Relaxing Standard Hedging Assumptions in the Presence of Downside Risk

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Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management

St. Louis, Missouri, April 18-19, 2005

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Relaxing standard hedging assumptions in a downside risk framework

The purpose of this study is to analyze how the introduction of a downside risk measure and less restrictive assumptions can change the optimal hedge ratio in the standard hedging problem. Based on a dataset of futures and cash prices for soybeans in the U.S., the empirical findings indicate that optimal hedge ratios change dramatically when a one-sided risk measure is adopted and standard assumptions are relaxed. Further, the results suggest that in a downside risk framework with realistic hedging assumptions there is little or no incentive for farmers to hedge.

Keywords: downside risk, hedging, futures markets

INTRODUCTION

Hedging models traditionally adopt variance as a measure of risk, and minimum-variance hedge ratios are calculated for agents to follow. While the minimum-variance hedge ratios are tractable and easy to estimate, the underlying assumptions may not be consistent with hedgers' observed behavior, which may explain why estimated and observed hedge ratios differ (Peck and Nahmias, 1989; Collins, 1997).

One problem with the traditional hedge ratios is that the measure of risk implies that agents consider positive and negative deviations from the expected return as equally undesirable events. However, risk is frequently perceived by agents as a failure to achieve a certain level of return (Unser, 2000). In this context, downside risk measures, which assume that returns below a certain target involve risk and returns above this target represent better investment opportunities, can be highly relevant (Grootveld and Hallerbach, 1999). In recent survey papers, Lien and Tse (2002) argued that a one-sided measure is more relevant in a hedging context than the traditional two-sided measure represented by the variance, and Chen, Lee and Shrestha (2003) emphasized that one-sided risk measures like the semivariance are consistent with the risk perceived by individuals.

Another problem with standard hedging models is the restrictive set of assumptions about agents' behavior. Chen, Lee and Shrestha (2003) point out that almost all work to calculate hedge ratios fails to incorporate transaction costs and the possibility of investments in assets other than cash and futures positions. In general, minimum-variance hedge ratios are calculated assuming that borrowing, lending and investing in alternative activities are not allowed, there are neither initial margin deposits nor brokerage fees in futures trading, and production is deterministic. But these assumptions are excessively restrictive and may not reflect agent's situations. By definition, there are margin deposit requirements and brokerage fees in futures markets. Further, opportunities to borrow and lend, as well as to invest in alternative investments, are at times available to hedgers. Lence (1996) conducted a study in which several assumptions were relaxed and his results showed that hedge ratios can change dramatically under more realistic assumptions. But as Chen, Lee and Shrestha (2003) emphasize Lence's findings are based on a specific utility function, a given set of return distributions, and it remains to be seen if the findings hold for downside risk hedge ratios.

In this paper, we analyze how estimated hedge ratios and the opportunity cost of hedging are affected when risk is measured in a downside framework, and the behavioral assumptions of the standard model are relaxed by allowing alternative investments and the introduction of brokerage fees. Hedge ratios are calculated under utility maximization based on a constant relative risk aversion (CRRA) utility function which allows the absolute level of risk aversion to change with wealth. A data set of U.S. soybean futures and cash returns, and the S&P500 returns between 1990 and 2004 are used in the analysis.

REVIEW OF LITERATURE

The notion of one-sided risk has been discussed since the early 50's, and has evolved into several downside-risk measures. Recently, the lower partial moment (LPM) is the measure that has been mostly used in the literature. The lower partial moment of order a with target d is defined as:

$$LPM_a(R;d) = \int_{-\infty}^{d} (R-d)^a \, dG(R), \tag{1}$$

where R is the investment return, d is the target return, and *G(R)* is the cumulative distribution function of *R*. The parameter a reflects the order of the partial moment, and can be seen as a measure of risk aversion. A value of $a < 1$ implies a risk-preference behavior, while $a > 1$ imply a risk-aversion behavior (Grootweld and Hallerbach, 1999). For $a > 1$, higher values of a indicate that the agent is more concerned with the magnitude of the deviation below the target, whereas small values of a indicate that the agent is not particularly interested in the amount of loss incurred by the deviation below the target (Fishburn, 1977).

Several risk measures commonly adopted are special cases of the LPM. For $a = 0$, the term in parenthesis in expression (1) becomes 1, and the measure is the probability of falling below the target. If the target is set to zero ($d = 0$), then the measure is just the probability of loss. When $a = 1$, the lower partial moment represents the expected deviation of returns below the target. For $a = 2$, the measure is similar to the variance, but with deviations computed only for observations below the target return. If the target is set to the mean return, then the lower partial moment of order two (LPM_2) is the semi-variance. Moreover, if the target is set to the mean return and returns are symmetrically distributed, the LPM_2 is proportional to the variance, i.e., both risk measures would lead to the same ordering of risky assets (Eftekhari, 1998).

