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## **Weather Derivatives in the Presence of Index and Geographical Basis Risk: Hedging Dairy Profit Risk**

by

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# Weather Derivatives in the Presence of Index and Geographical Basis Risk: Hedging Dairy Profit Risk

## Practitioner's Abstract

*Weather conditions pose unique risks to dairy producers. Weather derivatives represent a potentially promising approach to augment dairy producers' risk management against adverse weather events. This study examines the effect of basis risk in weather derivatives, and whether the existence of basis risk mitigates the usefulness of weather derivatives for dairy risk management. Assuming a representative dairy producer has access to both weather derivatives and traditional heat abatement equipment, a closed-form solution for his/her optimal portfolio choice problem in the presence of basis risk is derived within a mean-variance utility framework. First-, second-, and third- degree stochastic dominance criteria are used to test the risk management effectiveness with less restrictive assumptions. Also proportional transaction costs are imposed on weather derivative prices calculated on the basis of actuarial fairness to allow the desirability of these contracts to their issuers.*

**Keywords:** basis risk, mean-variance efficiency, profit risk, stochastic dominance, weather derivatives

## Introduction

Weather conditions are the primary dairy production risk. Hot and humid weather induces heat stress, which reduces both the quantity and quality of dairy production (Barth; Thompson). Heat stress is measured by temperature-humidity index (THI, also commonly known as the 'heat index'). Traditional heat abatement technologies control the environment through ventilation, misting or evaporative cooling (Turner et al.; Lin et al.). Adoption of abatement equipment, however, is hindered by its high initial cost and possibly long payback period, especially for small- and medium-scale firms. Moreover, the abatement equipment is only seasonally useful. Weather derivatives provide an alternative method of dairy producers' risk management. Instead of reducing production losses, weather derivatives make payments based upon observed weather conditions over a period of time so that they offer the potential to offset profit losses caused by adverse weather events.

The objective of this study is to examine the effect of basis risk in weather derivatives, and whether the existence of basis risk mitigates the usefulness of weather derivatives for dairy risk management.

Previous research has identified the problem of basis risk in weather derivatives (Turvey, 2001; Vedenov and Miranda, 2001). Little theoretical or empirical work has been done to examine the effect of basis risk on weather derivatives. However, Dischel

(2000) analyzes the correlation of precipitation in three nearby cities in Washington and finds that there is strong correspondence of rainfall between these cities. He suggests that precipitation basis risk might be overstated and useful correlations among sites might be quantified.

In this research, we investigate two kinds of basis risk. One is geographical basis risk, which occurs from the difference in location between the reference site of weather derivatives and the actual production area. The other is reference-index basis risk, which occurs because weather derivatives are typically based upon temperature, yet biological stresses occur as a function of THI. In other words, we study the cases that a dairy farmer is not able to buy weather derivatives of his/her production area and the weather derivatives available are written on temperature only, instead of THI itself.

Hedging against one instrument's risk using different but related derivatives is formally called cross-hedging. This often happens when there are not derivatives for the instrument being hedged, or when a suitable derivatives contract exists but the market is highly illiquid. The success of cross-hedging depends completely on how strongly correlated the instrument being hedged is with underlying indices of the derivatives. Thus the effect of using weather derivatives in temperature to cross-hedge dairy profit risk will depend on the degree of correlation between temperature and heat stress and the correlation of weather conditions between production area and weather derivative reference location.

The analysis is conducted by first constructing two profit models. One is for a representative producer's profit without using weather derivatives or abatement technologies. The other is for his profit with using both of these two instruments. The producer's optimal portfolio choice is then derived in a utility maximization framework. From the utility framework, the benefit of using weather derivatives for managing risk is measured. The assumptions implicit in this paper are that (i) the producer has Pratt's absolute risk aversion and choose mean-variance efficient portfolios with a one-period horizon; (ii) weather conditions are the only systematic risk factor to all producers in summer.

## **Background**

Weather derivatives as relatively new financial products possess several unique properties distinguishing them from other derivatives. First, their underlying (weather) is not a tradable asset. Second, they hedge against volumetric risk instead of price risk. The indemnity is calculated based on a weather index (Cooling/Heating Degree-Days, rainfall, etc) rather than asset price. Third, they are not as liquid as traditional standard derivatives. If we assume away transaction costs, the traditional financial derivative markets are liquid. Weather, by its nature, is location-specific. Different locations have different weather conditions whether at the same time or across time.

Due to their properties, weather derivatives have advantages over traditional financial derivatives in the view of hedging against weather risk. Because there is no need to prove damage to claim payoffs, there is little moral hazard. Furthermore, as weather information is almost perfectly symmetric, adverse selection is eliminated. At the same time, the use of weather derivatives is accompanied by basis risk caused by the fact that an end-user's location often is not the same location as the reference location of the weather derivatives he holds.

Economic losses are induced in the dairy industry when effective temperature conditions are out of dairy cows' thermal comfort zone. According to St-Pierre, Cobanov and Schnitkey (SCS, 2003), heat stress in dairy cattle is a function of the Temperature-Humidity Index (THI). Johnson reports that a THI higher than 72 degrees is likely to have adverse effects on per-cow yield. In SCS, it is suggested that the threshold of THI to trigger heat stress should be lowered to 70 degrees accounting for the lower heat tolerance of the current selection of dairy cows. So 70 degrees is used as a threshold for risk from heat stress,  $THI_{threshold}$ . According to the National Oceanic Atmospheric Administration (NOAA), the standard formula of THI is:  $THI = T - (0.55 - 0.55 RH)$  ( $T - 58$ ), where  $T$  is temperature in degrees Fahrenheit and  $RH$  is relative humidity in percent. Since  $RH$  is expressed as a percentage, it is easy to see that THI is positively correlated with temperature.

THI is varying in a day along with change of temperature and relative humidity. The maximum THI is in the afternoon, when the temperature is highest and relative humidity is lowest; and the minimum THI is in the night, when the temperature is lowest and relative humidity is highest. In this paper, daily THI refers to daily maximum THI. If maximum THI is lower than 70 degrees in a day, there is no heat stress for dairy cows.

## Theoretical Framework

### *Setting*

Consider a dairy producer who produces without using abatement equipment or weather derivatives. His profit is  $\tilde{y} = P \cdot \tilde{Q} - TC$ , where  $P$  is milk price,  $\tilde{Q}$  is the stochastic yield, and  $TC$  denotes a total cost. For analytical simplicity, it is assumed there is no price risk; therefore price is normalized to unity. The tilde ( $\sim$ ) denotes a random variable.

