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Pricing Weather Derivatives for Agricultural Risk Management

Practitioners' Abstract

Existing derivative pricing methods cannot be used to price weather derivatives due to the absence of a hedgeable commodity underlying weather risk and the complexity of weather processes. This study develops a pricing model that considers weather derivatives to be the same as any other financial asset. In this way, the price of a weather derivative is an equilibrium price consistent with both the potential payout at expiry and the market price of risk. We apply this model to the pricing of weather derivatives in the Central Valley of California and find significant differences in prices obtained under alternative weather process assumptions.

keywords: derivative, Monte Carlo, pricing, risk, weather

Introduction

The inadequacy of existing derivative pricing models based on historical simulation methods is by now relatively well understood (Cao and Wei; Turvey; Dischel 1998b). However, the use of more theoretically consistent pricing models based on Black and Scholes is also not possible due to the fact that a temperature index – the underlying “commodity” for weather derivatives – is not a hedgeable commodity. Without recourse to these pricing methods, recent research has sought to use some variant of equilibrium model that values weather derivatives within a general contingent claims framework (Cao and Wei; Pirrong and Jermakatan). Critical to this approach, however, is the ability to estimate and incorporate the market price of risk.

If traders cannot hedge their risk they must be compensated for bearing it, so the price of the derivative must reflect a potentially substantial risk premium. However, there is little theoretical or empirical work in the literature that addresses the critical problem of estimating the market price of risk. Many authors either assume it away, or provide justification for why it may be insignificant. Turvey, for example, relies on an assumption that the stochastic process for a temperature index exhibits independent increments. By ruling out either mean-reversion or time-varying volatility, he creates a simple environment in which a zero weather-beta is, in fact, possible. On the other hand, Alaton, Djehiche and Stillberger and Cao and Wei provide empirical evidence to the contrary, at the same time confirming Pirrong and Jermakayan's belief that weather indices are likely highly non-linear, seasonal and mean-reverting. If this is the case, then there is no stochastic representation that can provide an analytical solution to the valuation problem and, more importantly, analysts must establish the contribution of weather to the riskiness of an otherwise well-diversified portfolio. Therefore, the value of a weather derivative that follows a typical weather process must be found using an equilibrium valuation technique and appropriate numerical solution methods.

Determining the value of weather derivatives is the primary objective of this research. In order

to achieve this broader objective, however, we set two other, specific objectives: (1) establishing the nature and existence of the link between weather and crop yield, and (2) estimating the market price of risk. We also review previous work within this same research program which estimates the appropriate weather process for a Central California cooling degree day (CDD) index. Because this process is highly complex, it is clear from this first-stage of research that Black-Scholes option pricing models will not suffice. Consequently, we develop an equilibrium valuation model that provides implicit estimates of the market price of risk. The complexity of the underlying weather process also means that Monte Carlo simulation methods must be used to provide estimates of all derivative prices and risk premia. In order to establish a link between yield and weather, we employ a case study consisting of Central California tree fruit – perhaps the most likely users of weather derivatives given their proximity to many natural counterparties (amusement parks, hotel chains, etc.) – to compare estimates among alternative weather process assumptions. The paper concludes by commenting on the likely usefulness of weather derivatives for managing weather-borne risk and suggests some avenues for future research in this area.

A Pricing Model for Weather Derivatives

Overview of Modeling Method

Pricing any derivative security necessarily begins by developing an understanding of the precise statistical form of the underlying price or index process. Consequently, this section begins with a brief review of our previous weather-process estimation efforts as well as a description of the weather data used. A discussion of the theoretical and practical issues involved in creating a weather derivative pricing model is then followed by an explanation of our approach and a discussion of our results.

Data Sources and Sample Description

The weather data for this study are from the U.S. National Climatic Data Center (NCDC) for a weather station located in Fresno, CA. Estimates of the CDD data process are obtained using 30 years of daily average temperatures. Although there are many more years' of data from the NCDC, the sample used in this study consists of 30 years in order to gain as much estimation efficiency as possible while minimizing the “heat island” effect that arises in arid and semi-arid areas with the creation of heat-retention trapping buildings, roads and artificial parks (Dischel 1998a). The weather data consist of daily maximum, minimum and average temperature as well as daily precipitation for the entire year. However, the CDD index is defined so that it describes temperature over an entire growing season that runs from May through July – the critical phase of final fruit development for the soft fruit (peaches, plums, and nectarines) and table grapes grown in the Fresno area. Specifically, the CDD index is defined as the cumulative sum of the extent to which daily average temperatures exceed a 65 degree Fahrenheit benchmark:

$$(1) \quad w_t = \sum_{\tau=1}^T (\bar{x}_\tau - 65),$$

where \bar{x}_t is the average daily temperature on day t measured in degrees Fahrenheit. Although the temperature series is not directly applicable to any particular grower, primarily because it is gathered at the Fresno Air Terminal, the proximity of many growers to Fresno and the relative topographical homogeneity of the surrounding area should minimize the basis risk that would likely exist for growers located farther away from the weather station.