In contrast to the variance, the LPM offers flexibility in modeling risk behavior. While the variance as a measure of risk imposes that any deviation from the expected return is considered an undesirable event, the LPM assumes that only deviations below a certain target is taken as risk, and this target can be the expected return or any other one defined by the hedger. Moreover, the LPM allows for different levels of risk aversion, while this is not an explicit issue with the variance.

Although the idea of downside risk has been identified for some time, not many studies have been performed calculating hedge ratios in a mean-downside risk framework, and even less using agricultural commodities as the asset to be hedged. Still, many studies have been developed using the concept of downside risk in pricing models and in the estimation of minimum-downside risk hedge ratios. Since there is no analytic solution for the minimum-downside risk hedge ratio, various researchers have adopted different methods to calculate this ratio.

Several studies have focused on calculating optimal hedge ratios by minimizing downside risk and comparing them to minimum-variance hedge ratios. Eftekhari (1998) minimizes the lower partial moment of order two (LPM_2) with target set to zero to calculate the optimal hedge ratio for the FTSE-100 stock index from 1985 to 1994. Using continuously compounded returns on spot and nearby futures prices, he adopted two hedging horizons (one- and two-week) and a dynamic strategy based on rolling windows. The general result is that minimum-LPM hedge ratios are slightly smaller and tend to yield a better risk/return combination than the minimum-variance hedge ratios. In terms of hedging effectiveness, the LPM approach usually led to somewhat smaller risk than in the variance approach.[1] Similarly, Lien and Tse (2000) calculated the minimum-LPM and the minimum-variance hedge ratios for the Nikkei Stock Average index over 1-week hedging horizons from January 1988 to August 1996. Three orders of the LPM were used (1, 2, and 3), and the target returns ranged from –1.5% to +1.5%. In general the optimal hedge ratio increased as the order of the LPM increased, as well as when the target return increased. Their findings suggest that the minimum-LPM and the minimum-variance hedge ratios may differ sharply, particularly when the hedger is willingly to absorb small losses and very cautious about large losses, i.e., when the target return is small and the order of the LPM is large. Turvey and Nayak (2003) calculated minimum-semivariance[2] hedge ratios for Kansas wheat hedged on the Chicago Board of Trade wheat futures contract, and Texas steers hedged on the Chicago Mercantile Exchange live cattle futures contract using several targets. Daily price observations were used for wheat (1980-2000) and for steers/live cattle (1989-2000). Their results were consistent with previous studies in the sense that minimum-semivariance hedge ratios were usually smaller than the minimum-variance hedge ratios, but the difference between the two ratios varied depending on the target and the distribution of risk. Moreover, the minimum-semivariance hedge was found to offer a better protection against downside risk than the minimum-variance hedge.

A different approach was followed by Chen, Lee and Shrestha (2001), who adopted a mean-downside risk framework to estimate optimal hedge ratios. They argued that hedge ratios obtained by simple minimization of the generalized semivariance (GSV) [3] might not be consistent with the concept of stochastic dominance, since they are usually dependent on the target return. Consequently, they argued that these hedge ratios should be calculated using utility maximization in a mean-risk framework. Using an empirical distribution-based technique as the estimation procedure, the authors calculated the mean-GSV hedge ratios for the S&P500 index with two targets (zero, and the sample average of the S&P500 spot price changes) and a range of values for the parameter α from 1.25 to 60, and compared them with the minimum-GSV hedge ratio.[4] Their results showed that, as the order of the GSV increased, both the mean-GSV and the minimum-GSV hedge ratios tended to become smaller and converge to a level close to 0.7 under both targets. Further, the mean-GSV hedge ratios were usually smaller than the minimum-GSV hedge ratios, and showed less variability for lower levels of risk.

Another area of recent investigation has focused on identifying the implications of relaxing

1 These results do not necessarily hold when the sample size is small and when hedges are adjusted frequently.

2 Their definition of semivariance is basically the second-order lower partial moment defined in equation (1).

3 They adopt the same definition of the lower partial moment for the GSV.

4 Four other models were examined: minimum variance hedge ratio, mean-extended-Gini hedge ratio, Sharpe ratio-based hedge ratio, and mean-variance hedge ratio

assumptions of the traditional mean-variance hedging model. Based on an expected utility maximization framework, Lence (1996) incorporated the possibility of lending, borrowing, and investing in alternative investments. Transaction costs in futures markets (initial margins and brokerage fees) and stochastic production were also included. Using a CARA utility function and calibrating the model for grain storage, Lence considered three levels of risk aversion and three hedging horizons in the simulations. His results showed that the maximum-expected-utility hedge ratios obtained from relaxing the conventional assumptions can differ substantially from the standard minimum-variance hedge ratios, and in some cases optimal hedges were close to zero. His findings suggested that the mean-variance hedge ratios can be far from optimum in the presence of alternative investments and stochastic production. Optimal hedge ratios were also found to be very sensitive to transaction costs.