Suppose expected profit of a producer is his historical average,  $\mu$ , so the difference between  $\tilde{y}$  and  $\mu$  is his profit risk. The profit risk is orthogonally decomposed into two parts. One is systematic risk which comes from weather condition; the other is nonsystematic risk which reflects the individual's production variability not arising from weather and is assumed uncorrelated with weather conditions. Equations (1)-(11) give

the daily models, because equation (3), by its nature, is for daily calculation. However, they can be easily transformed into yearly models by using yearly cumulative values of the variables.

$$(1) \quad \tilde{y} = \mu + \theta \cdot f(\tilde{x}) + \tilde{\varepsilon},$$

where

$$(2) \quad \tilde{x} = E(\tilde{z}) - \tilde{z}$$

$$(3) \quad \tilde{z} = \max(\widetilde{THI} - THI_{threshold}, 0)$$

$$(4) \quad \theta = \text{cov}(\tilde{y}, f(\tilde{x})) / \text{var}(f(\tilde{x}))$$

$$(5) \quad E(\tilde{y}) = \mu, \quad E(\tilde{\varepsilon}) = 0, \quad \text{var}(\tilde{\varepsilon}) = \sigma_{\tilde{\varepsilon}}^2, \quad \text{cov}(\tilde{z}, \tilde{\varepsilon}) = 0, \quad \text{cov}(\tilde{x}, \tilde{\varepsilon}) = 0.$$

The coefficient  $\theta$  quantifies the sensitivity of the producer's individual profit to systematic risk. The factor  $\tilde{z}$ , which is common to all producers in a region, measures the degree of heat stress, and the factor  $\tilde{x}$  denotes the weather condition compared to its expectation. If  $\tilde{z}$  is lower than  $E(\tilde{z})$ , it means the heat stress is milder than its expectation. In this case,  $\tilde{x}$  is positive. And  $f(\tilde{x})$  captures systematic risk and increases with  $\tilde{x}$ . Also for analytical simplicity, the functional form of  $f(\tilde{x})$  is assumed to be linear, i.e.  $f(\tilde{x}) = \alpha \cdot \tilde{x}$ , where  $\alpha$  is a positive parameter of the linear relationship. The final term  $\tilde{\varepsilon}$  is a nonsystematic risk component.

Then equation (1) becomes,

$$(6) \quad \tilde{y} = \mu + \theta \cdot \alpha \cdot \tilde{x} + \tilde{\varepsilon} = \mu + \beta \cdot \tilde{x} + \tilde{\varepsilon}$$

where

$$(7) \quad \beta = \text{cov}(\tilde{y}, \tilde{x}) / \text{var}(\tilde{x}).$$

Since here the risk is from excessively high THI, weather derivatives that will be used are focused on weather call options. The payoff from a weather call option is:

$$(8) \quad \tilde{n} = \max(\tilde{I} - I_{threshold}, 0)$$

where  $\tilde{I}$  is the stochastic value of a weather index, and  $I_{threshold}$  is the strike level. Equation (8) captures the presence of both index and geographical basis risk. If the reference index  $\tilde{I}$  is temperature rather than THI, it reflects index basis risk. If the reference index  $\tilde{I}$  is weather condition of a location other than the production area, then geographical basis risk exists. Note if  $\tilde{I}$  is THI of the production area, there is no basis risk and  $\tilde{n} = \tilde{z}$ .

Transaction costs are imposed on the option premium. So purchasing weather options can decrease the producer's expected profit. The option premium equals the

expected payoff plus proportional transaction costs:

$$(9) \quad \pi = (1 + \gamma)E(\tilde{n})$$

where the loading rate  $\gamma > 0$  reflects transaction costs related to administrative and implementation fees and the desirability to the issuers. If  $\gamma$  is zero, the weather options are actuarially-fairly priced.

Also suppose that the producer is free to choose his abatement equipment investment  $\eta$  ( $\eta \geq 0$ ; and  $\eta = 0$  means he does not install abatement equipment). By using abatement equipment, the production loss from heat stress can be reduced. The biological functional form of the effectiveness of abatement equipment is formulated as:

$$(10) \quad \Delta loss = g(\eta, \widetilde{THI}) = (a + b \cdot \widetilde{THI}) \cdot \sqrt{\eta}$$

where  $\Delta loss$  is the reduced profit loss, i.e. the profit gain from using abatement,  $\eta$  is abatement investment,<sup>1</sup> and  $a$  and  $b$  are parameters.

It is easy to see that  $\Delta loss$  is increasing with  $\eta$  and  $\widetilde{THI}$ . When  $\eta = 0$ ,  $\Delta loss$  is also equal to 0. And with fixed  $\eta$ ,  $\Delta loss$  is increasing with  $\widetilde{THI}$ . That is because although the profit is low when  $\widetilde{THI}$  is high, the reduced profit loss will be high with abatement equipment; on the other hand, when  $\widetilde{THI}$  is low (i.e. weather is good), the abatement equipment is not of much use, so the reduced loss is low. Thus the parameter  $b$  is positive. Since the net profit from using abatement technology is  $(a + b \cdot \widetilde{THI}) \cdot \sqrt{\eta} - \eta$ , that is to say if  $\widetilde{THI}$  is high enough, the net profit from investing abatement equipment will be positive; otherwise, the net profit is negative.

With weather options and abatement equipment, the producer's net profit equals:

$$(11) \quad \tilde{y}^{net} = \tilde{y} + \phi \cdot (\tilde{n} - \pi) + \Delta loss - \eta$$

where  $\phi$  is weather option purchase amount. Therefore, there are two elements that the producer is free to choose: spending on weather options,  $\phi$ , and spending on abatement,  $\eta$ . It is assumed these two choices are determined simultaneously in a portfolio taking the remaining parameters as given.

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<sup>1</sup>Since abatement equipment is useful for many years once installed, the installation cost is annualized at a certain rate (say 15%) for yearly analysis. When "burn-rate" method is used to forecast weather, the expected THI will vary little over years. So, the producer's yearly optimal decision on weather option purchase amount and abatement investment will not change much over years once determined based on current information.

## Optimal Decisions

The producer's optimal portfolio choice of weather option purchase and abatement investment is derived using a utility maximization model. The producer is assumed to have a mean-variance utility function<sup>2</sup> of

$$(12) \quad U = E(\bullet) - \frac{1}{2}A \cdot \text{var}(\bullet)$$

where  $A$  is an index of agents' aversion to taking on risk. Then the representative producer's objective is to choose his optimal option purchase  $\phi$  and abatement spending  $\eta$  to maximize his utility from using weather options and abatement equipment<sup>3</sup>:

$$(13) \quad \max_{\phi, \eta} U^{net} = E(\tilde{y}^{net}) - \frac{1}{2}A \cdot \text{var}(\tilde{y}^{net}).$$

Specifically,

$$(14) \quad \begin{aligned} U^{net} &= E(\tilde{y}) + \phi E(\tilde{n} - \pi) + E(\Delta loss - \eta) - \frac{1}{2}A \cdot [\text{var}(\tilde{y}) + \phi^2 \text{var}(\tilde{n}) \\ &\quad + \text{var}(\Delta loss) + 2\phi \text{cov}(\tilde{y}, \tilde{n}) + 2\text{cov}(\tilde{y}, \Delta loss) + 2\phi \text{cov}(\tilde{n}, \Delta loss)] \\ &= \mu - \phi \gamma \mu_{\tilde{n}} + (a + b \mu_{\widetilde{THI}}) \sqrt{\eta} - \eta - \frac{1}{2}A \cdot [\beta^2 \sigma_{\tilde{z}}^2 + \sigma_{\tilde{\varepsilon}}^2 + \phi^2 \sigma_{\tilde{n}}^2 + b^2 \eta \sigma_{\widetilde{THI}}^2 \\ &\quad - 2\beta \phi \sigma_{\tilde{z}, \tilde{n}} - 2\beta b \sqrt{\eta} \sigma_{\widetilde{THI}, \tilde{z}} + 2\phi b \sqrt{\eta} \sigma_{\widetilde{THI}, \tilde{n}}] \end{aligned}$$

where  $\mu = E(\tilde{y})$ ,  $\mu_{\tilde{n}} = E(\tilde{n})$ ,  $\mu_{\widetilde{THI}} = E(\widetilde{THI})$ ;  $\sigma_{\tilde{z}}^2 = \text{var}(\tilde{z})$ ,  $\sigma_{\tilde{n}}^2 = \text{var}(\tilde{n})$ ,  $\sigma_{\widetilde{THI}}^2 = \text{var}(\widetilde{THI})$ ;  $\sigma_{\tilde{z}, \tilde{n}} = \text{cov}(\tilde{z}, \tilde{n})$ ,  $\sigma_{\widetilde{THI}, \tilde{z}} = \text{cov}(\widetilde{THI}, \tilde{z})$ ,  $\sigma_{\widetilde{THI}, \tilde{n}} = \text{cov}(\widetilde{THI}, \tilde{n})$ . And all these are positive numbers.<sup>4</sup>