In order to establish the relationship between this temperature index and crop yields, the yield model uses annual county-average yield data for peaches, nectarines and raisin grapes as reported to the California Department of Food and Agriculture (CDFA) by the county commissioner's office. County-level yield data necessarily induces some aggregation bias, but avoids idiosyncratic yield variations that would arise with farm-level, panel data on crop yields. Table 1 provides summary statistics for the CDD index for a series of 5-year intervals from 1970 to 2000. Note that, contrary to what many believe, these data do not suggest that a heat island effect has been responsible for a general rise in temperatures. Several alternative forms for the underlying weather process are estimated with these data.

Alternative Stochastic Processes

Correctly characterizing the stochastic process of the underlying price or value index is critical to obtaining accurate derivative prices. In previous research, the authors tested from among several alternative stochastic processes for the Fresno CDD index. These results are summarized here before the equilibrium pricing model is presented. Based on prior research on weather derivatives, several possible alternative CDD processes were identified (Cao and Wei; Turvey; Alaton, Djehiche and Stillberger; Pirrong and Jermakayan). The seven candidates include: (1) geometric Brownian motion (GBM), (2) GBM with a log-normal jump (GBM-J), (3) mean-reverting GBM (MRGBM), (4) mean reverting GBM with a log-normal jump (MRGBM-J), (5) an autoregressive, conditional heteroskedastic (ARCH) version of (2) (GBM-J-ARCH), and (6) an ARCH version of (4) (MRGBM-J-ARCH) and (7) a generalized autoregressive, conditional heteroskedastic version of (4) (MRGBM-J-GARCH). In each of the cases (5) and (6), the possibility that the volatility of weather processes may be time-varying is addressed by using a simple ARCH error structure similar to Jorion, although more complex, stochastic alternatives are not ruled out (Hull and White) in process (7). To determine which process provides a better fit to the weather data, nested models are compared using likelihood ratio tests in a pair-wise manner.

Because each of the first six models are special cases of the most general MRGBM-J-GARCH model, it suffices to describe only the final model here. In continuous-time notation, the stochastic weather process is written as:

$$(2) \quad \frac{dw_t}{w_t} = \kappa (\alpha_0 - \lambda \phi - \ln w_t) dt + \sigma_c dz + \phi dq,$$

where w is the CDD index, F_c is the variance of the weather process conditional on no discontinuities, δ is the rate of mean-reversion, q is the Poisson counter with mean arrival rate δ , and N is random percentage jump in the weather index conditional on a Poisson event and dz defines the Wiener process with properties: $E(dz) = 0$ and $E(dz^2) = dt$. Further, the variance term in (2) is written in terms of a GARCH (1,1) process as (Bollerslev):

$$(3) \quad \sigma_{\epsilon t}^2 = \theta_0 + \theta_1 v_{i,t-1}^2 + \theta_2 \sigma_{\epsilon,t-1}^2,$$

where $v_{i,t-1}^2$ is the variance of the log-relatives of the weather index.

Although there are several alternative ways of estimating the process defined in (3), the only feasible approach here, given the absence of a consistent, reliable series of CDD option prices that would allow the estimation of implicit parameters, is the maximum likelihood (ML) approach of Ball and Torous (1985); Jorion; and Jarrow and Rosenfeld. With this approach, each of the specifications defined above are tested against each other using a series of likelihood-ratio (LR) tests. These specifications include the six alternatives defined above: (1) GBM, (2) GBM-J, (3) MRGBM, (4) MRGBM-J, (5) GBM-J-ARCH, and (6) MRGBM-J-ARCH.¹ The most general of these specifications, including mean reversion, log-normal jumps and ARCH errors is:

$$(4) \quad L(\boldsymbol{\beta} | \boldsymbol{\beta}) = -T\lambda - \frac{T}{2} \ln(2\pi) + \sum_{t=1}^T \ln \left[\sum_{n=0}^N \frac{\lambda^n}{n!} \frac{1}{\sqrt{h_t + \delta^2 n}} \exp \left(\frac{-(v_t - (\alpha_0 + h_t/2 + n\delta^2/2 - n\phi - \ln w_t)(1 - e^{-\kappa}))^2}{2(h_t + \delta^2 n)} \right) \right],$$

for M observations of log-relatives of the weather index: $(v_t = \ln(w_{i,t}/w_{i,t-1}))$ where δ is the Poisson intensity parameter, σ_c^2 is the volatility of the discrete part, h_t the volatility of the continuous part, and N is the mean jump size.² Following Ball and Torous (1985), n is defined as the random realization of a