In the present study, we combine the notion of downside risk and relaxation of the standard assumptions. A downside risk framework is adopted with low target returns, and several of the standard hedging assumptions are relaxed. The research method and the data are discussed in the next sections.

RESEARCH METHOD

The analysis is based on a risk-averse farmer, who takes a short position in the futures market to hedge stored soybeans. The farmer is assumed to maximize the expected utility of final wealth $W_1 = W_0 \cdot r_h$, where W_1 and W_0 are final and initial wealth, respectively, and r_h is the return from the farmer's hedged portfolio. Two standard hedging assumptions are relaxed as brokerage fees are introduced and an alternative investment is allowed. These two assumptions are relaxed one at a time and then together, which yields the four different models in equations (2) through (5).

$$r_h = r_c + (1 - r_f) \cdot h \tag{2}$$

$$r_h = r_c + (1 - r_f - b) \cdot h \tag{3}$$

$$r_h = (1 - s_A) \cdot [r_c + (1 - r_f) \cdot h] + s_A \cdot r_A \tag{4}$$

$$r_h = (1 - s_A) \cdot [r_c + (1 - r_f - b) \cdot h] + s_A \cdot r_A \tag{5}$$

where r_c is the return on the cash position[5], r_f is the return on the futures position, r_A is the return on the alternative investment, s_A is the share of the farmer's wealth invested in the alternative investment, h is the hedge ratio, and b is the brokerage fee as a proportion of the initial futures price.[6]

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[5] $r_c = \frac{C_1}{C_0}$, where C_0 and C_1 are the respective cash prices at the beginning and at the end of the hedging period. The returns on the futures position and on the alternative investment are calculated in the same way.

[6] $b = \frac{B}{F_0}$, where B is the brokerage fees in US$/contract and F_0 is the futures price at the beginning of the hedging period, such that $1 - r_f - b = 1 - \frac{F_1 + B}{F_0}$.

Equation (2) represents the standard hedging model. The first assumption to be relaxed is the absence of transaction costs (equation 3). Five levels of brokerage fees (0.0005, 0.001, 0.00125, 0.0025, and 0.005) are introduced in the model, which are taken as a proportion of the initial futures prices. Brokerage fees have declined during the period of this study. In 1990 brokerage fees would commonly be between 0.00125 and 0.005, while by 2004 these values would be between 0.0005 and 0.00125.[7] The second assumption relaxed allows an alternative investment (equation 4), which means that part of the farmer's wealth can be invested in assets other than soybeans. The returns on the S&P500 index are used to reflect returns on alternative investments available to the farmer. Three values for the share of farmer's wealth invested in other assets are assumed: 0.10, 0.25, and 0.50. Finally, the fourth model relaxes the two assumptions simultaneously (equation 5); five levels of brokerage fees and three investment scenarios are used in this case.

The optimal hedge ratio is calculated assuming utility-maximization of the farmer's final wealth. Since the joint distribution of cash, futures, and S&P500 returns is elliptically symmetric, and final wealth satisfies the location-scale condition, expected utility can be written as a function of the first two moments of the return distribution (Chamberlain, 1983; Meyer, 1987). A constant relative risk aversion (CRRA) location-scale objective function is used (Nelson and Escalante, 2004):

$$E[U(W_1)] = V(m_h, LPM) = -\frac{1}{m_h^2 - g \cdot LPM_2(r_h, d)} \tag{6}$$

where m_h is the mean return on the hedged portfolio, ã is the coefficient of relative risk aversion, and $LPM_2(r_h; d)$ is the second-order lower partial moment of the portfolio return r_h with target d. The CRRA utility function is consistent with agents' observed behavior since it exhibits constant relative risk aversion and decreasing absolute prudence. Unlike the constant absolute risk aversion utility function, the CRRA utility function also exhibits risk vulnerability which Gollier and Pratt (1996) argue is a "natural" restriction of utility functions. Risk vulnerability means that the addition of an unfair background risk to initial wealth causes risk-averse decision makers to become more risk averse toward any other independent risk. In a price hedging context, this implies that an increase in revenue variability caused by stochastic production should increase the optimal hedge. The coefficient of relative risk aversion ã is specified to be 3 which is slightly more risk averse than average estimates of farmer risk preferences.[8] Nelson and Escalante (2004) found coefficients of relative risk aversion derived from historical financial attributes of Illinois farms to range from 0.27 to 4.95. The order two of the lower partial moment is chosen because it is most comparable to the traditional measure of variance. The targets are arbitrarily set at five levels: zero and four percentiles of the return distribution: 50^{th}, 25^{th}, 10^{th}, and 5^{th}. A target equal to zero means that the hedger is only concerned with negative returns, while targets set to lower percentiles imply the hedger is mainly concerned with extreme losses.