Take first order condition with respect to  $\phi$  and  $\eta$  respectively,

$$(15) \quad \gamma \mu_{\tilde{n}} + A[\phi \sigma_{\tilde{n}}^2 - \beta \sigma_{\tilde{z}, \tilde{n}} + b \sqrt{\eta} \sigma_{\widetilde{THI}, \tilde{n}}] = 0,$$

$$(16) \quad \frac{1}{2}(a + b \mu_{\widetilde{THI}}) \eta^{-\frac{1}{2}} - 1 - \frac{1}{2}A[b^2 \sigma_{\widetilde{THI}}^2 - \beta b \sigma_{\widetilde{THI}, \tilde{z}} \eta^{-\frac{1}{2}} + \phi b \sigma_{\widetilde{THI}, \tilde{n}} \eta^{-\frac{1}{2}}] = 0.$$

Then equation system of (15) and (16) can be solved simultaneously.

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<sup>2</sup>This framework is equivalent to expected utility maximization if (net) profit is distributed normally and producers' utility function is exponential. But Meyer has shown that the mean-variance model is consistent with expected utility model under much weaker restrictions. See Pratt (1964) and Meyer (1987).

<sup>3</sup>Theoretically, if  $\phi$  in the optimal portfolio is negative, that means the decision maker can benefit from selling weather call options.

<sup>4</sup>Even if there exists index and geographical basis risk,  $\text{cov}(\tilde{z}, \tilde{n})$  and  $\text{cov}(\widetilde{THI}, \tilde{n})$  are still positive as long as weather conditions of production area and reference location covary positively.

It follows from (15) that

$$(17) \quad \phi^* = -\frac{\gamma\mu_{\tilde{n}}}{A\sigma_{\tilde{n}}^2} + \beta\frac{\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2} - b\frac{\sigma_{\widehat{THI},\tilde{n}}}{\sigma_{\tilde{n}}^2}\sqrt{\eta}.$$

Substituting (17) into (16) yields

$$(18) \quad \sqrt{\eta^*} = \frac{a + b\mu_{\widehat{THI}} + \gamma\mu_{\tilde{n}}b\frac{\sigma_{\widehat{THI},\tilde{n}}}{\sigma_{\tilde{n}}^2} + A\beta b[\sigma_{\widehat{THI},\tilde{z}} - \frac{\sigma_{\widehat{THI},\tilde{n}}\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2}]}{2 + Ab^2[\sigma_{\widehat{THI}}^2 - \frac{\sigma_{\widehat{THI},\tilde{n}}^2}{\sigma_{\tilde{n}}^2}]}.$$

### ***Comparative Static Analysis***

It follows from (17) that:

**Proposition 1 .** *The optimal weather option purchase amount is decreasing with abatement equipment investment. Thus it indicates that weather options can act as a substitute for abatement equipment in dairy risk management strategies.*

In (18), it is not difficult to see that the denominator is positive, because  $b^2[\sigma_{\widehat{THI}}^2 - \frac{\text{cov}^2(\widehat{THI},\tilde{z})}{\sigma_{\tilde{z}}^2}] = b^2\sigma_{\widehat{THI}}^2[1 - \frac{\rho_{\widehat{THI},\tilde{z}}^2\sigma_{\widehat{THI}}^2\sigma_{\tilde{z}}^2}{\sigma_{\widehat{THI}}^2\sigma_{\tilde{z}}^2}] = b^2\sigma_{\widehat{THI}}^2(1 - \rho_{\widehat{THI},\tilde{z}}^2) > 0$ , since the correlation coefficient  $\rho_{\widehat{THI},\tilde{z}} \in (0, 1)$ .

Since  $\gamma, \mu_{\tilde{n}}, b, \sigma_{\widehat{THI},\tilde{n}}, \sigma_{\tilde{n}}^2$  are all positive, then

$$\frac{d\sqrt{\eta^*}}{d\gamma} = \frac{\mu_{\tilde{n}}b\frac{\sigma_{\widehat{THI},\tilde{n}}}{\sigma_{\tilde{n}}^2}}{2 + Ab^2[\sigma_{\widehat{THI}}^2 - \frac{\sigma_{\widehat{THI},\tilde{n}}^2}{\sigma_{\tilde{n}}^2}]} > 0.$$

It follows that:

**Proposition 2 .** *The optimal abatement investment is positively related to the weather option loading rate  $\gamma$ .*

The intuition of Proposition 2 is that higher transaction costs make weather options less attractive, and thus abatement equipment becomes more attractive to dairy producers.

From (17) and Proposition 2, it can be shown that

$$\frac{d\phi^*}{d\gamma} = -\frac{-\mu_{\tilde{n}}}{A\sigma_{\tilde{n}}^2} - b\frac{\sigma_{\widetilde{THI},\tilde{n}}}{\sigma_{\tilde{n}}^2} \frac{d\sqrt{\eta^*}}{d\gamma} < 0.$$

**Proposition 3 .** *The optimal weather option purchase amount is negatively related to the weather option loading rate  $\gamma$ .*

It is not straightforward to see the impact of a small change of  $\beta$  on  $\sqrt{\eta^*}$ , and  $\phi^*$ . From (18),

$$\frac{d\sqrt{\eta^*}}{d\beta} = \frac{A\beta b[\sigma_{\widetilde{THI},\tilde{z}} - \frac{\sigma_{\widetilde{THI},\tilde{n}}\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2}]}{2 + Ab^2[\sigma_{\widetilde{THI}}^2 - \frac{\sigma_{\widetilde{THI},\tilde{n}}^2}{\sigma_{\tilde{n}}^2}]}, \text{ thus}$$

$$\text{sign} \left[ \frac{d\sqrt{\eta^*}}{d\beta} \right] = \text{sign} \left[ \sigma_{\widetilde{THI},\tilde{z}} - \frac{\sigma_{\widetilde{THI},\tilde{n}}\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2} \right] = \text{sign} \left[ \rho_{\widetilde{THI},\tilde{z}} - \rho_{\widetilde{THI},\tilde{n}}\rho_{\tilde{z},\tilde{n}} \right].$$

By the definitions of  $\tilde{z}$  and  $\tilde{n}$  in (3) and (8), it is a reasonable assumption that  $\rho_{\widetilde{THI},\tilde{z}} > \rho_{\widetilde{THI},\tilde{n}}$ .<sup>5</sup> Together with the fact that  $\rho_{\tilde{z},\tilde{n}}$  is less than 1, we have  $\frac{d\sqrt{\eta^*}}{d\beta} > 0$ .

**Proposition 4 .** *The optimal abatement investment is increasing with  $\beta$ , under an unrestrictive assumption on the relationship of weather condition in two locations.*

**Proposition 5 .** *The optimal weather option purchase amount is increasing with  $\beta$ , under an unrestrictive assumption on the relationship of weather conditions of production area and reference location.*<sup>6</sup>

The proof of Proposition 5 is found in the appendix.