¹ Although estimating the GARCH(1,1) model described in the conceptual development may be preferable, it would not converge in this problem. This is perhaps understandable given the demands that GARCH models make on the data, the amount of complexity elsewhere in our model and the fact that a GARCH process would likely explain many of the same innovations defined within the model as jumps.

² This assumption is more general than Ball and Torous (1985), who assume a jump size of mean zero.

shock to revenue, and N is fixed at a value likely to include all possible occurrences of a shock, and (4) is maximized with respect to the remaining parameters. Each of the other likelihood functions are defined in a similar way, but with appropriate restrictions to arrive at the particular maintained form of the stochastic process for w_t .

Table 2 shows the results obtained by comparing models of increasing generality. Clearly, these tests show that the MRGBM-J-ARCH model is the preferred alternative. As explained in more detail below, the critical implication of this finding is that the market price of risk must be estimated as part of an equilibrium valuation framework.

Equilibrium Pricing Model

Several recent studies describe applications of the equilibrium pricing approach of Lucas or Cox, Ingersoll and Ross to either weather (Cao and Wei) or energy (Pirrong and Jermakayan) derivatives. When it is not possible to construct a riskless hedge and, thereby, apply usual no-arbitrage pricing arguments, a derivative security's price must reflect an underlying equilibrium described by the fundamental valuation partial differential equation (PDE). With this approach, the terminal value required to solve the PDE is found from a structural model of the returns available to the holder of the security. In the weather derivative case, therefore, the equilibrium price is derived from the payoff to growing an agricultural commodity subject to the weather risks described herein. For most agricultural commodities, output markets are competitive and global in nature, so the price is assumed to be parametric to the typical grower's level of sales. Output, however, depends on the grower's production technology, endogenous input choices, and fixed factors such as land quality, managerial skill and, of course, weather factors. With aggregate data, obvious production factors such as growers' input, skill or land quality cannot be modeled so yield fluctuations are assumed to be entirely determined by weather factors, namely heat and precipitation. A common production function that is both parsimonious and captures the diminishing marginal returns to production inputs, including heat and precipitation, is the Cobb-Douglas (double-log):

$$(5) \quad \ln y_t = \beta_0 + \sum_j \beta_j \ln x_j + \sum_k \beta_k \ln z_k + \sum_l \beta_l \ln w_l + \beta_m t + \epsilon_t,$$

for j variable inputs, x , k fixed inputs, z , a linear time trend, t , that is intended to proxy the rate of technological improvement and weather indices (w) for both temperature (CDD) and precipitation and an i.i.d. random error term, ϵ_t . In the California nectarine example described below, data are only available for weather indices so estimates are obtained for the impact of weather on yield alone. By estimating this equation over a number of years, we obtain parameters that are useful in forming yield expectations. Multiplying predicted yields by expected prices in the current year, this approach links weather to the value of production and, thereby, determines the value at expiry of any value of the CDD index. This terminal value is necessary to solve for equilibrium weather derivative prices. The equilibrium model must, however, take into account the idiosyncracies of both weather indices and the

derivatives linked to them.

Namely, to serve as the basis for an effective weather derivative trading program, an appropriate pricing model must be one that: (1) is consistent with the underlying weather index process, (2) reflects the fact that a degree-day index for most agricultural regions is not a tradable asset, but is rather a state variable, and (3) accounts for the fact that a CDD index is a cumulative sum, so its current value depends on past values of itself. Beyond the complexity of the underlying weather process, the fact that the weather index is not a traded asset but rather a state variable necessitates the use of an equilibrium valuation approach.³ Specifically, the risks associated with weather are non-diversifiable so the derivative price must reflect a market price of risk. This rules out the use of a Black-Scholes type of model. Turvey addresses the absence of a hedgeable asset by applying the risk-neutral pricing model of Cox and Ross under the simplifying assumptions that the mean drift rate of the weather index and the “weather beta” are both zero.⁴ However, his approach relies on the assumption, for which he provides some support in Toronto HDD data, that the stochastic process for temperature index exhibits independent increments. This rules out either mean-reversion or time-varying volatility. On the other hand, Cao and Wei provide empirical evidence to the contrary, confirming Pirrong and Jermakayan’s belief that weather indices are likely highly non-linear, seasonal and mean-reverting. If this is the case, then there is no stochastic representation that can provide an analytical solution to the valuation problem. Therefore, the equilibrium value of a weather derivative that follows the process in (2) must be found using appropriate numerical solution methods.

