0	0	39	38	6.790	6.790	5.215	6.790	6.790	6.790	6.324	5.818
1980	0	0	0	0	0	0	37	0	7.510	7.510	7.510	7.510	7.510	7.510	6.493	7.510
1981	0	0	0	36	0	0	47	39	5.960	5.960	5.960	5.367	5.960	5.960	5.104	5.607
1982	0	0	0	35	0	0	41	39	5.000	5.000	5.000	5.233	5.000	5.000	5.720	5.512
1983	0	0	0	0	0	0	36	0	8.415	8.415	8.415	8.415	8.415	8.415	7.392	8.415
1984	0	0	0	36	0	0	51	39	5.770	5.770	5.770	4.969	5.770	5.770	4.477	5.101
1985	0	0	0	37	0	36	51	40	4.880	4.880	4.880	4.483	4.880	4.511	4.240	4.675
1986	0	0	0	35	0	35	46	39	4.795	4.795	4.795	4.657	4.795	4.723	4.647	4.898
1987	0	0	0	0	0	0	36	33	5.255	5.255	5.255	5.255	5.255	5.255	7.847	6.857
1988	0	0	0	0	0	0	35	0	7.885	7.885	7.885	7.885	7.885	7.885	6.392	7.885
1989	0	0	0	0	0	0	35	36	5.500	5.500	5.500	5.500	5.500	5.500	5.422	5.412
1990	0	0	0	0	0	0	35	36	6.010	6.010	6.010	6.010	6.010	6.010	5.258	5.171
1991	0	0	0	0	0	0	34	35	5.665	5.665	5.665	5.665	5.665	5.665	5.411	5.549
1992	0	0	0	0	0	0	34	37	5.170	5.170	5.170	5.170	5.170	5.170	5.413	5.292
1993	0	0	0	0	0	0	33	15	5.880	5.880	5.880	5.880	5.880	5.880	6.220	6.785
1994	0	0	0	0	0	0	33	36	5.220	5.220	5.220	5.220	5.220	5.220	5.141	5.177
1995	0	0	0	0	0	0	32	0	6.240	6.240	6.240	6.240	6.240	6.240	7.255	6.240
1996	0	0	0	0	0	0	0	0	7.245	7.245	7.245	7.245	7.245	7.245	7.245	7.245
1997	0	0	0	0	0	0	32	2	6.160	6.160	6.160	6.160	6.160	6.160	5.799	6.864
1998	0	0	0	0	0	37	51	41	4.915	4.915	4.915	4.915	4.915	3.699	3.871	3.568
1999	0	0	0	0	0	35	51	39	4.655	4.655	4.655	4.655	4.655	4.211	3.954	4.242
2000	0	0	0	0	0	0	45	39	4.725	4.725	4.725	4.725	4.725	4.725	4.351	4.043
2001	0	0	0	0	0	34	39	38	4.260	4.260	4.260	4.260	4.260	4.212	4.781	4.562
2002	0	0	0	0	0	0	32	0	5.180	5.180	5.180	5.180	5.180	5.180	5.591	5.180
2003	0	0	0	0	0	0	0	0	6.770	6.770	6.770	6.770	6.770	6.770	6.770	6.770
2004	0	0	0	0	0	0	32	22	4.975	4.975	4.975	4.975	4.975	4.975	5.581	5.653
2005	0	0	0	0	0	0	32	35	5.240	5.240	5.240	5.240	5.240	5.240	5.251	4.998
2006	0	0	0	0	0	0	32	16	5.265	5.265	5.265	5.265	5.265	5.265	6.310	6.438
32 year average									5.739	5.739	5.874	5.742	5.870	5.713	5.886	5.875