### ***Effect of Basis Risk***

If there is no basis risk, namely, the dairy farmer can buy weather derivatives written on THI of his production area,  $\tilde{n}$  will be the same as  $\tilde{z}$ , and thus  $\text{var}(\tilde{z}) = \text{var}(\tilde{n}) = \text{cov}(\tilde{z}, \tilde{n})$ , and  $\text{cov}(\widetilde{THI}, \tilde{z}) = \text{cov}(\widetilde{THI}, \tilde{n})$ . Therefore the optional portfolio choice in (17) and (18) becomes:

<sup>5</sup>This inequality holds for all of our weather data.

<sup>6</sup>The assumption is on the correlation coefficients, specifically,  $\frac{\rho_{\widetilde{THI},\tilde{z}}\rho_{\widetilde{THI},\tilde{n}}}{\rho_{\tilde{z},\tilde{n}}} < 1$ .

$$(19) \quad \phi^* = -\frac{\gamma\mu_{\tilde{z}}}{A\sigma_{\tilde{z}}^2} + \beta - b\frac{\sigma_{\widetilde{THI},\tilde{z}}}{\sigma_{\tilde{z}}^2}\sqrt{\eta}$$

$$(20) \quad \sqrt{\eta^*} = \frac{a + b\mu_{\widetilde{THI}} + \gamma\mu_{\tilde{z}}b\frac{\sigma_{\widetilde{THI},\tilde{z}}}{\sigma_{\tilde{z}}^2}}{2 + Ab^2[\sigma_{\widetilde{THI}}^2 - \frac{\sigma_{\widetilde{THI},\tilde{z}}^2}{\sigma_{\tilde{z}}^2}]}$$

Compared with (17) and (18), the existence of basis risk causes decrease in  $\phi^*$  and increase in  $\eta^*$ , ceteris paribus. From (19) and (20), an increase in  $\beta$  will not affect  $\eta^*$  but only increase  $\phi^*$ . That is because using weather options is the most efficient way to hedge against weather risk if there is no basis risk. And an increase in risk aversion degree  $A$  will reduce the optimal abatement investment and increase the weather option purchase amount. However, no analytical inference about the impact of a small change of  $A$  on  $\sqrt{\eta^*}$  and  $\phi^*$  can be derived with the presence of basis risk.

### ***Risk Management Effectiveness***

By substituting  $\phi$  and  $\eta^*$  in (17) and (18) back into utility function in (14), the maximized increased utility in certainty equivalent from using weather options and abatement equipment can be derived from:

$$(21) \quad \begin{aligned} \Delta U &= U^{net}(\phi^*, \eta^*) - U(0, 0) \\ &= -\phi\gamma\mu_{\tilde{n}} + (a + b\mu_{\widetilde{THI}})\sqrt{\eta} - \eta - \frac{1}{2}A \cdot [\phi^2\sigma_{\tilde{n}}^2 + b^2\eta\sigma_{\widetilde{THI}}^2 \\ &\quad - 2\beta\phi\sigma_{\tilde{z},\tilde{n}} - 2\beta b\sqrt{\eta}\sigma_{\widetilde{THI},\tilde{z}} + 2\phi b\sqrt{\eta}\sigma_{\widetilde{THI},\tilde{n}}]. \end{aligned}$$

It is also viable to compare it with the cases in which the producer only uses one of these two instruments. The simultaneous usage of weather options and abatement equipment will be more favorable.

### **Data**

For the empirical part of this study, we need to estimate equations (6) and (10). Three types of data are needed: weather data, profit data and abatement investment data. The 35-year (1949 to 1964 and 1984 to 2002) weather data of 14 weather stations in IL,

OH, NY, and WI are from the National Climate Data Center (NCDC), a subsidiary of the National Oceanic Atmospheric Administration (NOAA).<sup>7</sup> The weather data in each station include daily maximum and minimum temperature and daily maximum and minimum relative humidity. Daily temperature and dew point<sup>8</sup> both follow routinely seasonal patterns each year. So the “burn-rate” method works well with them for pricing weather options. Daily maximum temperature-humidity index (THI) is derived from daily maximum temperature and minimum relative humidity.

Corresponding to the weather data of each station, a representative producer’s milk loss from heat stress and reduced loss from using abatement equipment are generated by employing the results in SCS.<sup>9</sup> Abatement investment cannot change in a relatively long period once fixed. Also weather options are assumed to be written on summer basis, i.e. the payoff is cumulative  $\tilde{n}$  of a summer and premium is the expected payoff. Thus, equations (6) and (10) are estimated based on cumulative summer data. The summer period is set from May 1st to Oct. 31st every year, because 97% of heat stress occurs in this period.

In the 14 selected weather stations, a major city in each of the four states is selected as the weather options reference city of that state. The 10 stations that are left are treated as production areas. Table 1 lists the selected production areas and their corresponding weather options reference cities. Distance between each pair of production area and reference city is reported as well.

## Empirical Results

### *Estimate Beta in Equation (6)*

Note all results of the empirical illustration in this paper is for one dairy cow, namely, the herd size is normalized into unity. Following SCS,  $THI_{threshold}$  is set as 70 degrees. For each selected production area, the daily milk loss during summers of the 35 years and the corresponding daily  $\widetilde{THI}$  are calculated using the weather data and the SCS milk loss model. Then by accumulating the milk loss and  $\tilde{z} = \max(\widetilde{THI} - THI_{threshold}, 0)$  during each summer in the 35 years, 35 observations of accumulated milk loss and  $\tilde{x} = E(\tilde{z}) - \tilde{z}$  are obtained. From a least squares regression,  $\beta$  is estimated. Table 2

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<sup>7</sup>It is a quite common phenomenon that daily relative humidity data from 1965 to 1983 are missing across weather stations in NCDC database. The selected 14 stations are the only ones which have the least amount of missing data of 1949 to 1964 and 1984 to 2002 in the four states.

<sup>8</sup>Dew point measures how much water vapor is in the air. In many places, the air’s total vapor content varies only slightly during an entire day, and so it is the changing air temperature that primarily regulates the full variation in relative humidity. Related information can be found at: <http://www.usatoday.com/weather> .

<sup>9</sup>See the Appendix for detail.

shows the estimation results of the selected production areas. For example, the beta of a representative farmer in Summit County is 0.56 kg milk per cow, which means each degree of  $\tilde{z}$  beyond its mean will induce 0.56 kg milk loss. The milk price is set as \$0.287/kg as in SCS, so the profit loss is \$0.16 per degree of  $\tilde{x}$ . The average beta across the selected production areas is 0.62 kg milk per cow, and the standard deviation is 0.13 kg.

### ***Estimate Parameters in Equation (10)***

The daily summer weather data are put into the SCS abatement effect models<sup>10</sup> to calculate the daily reduced THI corresponding to five abatement levels. Multiplying the estimated  $\beta$  and milk price, we calculate the reduced profit loss (in dollars) due to abatement investment (in dollars). The reduced profit loss and THI are accumulated for each summer. Thus in each selected production area, there are 35 observations of accumulated reduced profit loss and accumulated THI for each of the five abatement investment levels. Parameters  $a$  and  $b$  in equation (10) are estimated by least squares regression. Table 3 shows the regression results.