The estimation of the optimal hedge ratio in the presence of downside risk follows Eftekhari (1998). First the hedge ratio is set to $h = 0$, and the values of expected return, lower

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[7] A hedger is assumed to pay between US$15 and US$25 per contract in brokerage fees currently which is about half the fees that existed in 1990.

[8] Qualitatively similar findings were found for simulations using relative risk aversion ranging from 1 to 5.

partial moment and expected utility are calculated. Then the hedge ratio is increased by a small fraction and these values are calculated again for the new hedge ratio. This process is repeated until the hedge ratio reaches a large enough number, which is arbitrarily set to $h = 1.50$. The value of h which yields the highest expected utility is considered the optimal hedge ratio. Mean-variance hedge ratios are estimated by this method using the CRRA utility function presented in equation (6) where the variance of the portfolio return s_h^2 is used as the second moment of the distribution. Finally, a minimum-variance hedge ratio is used for comparison, and is obtained by dividing the covariance between cash and futures returns by the variance of futures returns:

$$h = \frac{Cov\left(R_c, R_f\right)}{Var\left(R_f\right)}. \tag{7}$$

Opportunity costs of placing sub-optimal hedge ratios are also calculated. These costs represent the minimum return required by the hedger to accept placing a sub-optimal hedge, and can be estimated as follows:

$$E\left[U\left(R_h^{opt}\right)\right] = E\left[U\left(R_h^{sub} + OC\right)\right] \tag{8}$$

where R_h^{opt} is the return provided by the optimal hedge ratio, R_h^{sub} is the return provided by the sub-optimal hedge ratio, and OC is the opportunity cost.

DATA

The empirical simulations are conducted using futures and cash prices of U.S. soybeans, and quotes of the S&P500 index from January 1990 through June 2004. Three hedging horizons are adopted: 4, 12, and 24 weeks. The soybean prices and the S&P500 quotes were obtained from the Commodity Research Bureau (CRB), and correspond to midweek (Wednesday) closing prices. The cash prices refer to soybeans in Central Illinois. The futures prices refer to the contracts traded at the Chicago Board of Trade (CBOT). Contract months are January, March, May, July, August, September and November, and the nearby futures contract that corresponds to the length of the hedging horizon was used. The selected contract permits the hedger to maintain the position without having to roll over to a new contract. For example, if the agent with a 12-week hedging horizon placed a hedge on September 8, 1993, the date to lift the hedge would be February 23, 1994 and so the March contract is used to place the hedge. Following this procedure, the hedger avoids potential risk in rolling the hedge forward at the expiration of the November contract.

RESULTS

The discussion of the results focuses on the 4-week horizon. Results for the 12- and 24-week horizons are qualitatively similar and are not presented for brevity. Summary statistics for futures, cash, and S&P500 returns are presented in Table 1. It is assumed that futures markets are unbiased, i.e., the empirical distribution of the futures returns were adjusted such that $E\left(r_f\right) = 0$. All empirical distributions are somewhat leptokurtic. However, Jarque-Bera fails to reject normality for the cash and S&P500 returns. The p-value of the test statistic for futures returns is

0.079, which is not strong evidence against normality. A normal Q-Q plot of the futures returns data reveals that the distribution is fat tailed.

Table 1. Summary Statistics for Futures, Cash and S&P500 Returns (4-week horizon)

	futures	cash	S&P500
Mean	0.00%	0.58%	1.04%
Median	0.10%	1.16%	0.99%
Std. deviation	6.81%	6.66%	4.99%
Kurtosis	3.970	3.517	3.350
Skewness	0.196	-0.176	-0.158
Maximum	21.67%	19.60%	13.95%
Minimum	-18.02%	-18.79%	-12.57%
Sample size	126	126	126
JB test (p-value)	0.079	0.358	0.558
$\sqrt{LPM_2(r;\boldsymbol{d})}$			
ä = 0	4.72%	4.56%	3.04%
ä = 50th percentile	4.78%	5.16%	3.49%
ä = 25th percentile	2.95%	3.14%	2.22%
ä = 10th percentile	1.50%	1.30%	1.35%
ä = 5th percentile	0.92%	1.12%	0.88%
Correlation			
futures	1.000	0.919	-0.053
cash		1.000	-0.003
S&P500			1.000
Correlation-LPM$_2$	cash – futures	cash – S&P500	futures – S&P500
ä = 0	0.95	0.27	0.25
ä = 50th percentile	0.95	0.32	0.32
ä = 25th percentile	0.93	0.12	0.09
ä = 10th percentile	0.91	0.01	0.00
ä = 5th percentile	0.89	0.00	0.00

As a first assessment of the differences between two-sided and one-sided risk measures, the standard deviation is greater than the square root of the second-order lower partial moment[9] (LPM$_2$), particularly as the target is reduced (Table 1). The square of the second-order lower partial moment declines monotonically with a lowering of the target for all the returns. However, the correlations among the returns behave differently. The sample correlation between cash and futures returns decline modestly as the target is lowered, but the sample correlation between cash-S&P500 and futures-S&P500 drops to zero at the lowest targets. The changing correlations at different targets suggest that the ordering of risky assets and consequently the hedge ratios may change at the lower targets.