Table 5. Detailed of Comparisons Sales Dates and Net Returns for Wheat

Year	Sale Dates (Weeks from Harvest)								Per Bushel Net Returns ($/bu)							
	S1		S2		S3		S4		S1		S2		S3		S4	
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
1975	0	0	23	0	23	0	27	25	2.910	2.910	3.025	2.910	3.033	2.910	3.125	3.156
1976	0	0	24	0	23	0	51	32	3.360	3.360	2.153	3.360	2.147	3.360	1.803	2.256
1977	0	22	22	24	23	26	44	31	1.920	2.099	2.178	2.347	2.261	2.308	2.650	2.337
1978	0	0	0	0	0	0	25	27	2.900	2.900	2.900	2.900	2.900	2.900	2.976	2.951
1979	0	0	0	0	0	0	25	23	3.400	3.400	3.400	3.400	3.400	3.400	3.858	3.888
1980	0	0	0	0	0	0	25	21	3.400	3.400	3.400	3.400	3.400	3.400	4.158	4.214
1981	0	0	0	0	0	0	25	0	3.830	3.830	3.830	3.830	3.830	3.830	3.832	3.830
1982	0	0	0	0	0	0	25	25	3.440	3.440	3.440	3.440	3.440	3.440	3.255	3.255
1983	0	0	0	0	0	0	24	26	3.390	3.390	3.390	3.390	3.390	3.390	3.120	3.091
1984	0	0	0	0	0	0	24	26	3.340	3.340	3.340	3.340	3.340	3.340	3.213	3.155
1985	0	0	0	0	0	0	25	27	2.890	2.890	2.890	2.890	2.890	2.890	2.694	2.814
1986	0	0	0	0	0	25	48	31	2.230	2.230	2.230	2.230	2.230	2.015	2.254	2.015
1987	0	0	0	0	0	0	25	26	2.320	2.320	2.320	2.320	2.320	2.320	2.388	2.491
1988	0	0	0	0	0	0	24	22	3.050	3.050	3.050	3.050	3.050	3.050	3.445	3.495
1989	0	0	0	0	0	0	24	0	3.770	3.770	3.770	3.770	3.770	3.770	3.538	3.770
1990	0	0	0	0	0	0	26	29	2.940	2.940	2.940	2.940	2.940	2.940	2.214	2.120
1991	0	0	0	0	0	0	24	23	2.520	2.520	2.520	2.520	2.520	2.520	3.176	3.065
1992	0	0	0	0	0	0	24	0	3.480	3.480	3.480	3.480	3.480	3.480	3.092	3.480
1993	0	0	0	0	0	22	24	24	2.630	2.630	2.630	2.630	2.630	2.792	3.120	3.120
1994	0	0	0	0	0	0	24	22	3.100	3.100	3.100	3.100	3.100	3.100	3.445	3.598
1995	0	0	0	0	0	0	24	0	3.910	3.910	3.910	3.910	3.910	3.910	4.494	3.910
1996	0	0	0	0	0	0	24	0	5.370	5.370	5.370	5.370	5.370	5.370	4.012	5.370
1997	0	0	0	0	0	0	24	27	3.730	3.730	3.730	3.730	3.730	3.730	2.990	2.971
1998	0	0	0	0	0	23	51	30	2.700	2.700	2.700	2.700	2.700	2.526	1.917	2.441
1999	0	0	0	0	0	28	50	33	2.360	2.360	2.360	2.360	2.360	1.827	2.017	2.056
2000	0	0	0	0	0	22	25	27	2.400	2.400	2.400	2.400	2.400	2.421	2.731	2.584
2001	0	0	0	0	0	0	24	24	2.880	2.880	2.880	2.880	2.880	2.880	2.400	2.501
2002	0	0	0	0	0	0	19	0	2.850	2.850	2.850	2.850	2.850	2.850	4.322	2.850
2003	0	0	0	0	0	0	23	0	2.830	2.830	2.830	2.830	2.830	2.830	3.402	2.830
2004	0	0	0	0	0	0	23	0	3.500	3.500	3.500	3.500	3.500	3.500	3.117	3.500
2005	0	0	0	0	0	0	23	22	3.030	3.030	3.030	3.030	3.030	3.030	3.145	3.083
2006	0	0	0	0	0	0	23	0	4.540	4.540	4.540	4.540	4.540	4.540	4.506	4.540
32 year average									3.154	3.159	3.128	3.167	3.130	3.143	3.138	3.148

Table 6. Results of Paired Differences Test for M1 and M2, *t*-Ratio (1975-2006)

	S1	S2	S3	S4
Corn	1.82	1.79	0.76	1.27
Soybean	N/A	1.02	1.37	0.08
Wheat	1.00	1.03	0.29	0.13

Note: *t*-critical value with 30 degree of freedom at 5% significance level is 2.042.