### ***Risk Management Results***

With coefficient beta and parameters  $a$  and  $b$  can we calculate the optimal portfolio choice and investigate the risk management value of weather derivatives and abatement equipment using equations (17), (18) and (21). The loading rate  $\gamma$  in equation (9) is set as 5%. And the producer's risk aversion level, which is represented by Pratt's Absolute Risk Aversion (PARA), is set as 0.20.<sup>11</sup> Farmers are assumed to be able to choose strike levels of weather options.

Table 4 shows the effectiveness of risk management strategies under different scenarios. First, for the sake of comparison, suppose a representative farmer does not employ either weather derivatives or abatement equipment. With  $\beta$ ,  $a$  and  $b$ , his annual rev-

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<sup>10</sup>In SCS there are three abatement effect models corresponding to three abatement intensity levels. The first model is for only using fans or sprinklers; the second model is for a combination of fans and sprinklers; and the third model is for a specific system, the Korral Cool system, which is used in the Southwest and other dry and hot areas. In the research, we use the second model, and based on this model, we linearly simulate six abatement effect functions corresponding to six different fixed cost levels. See Appendix.

<sup>11</sup>See, for example, Pratt (1964). Note that in this paper, we make no assumption about whether the risk aversion parameter is constant, decreasing, or increasing with initial wealth levels. We are studying the case that a representative farmer faces an opportunity to buy weather options which will not change his expected wealth level and needs to decide how much money to invest on weather options. So we have an implicit assumption that changes of expectation and variance of profit due to using abatement equipment and weather options will not affect his risk aversion degree.

enue loss from heat stress can be calculated, which can then be put into mean-variance framework to calculate his utility loss. The increased utility by using these two instruments is calculated by equation (21) in certainty equivalent and presented in table 4. No analytical results of the optimal strike level can be derived. The increased utility corresponding to a series of alternative strike levels are examined and the optimal strike level is chosen as the one with the maximum increased utility. There are six different scenarios being examined:

1. no basis risk, no transaction costs;
2. only with reference-index basis risk, no transaction costs;
3. only with geographical basis risk, no transaction costs;
4. with both kinds of basis risk, no transaction costs;
5. with both kinds of basis risk and transaction costs;
6. only using abatement equipment, no weather derivatives used.

Table 4 displays that the increased utility is decreasing along the 6 scenarios. Take a representative farmer in Summit County of Ohio as an example. His yearly utility loss is \$91 in certainty equivalent without using any instruments.<sup>12</sup> In scenario 1, his increased utility is \$52.4. So, the optimal use of weather options and abatement equipment can reduce utility loss by 58%. In scenario 2, when the only available weather options is on temperature of Summit County, his increased utility is \$47.7. Then the reduced utility loss decreases from 58% to 53% because of the presence of reference-index basis risk. In scenario 3, there are no weather derivatives available on basis of Summit County’s weather indices and the farmer only has access to call options written on THI of Cleveland (around 50 miles away from Summit County). With the geographical basis risk ,his increased utility is \$47.21, which reduces utility loss by 52%. In the presence of both kinds of basis risk, his reduced utility loss is 48% and 46% with and without transaction costs. However, the reduced utility loss is 32% if only using abatement equipment, which is lower than those of other scenarios. The results of other production areas are similar. Figure 1 shows the average reduced utility loss across the selected production areas from above scenarios 1, 5 and 6. Therefore, although basis risk reduces the effectiveness of weather derivatives, weather derivatives can significantly improve upon the effectiveness of abatement equipment for reducing dairy profitability risk.

### ***Distance Effect***

It is interesting to know whether the distance in geographical basis risk plays a role in risk management effectiveness. In other words, we examine if the larger the distance

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<sup>12</sup>The annual net revenue from a dairy cow typically is around \$330, which is calculated based on Gayle S. Willett’s report “How Much Debt can a Dairy Cow Carry?” at <http://cru.cahe.wsu.edu/CEPublications/eb1762/eb1762.html>.

between the production area and weather options reference city, the less the effectiveness in using weather derivatives. The reduced effect due to basis risk and transaction costs is measured in percentage by dividing the difference between increased utility in scenario 1 ( $\Delta U$ ) and scenario 5 ( $\Delta U^{IGT}$ ) by increased utility in scenario 1.

$$\text{Reduced Effect} = \frac{\Delta U - \Delta U^{IGT}}{\Delta U} \times 100\%$$

Figure 2 shows the reduced effect against different distances in the 10 production areas and there is not a clear linear relationship between distance and reduced effect.

### *Different Risk Aversion Degrees*

So far, the producer's risk aversion degree, PARA, is set as 0.2. In this subsection, a series of different PARAs are examined. Again, Summit County is taken as an example. Corresponding to different PARAs, the percentage of increased utility in the utility loss without using the instruments is measured under three scenarios. Supposing the presence of both kinds of basis risk and transaction costs, the three scenarios are examined: (i) using abatement equipment alone; (ii) using weather options alone; (iii) using both instruments. The PARAs range from 0 to 0.55. Figure 3 shows the results. The optimal portfolio choices bring more utility than only using abatement equipment or weather options. If the producer's PARA is less than 0.37, using abatement equipment alone will bring more utility compared with using weather options alone; if his PARA is higher than 0.37, using weather options alone will be more favorable than using abatement equipment alone. An extreme case is that the producer is risk neutral, i.e., his PARA is zero. Then buying weather options will bring no benefit to him because weather options bear transaction costs.<sup>13</sup>

### *Cross-Validation and Stochastic Dominance*

Cross-Validation is an often-used data resampling method. The procedure is that 34 year data out of the 35 year data are used to estimate parameters in equations (6) and (10), and determine the optimal portfolio ( $\phi^*$  and  $\eta^*$ ) and strike level, which together with the one year data left are then put into equation (11) as an out-of-sample evaluation to obtain the net profit of using the portfolio. This procedure is repeated 35 times by successively omitting one of the 35 observations each time.

First, with the cross-validation results, the robustness of parameter estimation can be investigated. The results show that the estimates of  $\beta$ ,  $a$  and  $b$  are robust. For

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<sup>13</sup>In this case,  $\phi^*$  is zero if selling is forbidden to the producer.

instance, the mean of the 35 estimates of  $\beta$  in Summit County is 0.564kg/cow, and the standard deviation of these estimates is 0.0342. The means and standard deviations of these parameter estimates are very close to the estimates and standard errors reported in table 2 and 3. To save space, we have omitted reporting the detailed results.

Then, with the cross-validation results, we can test the effectiveness of using weather derivatives and/or abatement equipment by stochastic dominance criteria, which compare the performance of any two risk management strategies requiring much less restricted assumptions than a specific utility function. First degree stochastic dominance criterion (FSD) holds for all decision makers who prefer more to less. Second degree stochastic dominance (SSD) holds for all decision makers who have diminishing marginal utility, or equivalently a concave preference function. And third degree stochastic dominance (TSD) holds for all decision makers who are decreasingly averse to risk as they become wealthier.