There are two common ways of accomplishing this in practice. First, one can specify the fundamental differential equation that describes the equilibrium derivative price and, given the terminal value condition for a European call option, solve for V using a finite difference or some other numerical method (Pirrong and Jermakayan). In the current example, begin by defining the value of a CDD weather option as V and the risk free rate of return, r , so that the rate of return to holding the derivative over a period t_0 to t_N , after adjusting for the market price of risk, must equal the risk-free rate of return:

$$(6) \quad rV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial w_t} w_t [\alpha_w(w_t, t) - \psi(w_t, t) \sqrt{h_t(w_t, t)}] + \left(\frac{1}{2} \right) \left(\frac{\partial^2 V}{\partial w_t^2} \right) w_t^2 h_t(w_t, t) - \lambda_t [V(w_t) - V(1 - \phi)w_t] + \eta(t) w_t \frac{\partial V}{\partial I}$$

³ Although there are CDD and HDD futures contracts on the CME, these apply to only a handful of major metropolitan areas, so cannot be used to hedge weather risks for growers in Central California, or many other agricultural areas for that matter.

⁴ This latter assumption is justified on the grounds that it is unlikely that the market portfolio can have any impact on the number of degree days, but it is not necessarily true that weather events do not impact the market portfolio.

where the mean and variance terms are as defined above, $R(w_t, t)$ is the market price of risk, $0 = 1$ if $t \in [t_0, t_N]$ and zero otherwise, and:

$$(7) \quad I = \int w(\tau) d\tau.$$

If V is the value of a call option, then the terminal condition that allows us to solve (6) is:

$$(8) \quad V = \max[(w_T - K), 0],$$

where w_T is the value of the CDD index at expiry, T , and K is the agreed-upon strike level of the CDD index specified in the option contract. Second, Boyle, and many others since, show that it is also possible to find prices for complex derivatives using Monte Carlo simulation techniques. With this approach, the process for the underlying state variable is simulated a large number of times and the value of the derivative at expiry is calculated subject to a terminal value condition similar to (8) above. By averaging the implied derivative prices over all random draws, this method provides a very close estimate of the true derivative price. Because this technique offers a flexible method of accommodating complexities that would otherwise make the problem intractable, we use this approach in pricing California weather derivatives. This approach also provides a relatively straightforward means of estimating the market price of risk.

Pirrong and Jermakayan estimate R by comparing forward electricity prices to spot prices and estimating the value of R implicitly as a function of time only. Alaton, Djehiche and Stillberger adopt a similar approach in a weather derivative context by estimating the market price of risk from existing derivative contracts. On the other hand, Turvey assumes the market price of risk is zero by the arguments above. In the absence of any comparable underlying asset price series, Cao and Wei develop a general equilibrium contingent claim model wherein they estimate the market price of risk as an implicit parameter. Specifically, they generate forward prices under conditions where the market price of risk would be zero (no contemporaneous or lagged correlation between weather and aggregate economic output) and compare these to prices generated under more general risk assumptions. They interpret significant differences between the two as evidence of a positive market price of risk. This study adopts Cao and Wei's approach within the general framework developed above and uses Monte Carlo solution techniques to estimate the market price of risk for each weather model.

In order to calibrate the payoff function, estimates of the relationship between California nectarine yields and season-ending CDD values are obtained for the 1980 - 2001 time period. If the market price of risk is zero, then the payoff will be independent of weather. By estimating a second yield function wherein yields are functions of time alone ie. independent of the weather index, and re-calculating the implied derivative prices, we are able to estimate the market price of risk. In other words, estimates of the market price of risk are obtained by comparing the derivative prices that emerge from pure time-series behavior of yields to those where yields depend on a weather index.