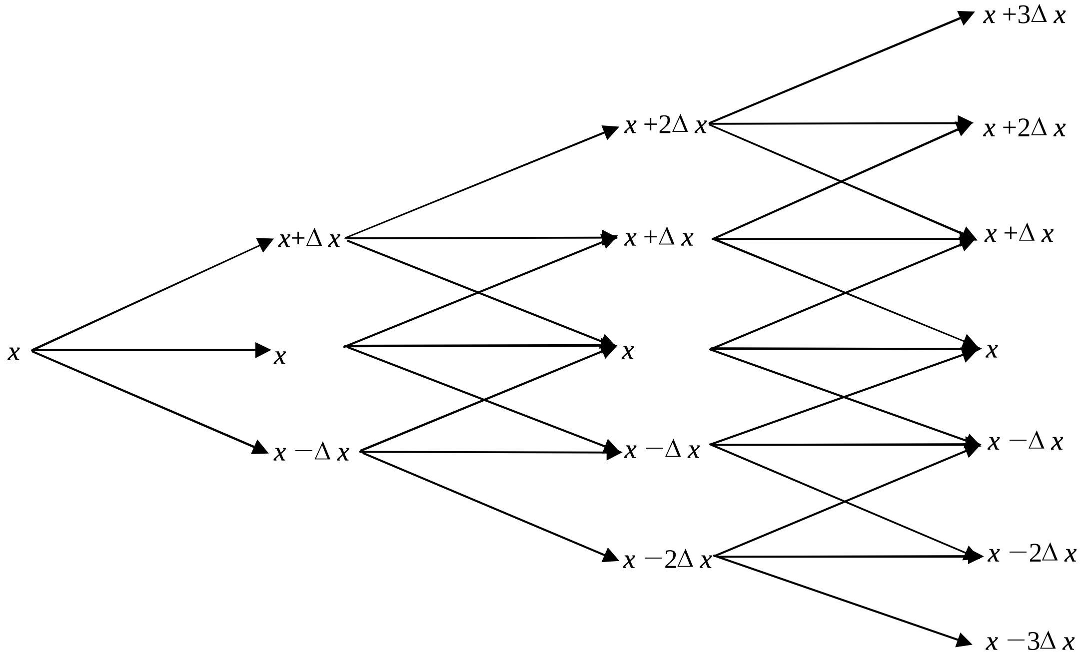

Figure 1. An Example of Trinomial Lattice

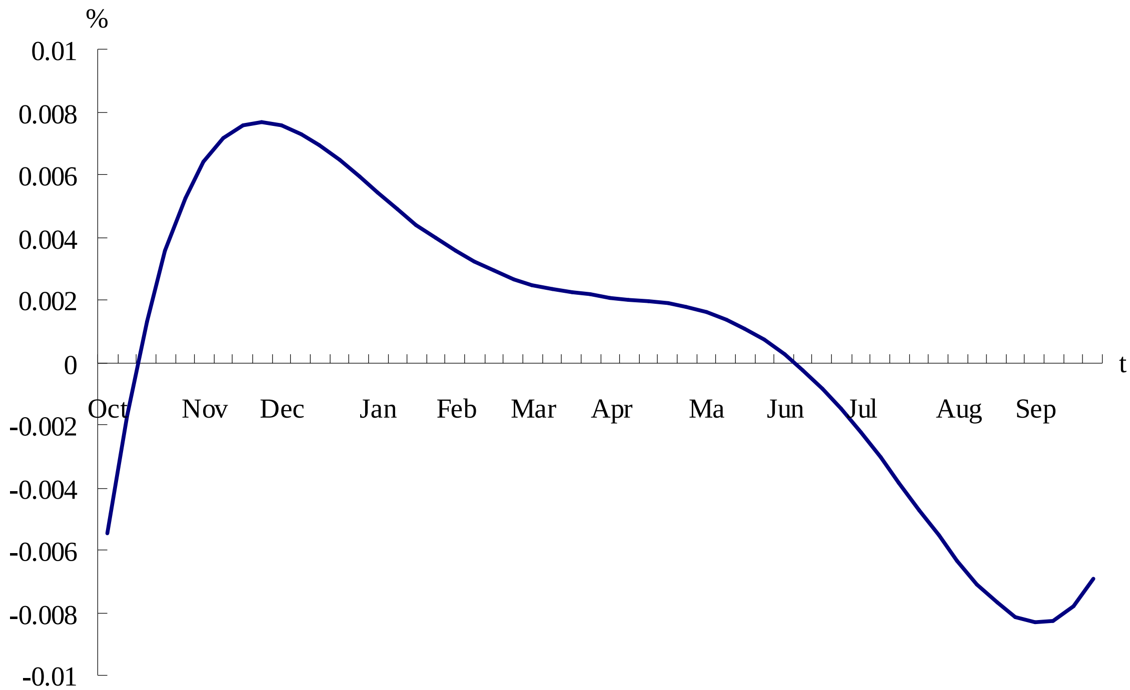

Figure 2. Seasonality of Change in Price for Corn

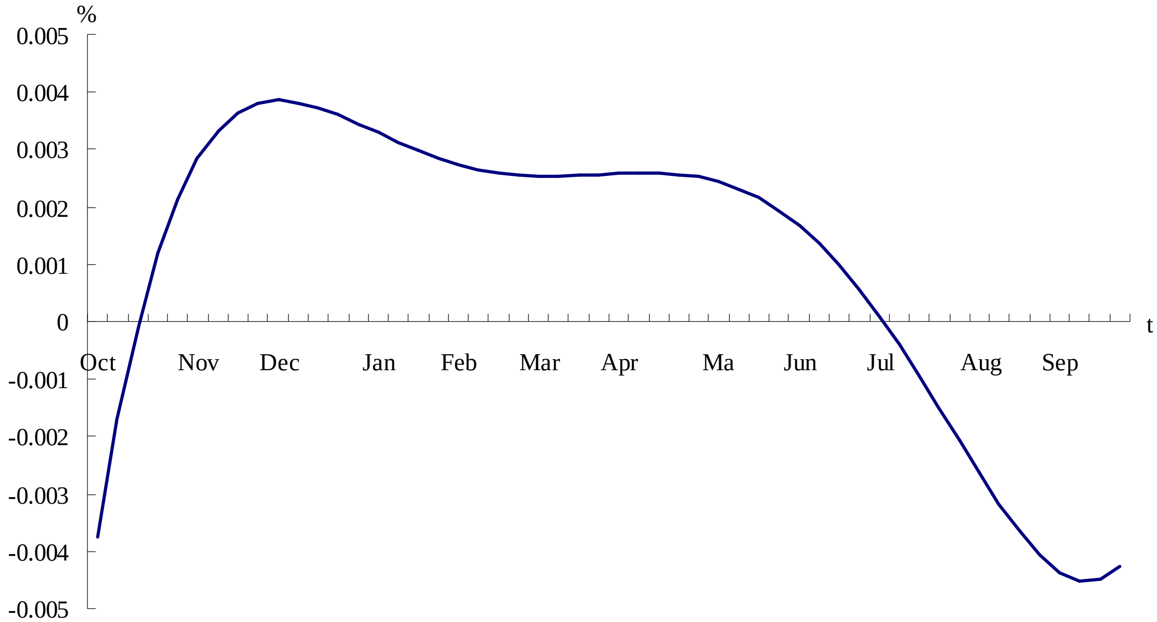

Figure 3. Seasonality of Change in Price for Soybean