The traditional minimum-variance hedge ratio is 0.89 in the 4-week hedging horizon (Table 2). Based on the CRRA utility function, the standard model[10] yields an optimal hedge ratio of 0.90 when the variance is adopted as the risk measure. Allowing for the presence of downside risk in the standard model, the optimal hedge ratio becomes smaller as the target return is set at

[9] The square root of the second-order lower partial moment in a downside risk context is equivalent to the standard deviation in a variance context.
[10] The standard model assumes no borrowing and lending, no transaction costs, and deterministic production.

lower levels of the distribution of returns.[11] In the extreme situation where the hedger is concerned only with losses below the 10th and 5th percentiles of the return distribution, the optimal hedge ratios are 0.59 and 0.68 respectively.

Table 2. Optimal Hedge Ratios at the 4-week Horizon

	Minimum variance	CRRA Mean-variance	CRRA downside risk with target set to: zero	50th percentile	25th percentile	10th percentile	5th percentile
Standard model	0.89	0.90	0.86	0.88	0.76	0.59	0.68
Standard model + brokerage fees							
b = 0.00050		0.86	0.78	0.82	0.62	0.23	0.17
b = 0.00100		0.83	0.71	0.77	0.50	0.05	0.00
b = 0.00125		0.81	0.68	0.73	0.45	0.00	0.00
b = 0.00250		0.72	0.49	0.58	0.22	0.00	0.00
b = 0.00500		0.54	0.10	0.20	0.00	0.00	0.00
Standard model + investment in alternative asset							
s_A = 0.10		0.89	0.86	0.88	0.76	0.69	0.71
s_A = 0.25		0.89	0.87	0.88	0.77	0.91	0.78
s_A = 0.50		0.86	0.90	0.88	0.88	1.12	1.30
Standard model + brokerage fees + investment in alternative asset							
s_A = 0.10							
b = 0.00050		0.85	0.78	0.81	0.59	0.16	0.09
b = 0.00100		0.81	0.70	0.75	0.45	0.00	0.00
b = 0.00125		0.79	0.65	0.72	0.40	0.00	0.00
b = 0.00250		0.69	0.44	0.55	0.14	0.00	0.00
b = 0.00500		0.49	0.00	0.12	0.00	0.00	0.00
s_A = 0.25							
b = 0.00050		0.84	0.77	0.80	0.55	0.01	0.00
b = 0.00100		0.79	0.67	0.72	0.37	0.00	0.00
b = 0.00125		0.77	0.62	0.69	0.30	0.00	0.00
b = 0.00250		0.64	0.35	0.47	0.00	0.00	0.00
b = 0.00500		0.40	0.00	0.00	0.00	0.00	0.00
s_A = 0.50							
b = 0.00050		0.79	0.74	0.76	0.47	0.00	0.00
b = 0.00100		0.72	0.58	0.64	0.17	0.00	0.00
b = 0.00125		0.68	0.49	0.58	0.04	0.00	0.00
b = 0.00250		0.50	0.07	0.22	0.00	0.00	0.00
b = 0.00500		0.14	0.00	0.00	0.00	0.00	0.00

Hedge ratios become smaller as the standard assumptions are relaxed. When brokerage fees are introduced hedge ratios drop quickly, turning to zero at higher fees and lower targets. Both higher fees and lower target returns cause hedge ratios to drop, but it appears that lower targets have a greater impact than higher fees. Using the variance as the risk measure, the optimal hedge

[11] In previous studies, the optimal hedge ratios in a downside risk framework have been estimated between 0.7 and 1.0 with target returns set to zero which corresponds to our estimate, 0.86.