Four risk management strategies to be compared include: (i) use none of the two instruments; (ii) use abatement equipment alone; (iii) use weather options alone; (iv) use weather options and abatement equipment simultaneously. Weather options here have two kinds of basis risk and transaction cost, i.e. under scenario 5. Comparison is made and yields similar results across the selected production areas. As an illustration, the results of Summit County are shown in table 5. In the sense of first and second degree stochastic dominance, using both instruments dominates using none of them and using weather options alone. In the sense of third degree stochastic dominance, using both instruments dominates all the other three strategies. This finding is consistent with that in the mean-variance utility framework. We also notice that the mean of 35 out-of-sample profit loss of Summit County is  $-\$49.7$  with a standard deviation of 20.3; the mean of 35 out-of-sample net profit loss with using the two instruments is  $-\$34.4$  with a standard deviation of 9.7. Thus, we see that using weather options and abatement equipment can significantly reduce both the mean and variance of profit loss from heat stress in summer. And we also observe that there are 27 out of the 35 years where the net profit from using the optimal portfolio is positive. The maximum is  $\$48.9$  and the minimum is  $-\$8.6$ . That means in most cases, optimally using weather options and abatement equipment can increase net profit. Moreover, negative net profit from using these instruments only happens when weather conditions favor milk production, namely the milk losses are relatively low. Therefore, using weather options together with abatement equipment can smooth the producer's yearly net revenue. That is a desirable result for a risk averse producer.

## Conclusion

This study examines the effect of basis risk in weather derivatives, which is one of the main concerns when investigating the viability and values of weather derivatives acting

as a risk management tool in weather sensitive industries. This research is conducted by exploring the use of weather derivatives together with traditional abatement technologies to manage dairy profit risk from heat stress and performing a theoretical and empirical analysis of the effect of two main kinds of basis risk in weather derivatives.

A representative dairy producer's profit risk is decomposed into systematic risk from weather conditions and idiosyncratic risk which is uncorrelated with weather conditions. With the access to weather options and abatement equipment, the producer's optimal portfolio choice of these two instruments is derived in a mean-variance utility maximization framework. And first-, second-, and third-degree stochastic dominance criteria are used to compare the different risk management strategies with unrestrictive assumptions. The results suggest that although basis risk reduces the effectiveness of weather derivatives, it still is a significant improvement to use weather derivatives along with abatement equipment in dairy profit risk management.

This paper provides an applicable link of the weather derivatives literature in agricultural economics to a real-world application in which an easily-quantifiable weather metric (daily THI in excess of a biological threshold) is the primary source of production risk for a major agricultural commodity. In addition, this research raises many questions of relevance to the economic community, such as the optimal contract design, whether the existence of these contracts reinforces economies of scale in dairy production, what level of sophistication is required to effectively utilize these tools, and finally, what size of a dairy is required to use weather derivatives. These questions may be of interest for further research.

## References

- Barth, C. L. "State-of-the-art for Summer Cooling for Dairy Cows." 52-61, in *Livestock Environment II, Proceedings of the Second International Livestock Environment Symposium*, Scheman Center, Iowa State University, April 20-23, 1982.
- Dischel, B. "Is Precipitation Basis Risk Overstated?" *Weather Risk Special Report, Energy and Power Risk Management/Risk*, Aug. 2000.
- Johnson, H. D. "Bioclimate Effects on Growth, Reproduction and Milk Production." p. 35, in Johnson, H. D. (ed.) *Bioclimatology and the Adaptation of Livestock*. Elsevier, Amsterdam, The Netherlands, 1980.
- Lin, J. C., B. R. Moss, J. L. Koon, C. A. Flood, R. C. Smith III, K. A. Cummins and D. A. Coleman. "Comparison of Various Fan, Sprinkler, and Mister Systems in Reducing Heat Stress in Dairy Cows." *Applied Engineering in Agriculture* 14(1998): 177-182.
- Miranda, M. "Area-Yield Crop Insurance Reconsidered." *American Journal of Agricultural Economics* 73(1991): 233-242.
- Meyer, J. "Two-Moment Decision Models and Expected Utility." *American Economic Review* 77(1987): 421-430.
- National Oceanic and Atmospheric Administration, "Livestock Hot Weather Stress." Reg. Operations Letter C-31-76. US Department of Commerce, National Weather Service, Central Region, Kansas City, Missouri, 1976.
- Pratt, J. "Risk Aversion in the Small and in the Large." *Econometrica* 32(1964): 122-136.
- St-Pierre, N. R., B. Cobanov and G. Schnitkey. "Economic Cost of Heat Stress: Economic Losses from Heat Stress by U.S. Livestock Industries." *Journal of Dairy Science* 86(2003): 52-77.
- Thompson, G. E. "Review of the Progress of Dairy Science Climatic Physiology of Cattle." *Journal of Dairy Research* 49(1973): 441-473.
- Turner, L.W., J. P. Chastain, R. W. Hemken, R. S. Gates and W. L. Crist. "Reducing Heat Stress in Dairy Cows through Sprinkler and Fan Cooling." *Applied Engineering in Agriculture* 8(1992): 251-256.
- Turvey, C. "Weather Derivatives for Specific Event Risks in Agriculture." *Review of Agricultural Economics* 33(2001): 335-351.
- Vedenov, D. and M. Miranda. "Rainfall Insurance for Midwest Crop Production." Selected Paper, AAEA Annual Meetings, Ames, IA 2001.

## Appendix

### *Proof of Proposition 5*

From equations (17) and (18),

$$\begin{aligned}
 (A.1) \quad \frac{d\phi^*}{d\beta} &= \frac{\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2} - b \frac{\sigma_{\widehat{THI},\tilde{n}}}{\sigma_{\tilde{n}}^2} \frac{Ab[\sigma_{\widehat{THI},\tilde{z}} - \frac{\sigma_{\widehat{THI},\tilde{n}}\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2}]}{2 + Ab^2[\sigma_{\widehat{THI}}^2 - \frac{\sigma_{\widehat{THI},\tilde{n}}^2}{\sigma_{\tilde{n}}^2}]} \\
 &= \frac{1}{\sigma_{\tilde{n}}^2} \left[ \sigma_{\tilde{z},\tilde{n}} - \frac{Ab^2\sigma_{\widehat{THI},\tilde{n}}[\sigma_{\widehat{THI},\tilde{z}} - \frac{\sigma_{\widehat{THI},\tilde{n}}\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2}]}{2 + Ab^2[\sigma_{\widehat{THI}}^2 - \frac{\sigma_{\widehat{THI},\tilde{n}}^2}{\sigma_{\tilde{n}}^2}]} \right] \\
 &= \frac{1}{\sigma_{\tilde{n}}^2} \left[ \sigma_{\tilde{z},\tilde{n}} - \frac{Ab^2\rho_{\widehat{THI},\tilde{n}}\sigma_{\widehat{THI}}\sigma_{\tilde{n}}\sigma_{\widehat{THI}}\sigma_{\tilde{z}}[\rho_{\widehat{THI},\tilde{z}} - \rho_{\widehat{THI},\tilde{n}}\rho_{\tilde{z},\tilde{n}}]}{2 + Ab^2\sigma_{\widehat{THI}}^2[1 - \rho_{\widehat{THI},\tilde{n}}^2]} \right]
 \end{aligned}$$

If the last item of the numerator in the big bracket, i.e.  $[\rho_{\widehat{THI},\tilde{z}} - \rho_{\widehat{THI},\tilde{n}}\rho_{\tilde{z},\tilde{n}}]$  is negative<sup>14</sup>, then it is easy to see that  $\frac{d\phi^*}{d\beta} > 0$ .