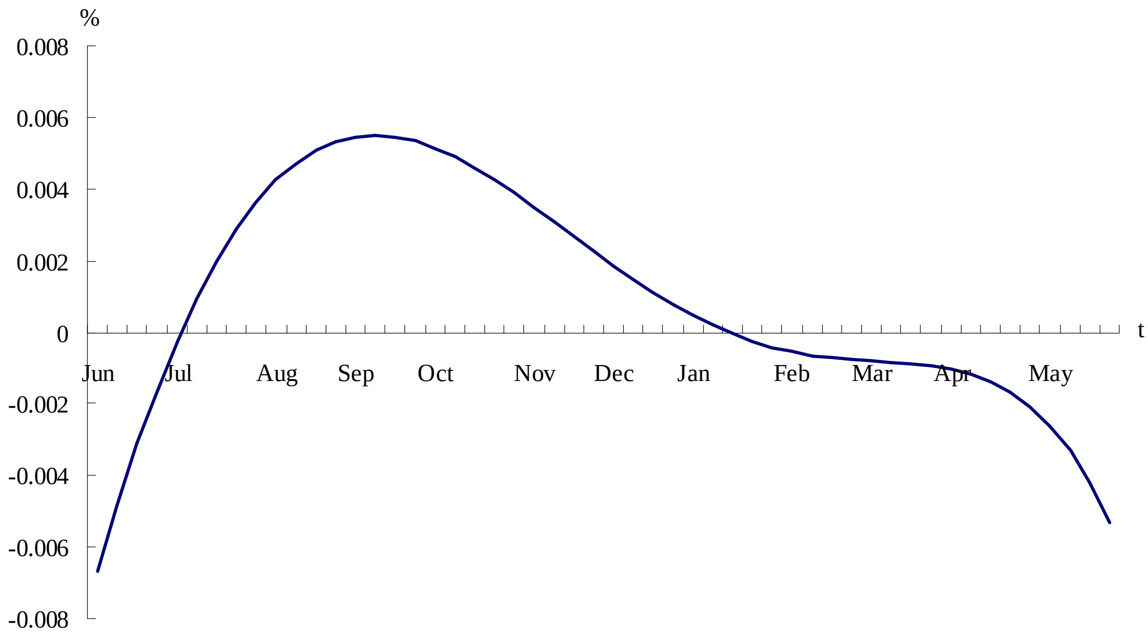

Figure 4. Seasonality of Change in Price for Wheat

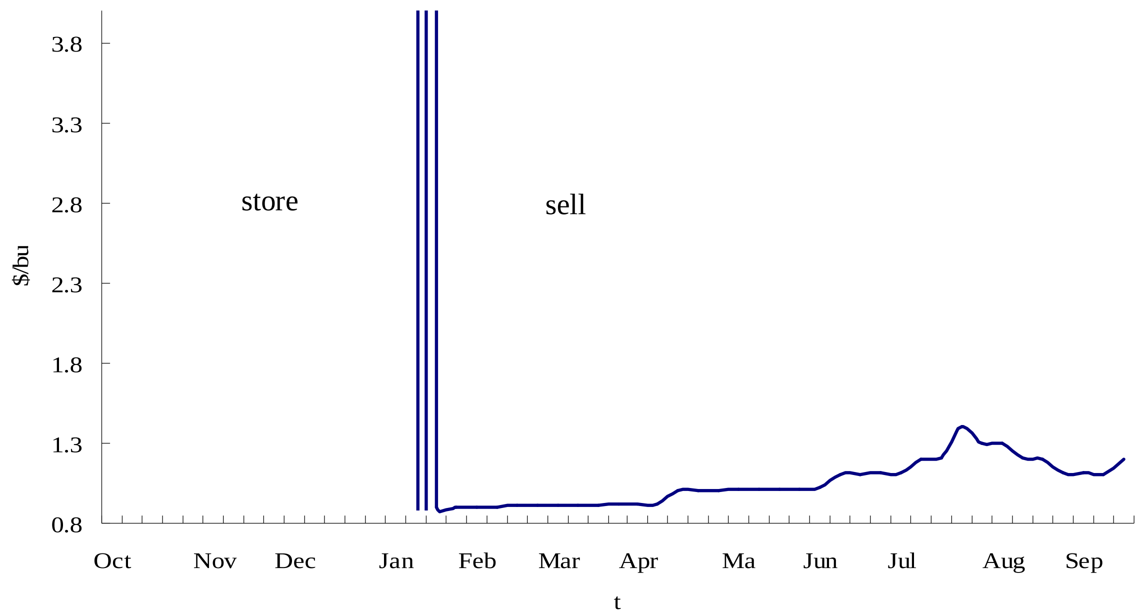

Figure 5. Cutoff Price of M1 for Corn

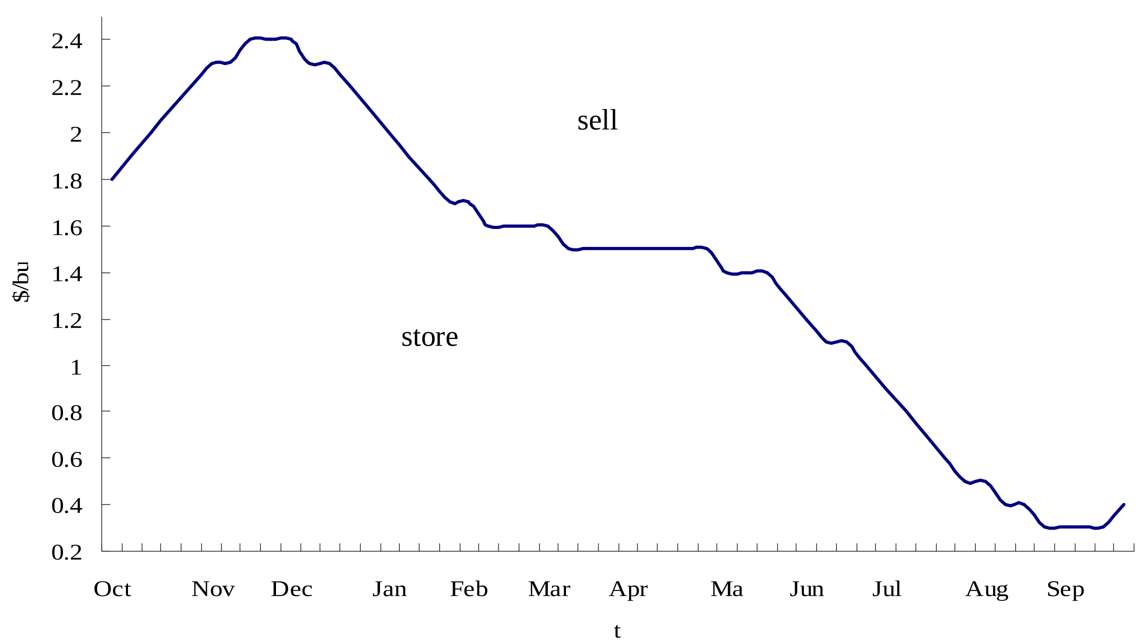

Figure 6. Cutoff Price of M2 for Corn