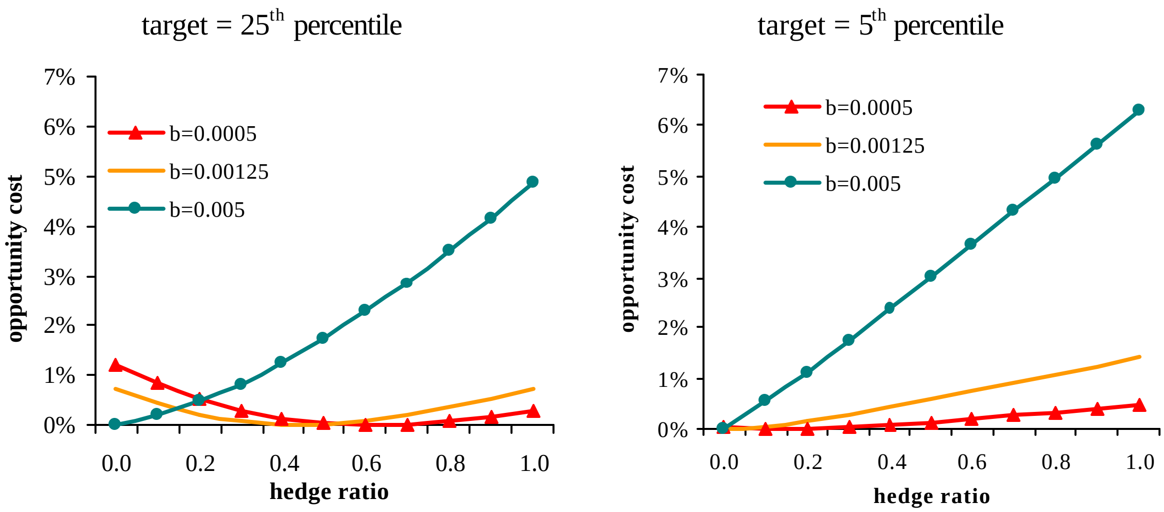
ratio reaches a low of 0.54 with the highest level of brokerage fee. In the presence of downside risk, the optimal hedge ratio can reach about 0.6 in the standard model, and quickly approaches zero even when low brokerage fees are combined with lower targets. The introduction of an alternative investment in the model at lowest level (S_A = 0.10) does not markedly change the hedge ratios developed under the standard model. However, as the level of the alternative investment increases LPM hedge ratios increase, particularly at lower target returns. For example, with the target return set to the 10th percentile, the optimal hedge ratio increases from 0.69 to 1.12 as the farmer's share of wealth invested in an alternative asset increases from 10% to 50%. Increases in the optimal futures positions may reflect a lowering of portfolio risk when the share of the alternative investment increases, particularly as the correlation between futures-S&P500 and cash-S&P500 returns decline.

The opportunity costs of not hedging are presented in Table 3 for the standard model. Under both the variance and the LPM risk measures, opportunity costs are small for hedge ratios close to the optimal level, indicating that minor departures from the optimal hedge ratio are not penalized severely. Comparing the risk measures, opportunity costs of not hedging are lower in the presence of downside risk, which is expected since downside risk hedge ratios are lower than mean-variance hedge ratios. For example, at h=0 in a variance context, the highest opportunity cost exists, 7.28% of the initial wealth, but this drops off quickly to 0.24% at the 5th percentile in the downside risk framework. Relatively low opportunity costs for downside risk measures are also observed when low brokerage fees are introduced in the model (Figure 1). At the 25th percentile, when brokerage fees are equal to or smaller than 0.125%, the opportunity costs of placing sub-optimal hedge ratios in the presence of downside risk barely surpass 1%. More generally, the downside risk framework implies very low opportunity costs of placing sub-optimal hedges when target returns and brokerage fees are low, suggesting that farmers are not penalized by hedging at a sub-optimal ratio or not hedging at all. In the presence of higher brokerage fees, the opportunity costs of hedging change substantially. With brokerage fees of 0.5%, opportunity costs increase monotonically and farmers are penalized heavily when their hedge deviates from its optimal zero value.

Table 3. Opportunity Costs of Hedging at the 4-week horizon – Standard Model (annual return as a percentage of initial wealth)