If  $[\rho_{\widehat{THI},\tilde{z}} - \rho_{\widehat{THI},\tilde{n}}\rho_{\tilde{z},\tilde{n}}]$  is positive, it follows from (A.1):

$$\begin{aligned}
 (A.2) \quad \frac{d\phi^*}{d\beta} &= \frac{1}{\sigma_{\tilde{n}}^2} \left[ \sigma_{\tilde{z},\tilde{n}} - \frac{Ab^2\sigma_{\widehat{THI}}^2\sigma_{\tilde{n}}\sigma_{\tilde{z}}[\rho_{\widehat{THI},\tilde{z}}\rho_{\widehat{THI},\tilde{n}} - \rho_{\widehat{THI},\tilde{n}}^2\rho_{\tilde{z},\tilde{n}}]}{2 + Ab^2\sigma_{\widehat{THI}}^2[1 - \rho_{\widehat{THI},\tilde{n}}^2]} \right] \\
 &> \frac{1}{\sigma_{\tilde{n}}^2} \left[ \sigma_{\tilde{z},\tilde{n}} - \frac{Ab^2\sigma_{\widehat{THI}}^2\sigma_{\tilde{n}}\sigma_{\tilde{z}}[\rho_{\widehat{THI},\tilde{z}}\rho_{\widehat{THI},\tilde{n}} - \rho_{\widehat{THI},\tilde{n}}^2\rho_{\tilde{z},\tilde{n}}]}{Ab^2\sigma_{\widehat{THI}}^2[1 - \rho_{\widehat{THI},\tilde{n}}^2]} \right] \\
 &= \frac{\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2} \left[ 1 - \frac{\rho_{\widehat{THI},\tilde{z}}\rho_{\widehat{THI},\tilde{n}} - \rho_{\widehat{THI},\tilde{n}}^2\rho_{\tilde{z},\tilde{n}}}{1 - \rho_{\widehat{THI},\tilde{n}}^2} \right] \\
 &= \frac{\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2} \left[ 1 - \frac{\frac{\rho_{\widehat{THI},\tilde{z}}\rho_{\widehat{THI},\tilde{n}}}{\rho_{\tilde{z},\tilde{n}}} - \rho_{\widehat{THI},\tilde{n}}^2}{1 - \rho_{\widehat{THI},\tilde{n}}^2} \right]
 \end{aligned}$$

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<sup>14</sup>This normally does not happen. Refer to the derivation of Proposition 4.

So if this inequality holds,

$$(A.3) \quad \frac{\rho_{\widetilde{THI}, \widetilde{z}} \rho_{\widetilde{THI}, \widetilde{n}}}{\rho_{\widetilde{z}, \widetilde{n}}} < 1,$$

then,

$$(A.4) \quad \frac{d\phi^*}{d\beta} > 0.$$

Hence, the inequality (A.3) is the sufficient condition for (A.4) to hold, i.e. Proposition 5. By the definitions of  $\widetilde{z}$ , and  $\widetilde{n}$  in (3) and (8), it is a reasonable assumption that  $\rho_{\widetilde{z}, \widetilde{n}} > \rho_{\widetilde{THI}, \widetilde{n}}$ . Together with the fact that  $\rho_{\widetilde{THI}, \widetilde{z}}$  is less than 1, we have the inequality (A.3). Actually this inequality holds for all of our weather data.

### ***Milk Loss Function***

The milk loss model in SCS (2003) is:  $MILK_{loss} = 0.0695 * (THI_{max} - THI_{threshold})^2 * Duration$ , where  $MILK_{loss}$  is in kilogram, and  $Duration$  is the proportion of a day where heat stress occurs (i.e.  $THI_{max} > THI_{threshold}$ ).

The process to calculate the  $Duration$  of heat stress:

$$\begin{aligned} & THI_{mean} = (THI_{max} + THI_{min})/2 \\ & \text{if } THI_{max} < THI_{threshold} \\ & \quad Duration = 0 \\ & \text{elseif } THI_{min} \geq THI_{threshold} \\ & \quad Duration = 24 \\ & \text{elseif } THI_{mean} > THI_{threshold} \\ & \quad Duration = (PI - 2 * \arcsin(\frac{THI_{threshold} - THI_{mean}}{THI_{max} - THI_{mean}})) / PI * 12 \\ & \text{else } Duration = (PI + 2 * \arcsin(\frac{THI_{mean} - THI_{threshold}}{THI_{max} - THI_{mean}})) / PI * 12 \\ & \text{end} \end{aligned}$$

where  $PI = 3.1415\dots$

### ***Abatement Effect Function***

In SCS, for a 50 m<sup>2</sup> cow pen, which can hold 7.1759 dairy cows, when the annualized fixed costs are \$310, the corresponding operating costs are \$0.0685/hour of operation. And the abatement effect is:  $\Delta THI = -17.6 + (0.36 * T) + (0.04 * H)$ , where  $\Delta THI$  is the change in apparent THI,  $T$  is ambient temperature (°C), and  $H$  is ambient relative humidity in percent.

Based on the above specifications, we linearly simulate six abatement effect functions corresponding to six fixed cost levels. The six fixed cost levels are 130, 190, 250, 370, 430, 490 dollars respectively. That is, all the parameters in a simulated model are proportional to those in the SCS model, with the proportion equal to the ratio of fixed cost levels.

We define the reduced profit loss by:

$$\Delta loss = \max(\min(THI_{max} - THI_{threshold}, \Delta THI), 0) * \beta * MILKprice.$$

Table 1: Production Areas and Options Reference Cities

State	County	Reference City	Distance (mile)
OH	Summit	Cleveland	49.4
OH	Franklin	Cleveland	125.9
OH	Trumbull	Cleveland	58.0
IL	Rock Island	Springfield	131.1
NY	Albany	Buffalo	252.1
NY	Broome	Buffalo	148.6
NY	Queens	Buffalo	291.9
NY	Monroe	Buffalo	62.9
NY	Onondaga	Buffalo	125.6
WI	Dane	Milwaukee	79.6

Note: County is the production area; Distance is between the the weather stations of the production area and of reference city.

Table 2: Beta Coefficient Estimation

State	County	$\beta$	$R^2$
OH	Summit	0.56 (0.034)	0.89
OH	Franklin	0.71 (0.048)	0.87
OH	Trumbull	0.52 (0.033)	0.88
IL	Rock Island	0.76 (0.057)	0.84
NY	Albany	0.61 (0.037)	0.89
NY	Broome	0.38 (0.020)	0.92
NY	Queens	0.84 (0.039)	0.93
NY	Monroe	0.60 (0.029)	0.93
NY	Onondaga	0.59 (0.028)	0.93
WI	Dane	0.63 (0.045)	0.85

Note: Numbers in parentheses are standard errors.

Table 3: Beta Coefficient Estimation

State	County	$a$	$b$	$R^2$
OH	Summit	-57.41 (3.45)	0.0051 (0.00027)	0.91
OH	Franklin	-48.13 (5.66)	0.0046 (0.00043)	0.90
OH	Trumbull	-56.66 (3.03)	0.0050 (0.00024)	0.91
IL	Rock Island	-60.23 (6.46)	0.0055 (0.00049)	0.91
NY	Albany	-51.34 (3.53)	0.0047 (0.00028)	0.91
NY	Broome	-40.88 (1.38)	0.0036 (0.00011)	0.90
NY	Queens	-60.14 (5.32)	0.0056 (0.00041)	0.92
NY	Monroe	-56.16 (2.80)	0.0050 (0.00022)	0.93
NY	Onondaga	-58.18 (2.67)	0.0052 (0.00021)	0.93
WI	Dane	-50.37 (3.97)	0.0046 (0.00031)	0.88

Note: Numbers in parentheses are standard errors.