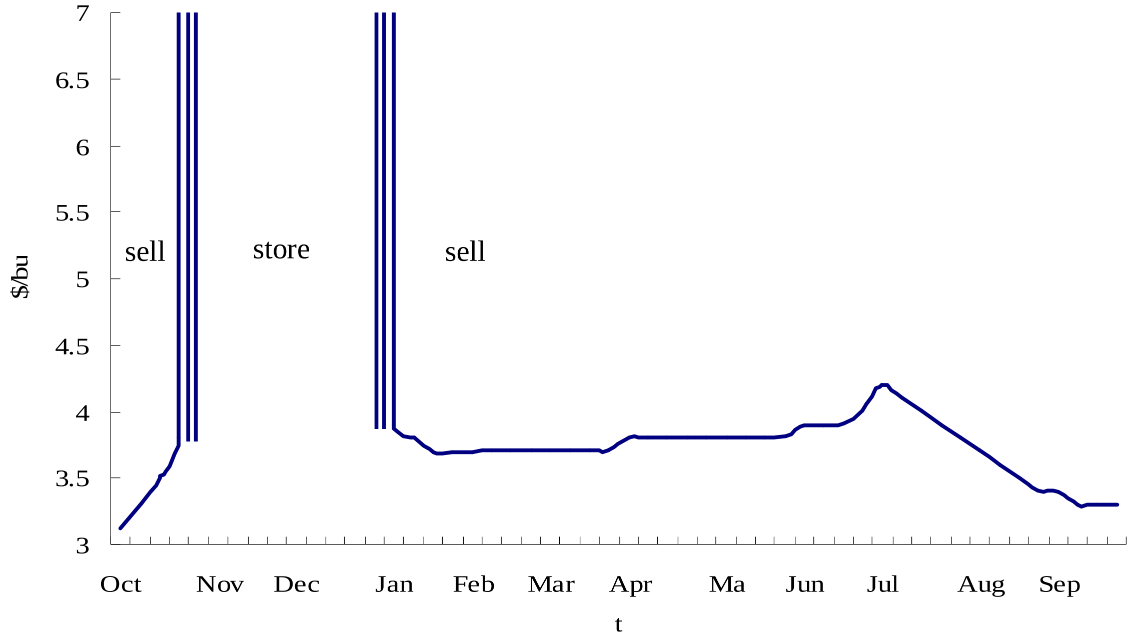

Figure 7. Cutoff Price of M1 for Soybeans

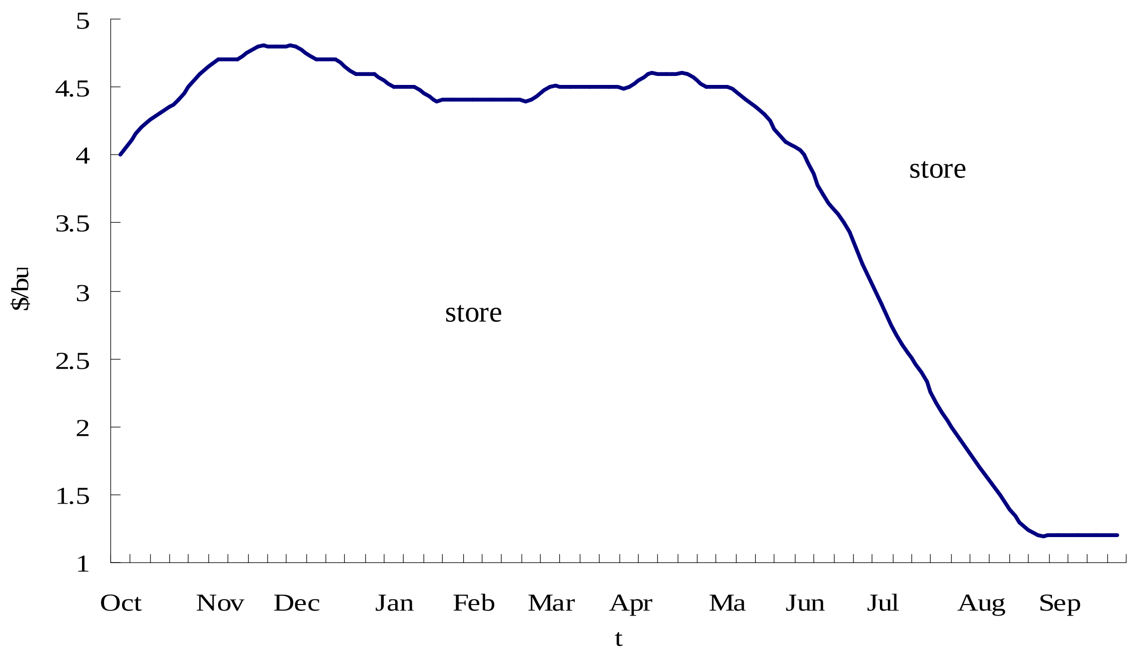

Figure 8. Cutoff Price of M2 for Soybeans

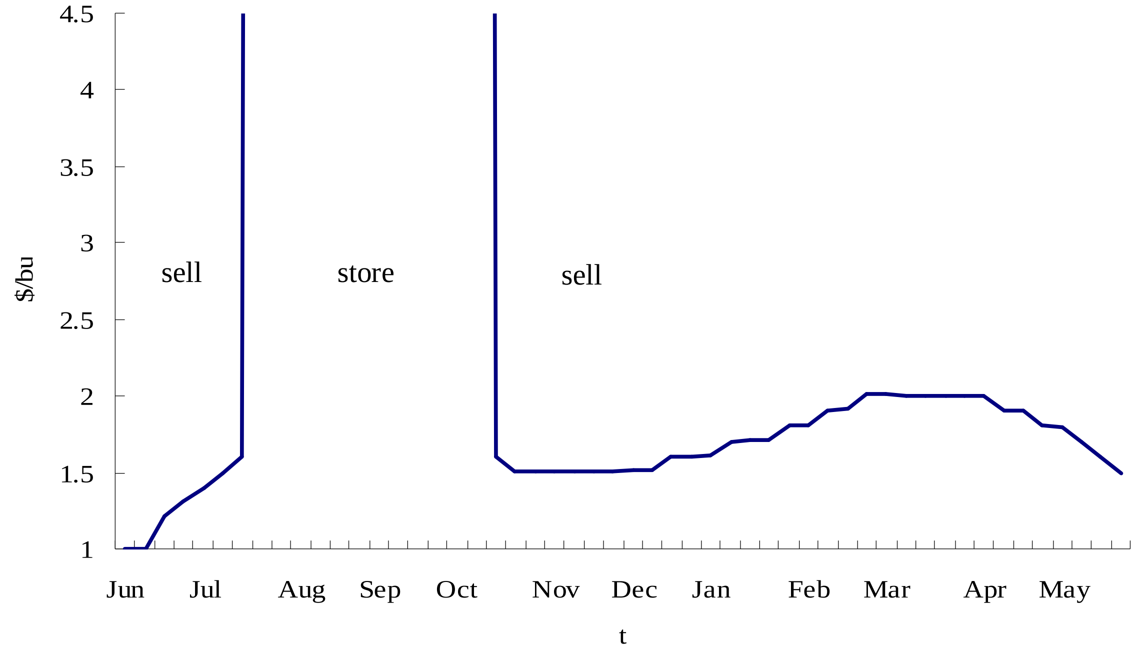

Figure 9. Cutoff Price of M1 for Wheat