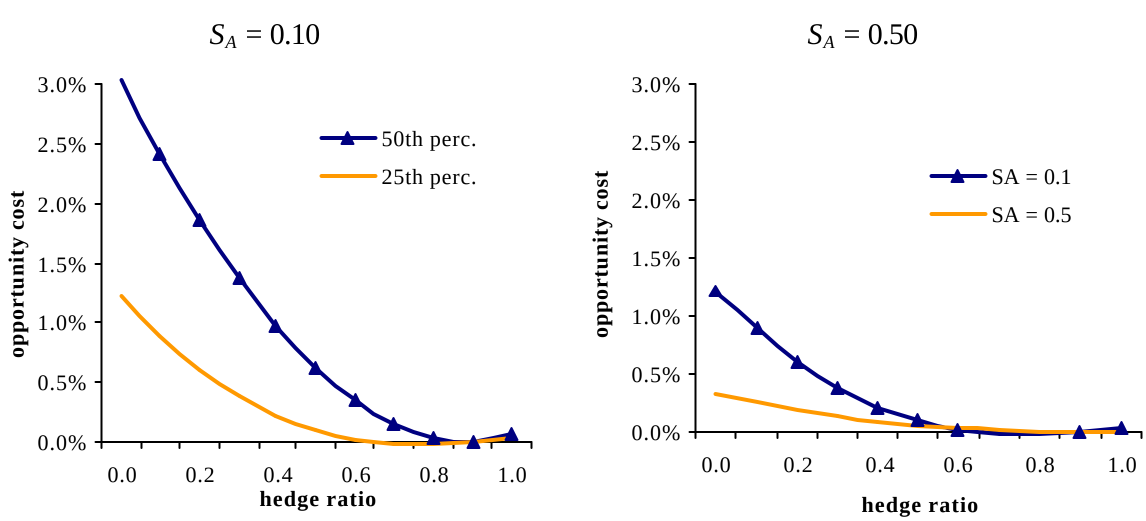
	CRRA Mean-variance	CRRA downside risk with target set to: zero	50th percentile	25th percentile	10th percentile	5th percentile
h = 0.0	7.28	3.09	3.70	1.65	0.33	0.24
h = 0.1	5.72	2.42	2.69	1.20	0.19	0.13
h = 0.2	4.42	1.83	2.29	0.83	0.10	0.06
h = 0.3	3.25	1.32	1.70	0.53	0.04	0.02
h = 0.4	2.21	0.89	1.19	0.31	0.01	0.01
h = 0.5	1.43	0.55	0.76	0.15	0.00	0.00
h = 0.6	0.78	0.29	0.42	0.05	0.00	0.00
h = 0.7	0.39	0.11	0.18	0.01	0.00	0.00
h = 0.8	0.13	0.01	0.03	0.00	0.00	0.00
h = 0.9	0.00	0.01	0.00	0.03	0.00	0.00
h = 1.0	0.13	0.09	0.09	0.08	0.00	0.00

Figure 1. Opportunity Costs of Hedging at the 4-week Horizon (annual return as a percentage of initial wealth) – brokerage fees



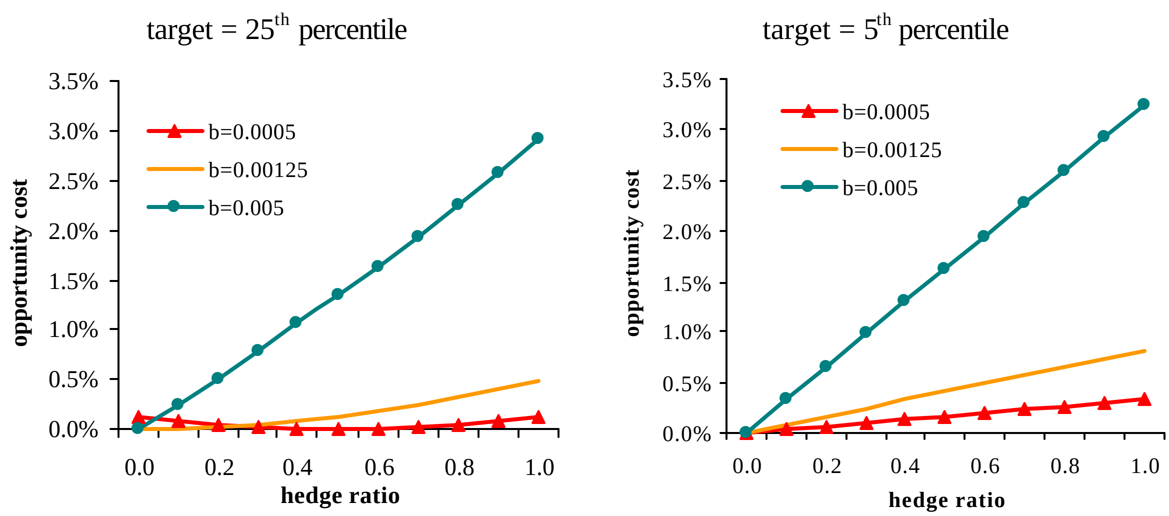
In the presence of an alternative investment, the opportunity costs of placing sub-optimal hedges are small. Although the introduction of alternative assets in the farmer's portfolio often leads to higher optimal hedge ratios, the opportunity cost of not hedging (h=0) never reaches more than 3% (Figure 2). When 50% of the farmer's wealth is invested in alternative assets – the case that yields the highest hedge ratios – and the target return is set to either the 50th or the 25th percentiles, the opportunity cost of not hedging is 0.97% and 0.33% respectively. For lower targets, the opportunity cost of not hedging is almost zero. While the opportunity cost can reach nearly 3.0% at the lowest level of investment and the 50th-percentile target, the opportunity cost of hedging barely reaches 1% at the 25th percentile and lower targets.