Table 4: Risk Management Effectiveness

State	County	$ULoss$	$\Delta U$	$\Delta U^I$	$\Delta U^G$	$\Delta U^{IG}$	$\Delta U^{IGT}$	$\Delta U^A$
OH	Summit	90.80	52.39 (71)	47.73 (76)	47.21 (74)	43.41 (80)	42.06 (81)	28.73
OH	Franklin	178.38	114.52 (71)	107.87 (77)	104.10 (70)	98.51 (76)	95.66 (77)	64.98
OH	Trumbull	78.98	44.25 (71)	41.04 (75)	39.82 (74)	35.13 (75)	33.87 (80)	24.02
IL	Rock Island	186.59	108.02 (72)	97.46 (76)	96.54 (75)	85.05 (76)	82.05 (80)	62.58
NY	Albany	79.31	44.62 (71)	41.11 (77)	40.67 (72)	39.09 (75)	37.39 (78)	25.16
NY	Broome	37.76	17.38 (71)	16.69 (73)	14.40 (71)	14.20 (75)	13.02 (79)	7.19
NY	Queens	155.83	98.62 (71)	91.81 (76)	82.69 (71)	79.25 (74)	77.05 (75)	62.15
NY	Monroe	82.61	50.96 (71)	47.60 (75)	48.65 (72)	45.75 (75)	43.70 (78)	26.01
NY	Onondaga	83.54	50.50 (71)	47.15 (75)	47.38 (72)	44.08 (75)	42.17 (78)	27.55
WI	Dane	114.44	67.14 (71)	61.07 (75)	59.55 (70)	57.35 (68)	54.16 (74)	33.32

Note:  $ULoss$  is the utility loss in dollar without using the two instruments;

$\Delta U$  is the increased utility in dollar using the two instruments without basis risk or transaction costs;

$\Delta U^I$  is the increased utility in dollar using the two instruments with index basis risk;

$\Delta U^G$  is the increased utility in dollar using the two instruments with geographical basis risk;

$\Delta U^{IG}$  is the increased utility in dollar using the two instruments with index and geographical basis risk;

$\Delta U^{IGT}$  is the increased utility in dollar using the two instruments with index and geographical basis risk and transaction costs;

$\Delta U^{IGT}$  is the increased utility in dollar from using abatement equipment alone.

Numbers in parentheses are optimal strike levels.

Table 5: Stochastic Dominance Comparison Results

FSD	SSD	TSD
Both $\succ$ None	Both $\succ$ None	Both $\succ$ None
Both $\succ$ Options	Both $\succ$ Options	Both $\succ$ Options
Abatement $\succ$ None	Abatement $\succ$ None	Both $\succ$ Abatement
	Abatement $\succ$ Options	Abatement $\succ$ None
		Abatement $\succ$ Options

Note: The sign “ $\succ$ ” means “stochastically dominates”; “Both” means using both instruments; “Abatement” means using abatement equipment alone; “Options” means using weather options alone; “None” means using none of the two instruments.

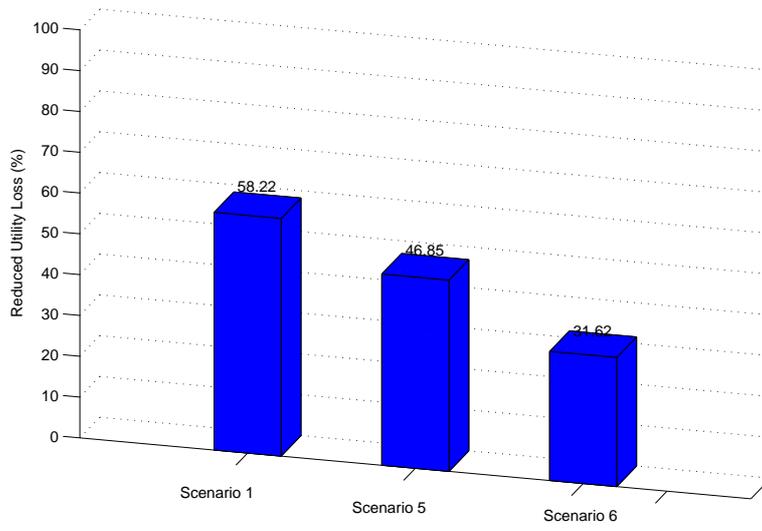


Figure 1: Effectiveness comparison

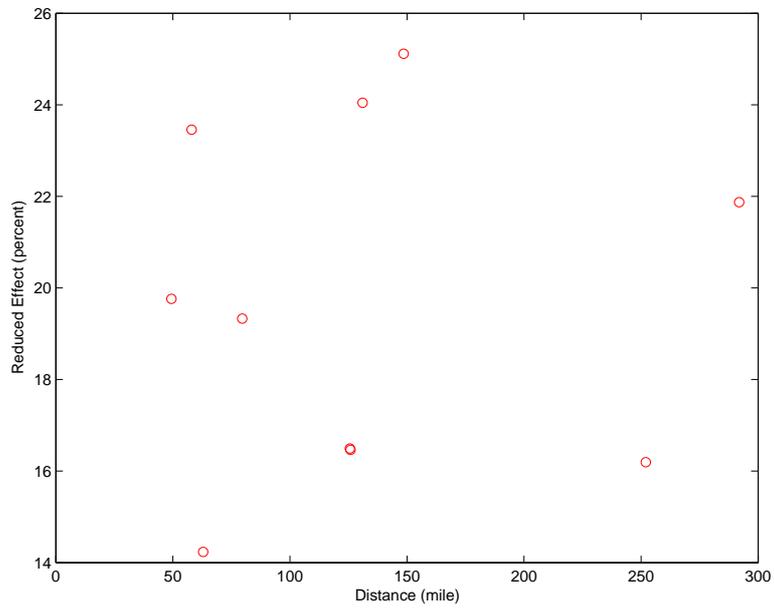


Figure 2: Distance effect

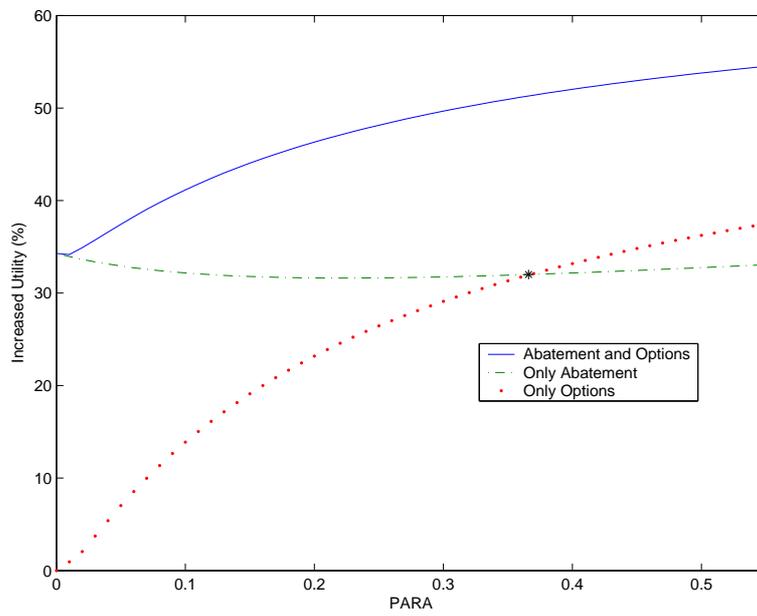


Figure 3: Increased utility with different PARAs