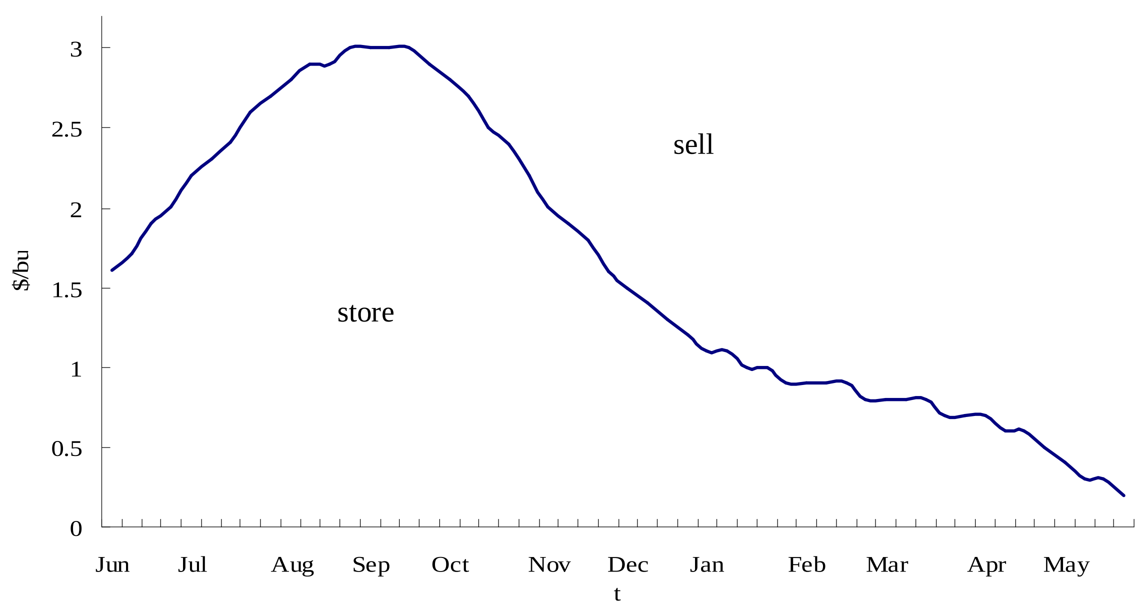

Figure 10. Cutoff Price of M2 for Wheat

References

Anderson, R.W. 1985. "Some Determinants of the Volatility of Futures Prices." *The Journal Futures Markets* 5:331–48.

Anderson, R.W., and J.-P. Danthine. 1983. "The Time Pattern of Hedging and the Volatility of Futures Prices." *The Review of Economic Studies* 50:249–66.

Anderson, K.B., and B.W. Brorsen. 2005. "Marketing Performance of Oklahoma Farmers." *American Journal of Agricultural Economics* 87:1265–71.