Figure 2. Opportunity Costs of Hedging at the 4-week Horizon (annual return as a percentage of initial wealth) – alternative investment



When brokerage fees are introduced together with the alternative investment, optimal hedge ratios are quickly driven towards zero, and similar to the previous discussion about the effects of brokerage fees on optimal hedge ratios, the opportunity cost of not hedging becomes zero and the opportunity cost of actually hedging becomes relatively high. When 50% of the farmer's wealth is invested in alternative assets and the targets are set to the 25th and the 5th percentiles, the opportunity cost of not hedging is zero in both cases, while the opportunity cost of hedging increases as higher hedge ratios are adopted (Figure 3).

Figure 3. Opportunity Costs of Hedging at the 4-week Horizon (annual return as a percentage of initial wealth) – brokerage fees and alternative investment (S_A = 0.50)



SUMMARY, DISCUSSION AND IMPLICATIONS

This paper analyzed how hedge ratios and the opportunity costs of placing sub-optimal hedges vary as a downside risk measure is introduced in the presence of transaction costs and alternative investments. The findings indicate that downside risk hedge ratios can differ substantially from the standard mean-variance hedge ratios at low targets and when the standard assumptions are relaxed. Although it might have been expected that low targets would automatically reduce hedge ratios our findings indicate that this is not the case. Hedge ratios in the standard model can increase at lower targets as the correlations among the variables in the downside portion of the return distribution change. These results support Lien and Tse (2002), and Demirer and Lien's (2003) discussion of the conceptual properties of the minimum-LPM hedge ratio, and Lien and Tse (2000) and Turvey and Nayak's (2003) empirical evidence using the minimum-LPM model that optimal hedge ratios do not necessarily decrease as target returns are set to lower levels. The introduction of transaction costs appears to have the largest effect on the optimal hedge ratio, particularly at lower target return levels. Using transaction costs that existed near the beginning of the sample period, hedge ratios decline quickly to zero as the target return declines. With more current transaction costs, hedge ratios decline towards zero but not as rapidly. In the presence of alternative investment opportunities, hedge ratios are not strongly affected relative to the standard model at lower levels of investment. However, hedge ratios increase as the level of the alternative investment increases and the target declines. When alternative investments and transaction costs are introduced simultaneously, the effect on the hedge ratios is dramatic and the optimal hedge ratios are driven to zero most quickly, particularly as the target returns decline.

With regards to the opportunity cost of not hedging the results are clear. In the presence of non-zero optimal hedges, the opportunity cost of not hedging is small. In the presence of a zero optimal hedge, which is primarily driven by higher transaction costs, the opportunity cost of actually hedging increases quickly, particularly at lower target returns. In these situations, there appears to be little incentive for farmers to hedge.

Recall Lence (1996) demonstrated that hedge ratios for a farmer can change dramatically when the assumptions used to estimate minimum-variance hedge ratios are relaxed. He found that the inclusion of transaction costs, alternative investments and stochastic production can cause minimum-variance and maximum-utility optimal hedges to differ substantially, and reduce the level of farmer hedging. Chen, Lee and Shretha (2003) questioned the robustness of Lence's findings due to the utility function, the specifics of the return distributions, and the definition of risk measure used in the analysis. How do our findings add to this dialogue? Our results, based on a CRRA utility function, the most recent returns for cash, futures and the S&P500, the LPM measure of downside risk, are quite compatible with Lence's conclusions and implications. Even though we do not consider the stochastic production case, our results clearly demonstrate the sensitivity of hedge ratios to deviations from the minimum- variance assumptions. Further, in our simulations the farmer has little incentive to hedge, and even when the incentive to hedge emerges the opportunity cost of not hedging is relatively small. Our findings also support the notion mean-variance utility-maximizing hedge ratios should be used with caution in the presence of downside risk. Variance-based hedge ratios are close to downside risk-based hedge ratios only under specific conditions. In our analysis, this occurs when the target return is set to either zero or the 50th percentile of the distribution and the standard assumptions hold. While variance-based hedge ratios are easier to calculate than downside-risk-based hedge ratios, their results tend to differ dramatically when more realistic models are used.

Finally, our findings might help explain the observed limited use of futures contracts for hedging by soybean and grain farmers. If producers are intensely concerned with downside risk and transaction costs are not negligible, then the opportunity costs of not hedging using futures contracts may be small and hedge ratios may be dose to zero. Interestingly, transaction costs which have been declining in importance only measure brokerage fees. Clearly, other costs including initial and maintenance margin deposits and the opportunity of following futures markets exist which can further reduce the motivation of producers to hedge. However, the recent increase in high-volume, larger grain producers may reduce per unit transaction costs and make hedging more attractive for these producers.

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