Bessembinder, H., J.F. Coughenour, P.J. Seguin, and M.M. Smoller. 1995. "Mean Reversion in Equilibrium Asset Prices: Evidence from the Futures Term Structure." *The Journal of Finance* 50:361–75.

Bessler, D.A., and T. Covey. 1991. "Cointegration: Some Results on U.S. Cattle Prices." *The Journal of Finances Markets* 11:461–74.

Brennan, M.J. 1991. "The Price of Convenience and the Valuation of Commodity Contingent Claims." *Stochastic Models and Option Values.* D. Lund and B. Oksendal, eds., pp. 33–71. New York: North-Holland Publishing Co.

Brennan, M.J., and E.S. Schwartz. 1985. "Evaluating Natural Resource Investments." *The Journal of Business* 58:135–57.

Cartea, A., and M.G. Figueroa. 2005. "Pricing in Electricity Markets: A Mean Reverting Jump Diffusion Model with Seasonality." *Applied Mathematical Finance* 12:313–35.

Chen, R.-R., and T.T. Yang. 1999. "A Universal Lattice." *Review of Derivatives Research* 3:115–33.

Cox, D.R., and H.D. Miller. 1965. *The Theory of Stochastic Processes.* New York: John Wiley and Sons.

Dixit, A.K., and R.S. Pindyck. 1994. *Investment Under Uncertainty*. Princeton, NJ: Princeton University Press.

Fackler, P.L., and M.J. Livingston. 2002. "Optimal Storage by Crop Producers." *American Journal of Agricultural Economics* 84:645–59.

Fama, E.F., and K.R. French. 1987. "Commodity Futures Prices: Some Evidence on Forecast Power, Premiums, and the Theory of Storage." *The Journal of Business* 60:55–73.

Hagedorn, L.A., S.H. Irwin, D.L. Good, and E.V. Colino. 2005. "Does the Performance of Illinois Corn and Soybean Farmers Lag the Market?" *American Journal of Agricultural Economics* 87: 1271–79.

Hull, J., and A. White. 1990. "Valuing Derivative Securities Using the Explicit Finite Difference Method." *Journal of Financial and Quantitative Analysis* 25:87–100.

Hull, J., and A. White. 1993. "One-Factor Interest-Rate Models and the Valuation of Interest-Rate Derivative Securities." *Journal of Financial and Quantitative Analysis* 28:235–54.

Kansas City Federal Reserve. 2008. "Bank Prime Loan." Available at http://www.federalreserve.gov/releases/h15/data/Monthly/H15_PRIME_NA.txt

Kenyon, D., K. Kling, J. Jordan, W. Seale, and N. McCabe. "Factors Affecting Agricultural Futures Price Variance." *The Journal of Futures Markets* 7:73–92.

Lence, S., K. Kimle, and M.L. Hayenga. 1993. "A Dynamic Minimum Variance Hedge." *American Journal of Agricultural Economics* 75:1063–71.

McDonald, R.L., and D.R. Siegel. 1986. "The Value of Waiting to Invest." *The Quarterly Journal of Economics* 101:707–27.

Schroeder, T.C., and B.K. Goodwin. 1991. "Price Discovery and Cointegration for Live Hogs." *The Journal of Futures Markets* 11:685–696.

Irwin, S.H., D.L. Good, and J.Martines-Filho and R.M. Batts. 2006. "The Pricing Performance of Market Advisory Services in Corn and Soybeans Over 1995-2004." AgMAS Project Research Report 2006-02, Department of Agricultural and Consumer Economics, University of Illinois at Urbana-Champaign. Available at http://www.farmdoc.uiuc.edu/agmas/reports/06_02/AgMAS06_02.pdf

Streeter, D.H., and W.G. Tomek. 1992. "Variability in Soybean Futures Prices: An Integrated Framework." *The Journal of Futures Markets* 12:705–30.

Tronstad, R., and C.R. Taylor. 1991. "Dynamically Optimal After-Tax Grain Storage, Cash Grain Sale, and Hedging Strategies." *American Journal of Agricultural Economics* 73:75–88.

U.S. Department of Agriculture (USDA), Agricultural Marketing Service (AMS). Available at http://www.ams.usda.gov/AMSv1.0/

U.S. Department of Agriculture (USDA), National Agricultural Statistic Service (NASS). Available at http://www.agstats.states.il.us/website/reports.htm

U.S. Department of Agriculture (USDA), National Agricultural Statistic Service (NASS). Available at http://www.nass.usda.gov/QuickStats/Create_Federal_All.jsp

Yoon, B.-S., and B.W. Brorsen. 2005. "Can Multiyear Rollover Hedging Increase Mean Returns?" *Journal of Agricultural and Applied Economics* 37:65−78.