Further, all Monte Carlo derivative pricing models assume a constant, risk-free interest rate of 5%, a 92 day growing season (May 1 - July 31), a long-run average price of \$6.55 per carton, an average yield of 9.08 tons per acre and an average price calculated over 1,000 simulations. The next section summarizes the results obtained from calculating derivative prices in this way and comparing prices among each alternative weather process.

Results and Discussion

In this section, we present and discuss the results obtained from: (1) estimating two-stage least squares estimates of the nectarine yields models, and (2) conducting Monte Carlo simulations of weather derivative prices. Following the presentation of these results, we offer some implications for real-world pricing of weather claims and their usefulness as risk management tools.

Determining the nature of the relationship between weather and yield is necessary for both practical and technical reasons. Clearly, if there is no functional relationship between weather and yield, weather derivatives cannot be effective yield-risk management tools. More formally, traders must be able to quantify the relationship between weather and the underlying value of the derivative if a meaningful market is to arise. Estimating this relationship using the model in (5), however, involves many practical considerations. First, because reliable yield data for Fresno County crops are available for only the last twenty years, yield models must necessarily be as parsimonious as possible to capture the fundamental underlying weather-yield relationship. Second, for most agricultural products, yield is a function of both temperature and precipitation. Although the theoretical relationship between yield and precipitation for tree fruit is not as straightforward as for annual cereal or oilseed crops, it is still likely to be an important factor. Valuing derivatives that represent composites of both types of weather, however, greatly increases the complexity of the problem. Despite the fact that nectarine yields in the current data do not respond to cumulative precipitation amounts, resolution of this problem is an interesting problem for extensions to the single-state variable methods developed here. The complexity of perennial crop yields raises a third important issue, namely that yield may be a non-linear function not only of the cumulative value of a temperature index, but also the timing and severity of weather events during the growing season. With only annual yield data available, however, econometric estimation at this level of detail is beyond any data that are available. Given each of these issues, however, the data and model shortcomings do not preclude estimation of reasonable yield functions, as evidenced by the various measures of goodness-of-fit reported in table 3.

Given the limited number of annual observations available, the coefficients of determination are reasonably high and the estimated parameters adhere to prior expectations of sign and significance. Initially, the study considered three commodities: raisin grapes, nectarines and peaches – due both to yield data availability and their importance to the Fresno county agricultural economy – but the results reported here concern only nectarines in order to keep the interpretation and discussion manageable. Consequently, the remainder of the analysis considers the value of weather derivatives to nectarine growers.

Using estimated yield function parameters, equilibrium derivative values are found first by assuming a positive market price of risk, and then a zero market price of risk. Table 4 presents the resulting weather derivative price estimates and implied estimates of the market price of risk. Several interesting results are apparent from this table. First, t-tests are used to determine whether there is a statistically significant difference between derivative prices calculated under an assumption of a positive market price of risk relative to zero risk. In each case, the estimates in table 4 suggest that there is indeed a statistically significant difference between the two estimates. While Cao and Wei find a small percentage of their example geographies where the market price of risk is significant – a finding supported by Alaton et al. in an entirely different context – the results reported here show that the market price of risk is not only statistically significant (relative to the benchmark derivative), but economically large as well, ranging from 6.68% in the preferred MRGBM-J-ARCH case to 7.22% when the CDD index follows a MRGBM-J process. This result is more akin to Pirrong and Jermakayan, who find the market price of risk to be a significant factor, albeit in the case of power, rather than weather derivatives.

Second, regarding the preferred model in table 3 as a benchmark, or the most accurate model in the absence of an exact solution to the pricing problem, it is apparent that many of the simpler models contain significant pricing errors. In particular, the non-mean reverting models miss the true derivative price by an average of 40%. Indeed, consistent with the results of Cao and Wei, Alaton, Djehiche and Stillberger, or Pirrong and Jermakayan, mean reversion appears to be the most important feature of the weather index process. Comparing the price implied by model (3) with (1), model (4) with (2) and model (6) with (5) is instructive as each of these comparisons represent models that differ only by mean reversion. In each case, the mean-reverting price is significantly higher than its non-mean-reverting counterpart (see tests in table 6). Dixit and Pindyck demonstrate the knife-edge nature of this result in the context of a real option model. While derivatives that admit the possibility of an explosive path away from the strike price will have a higher intrinsic value, low levels of mean reversion (as our data show) mean that volatility plays a greater role in determining the expected payoff.

Third, a similar comparison among like models reveals the effect of allowing for non-constant volatility. Namely, comparing model (2) with (5) and model (4) with (6) in tables 4 and 5 shows that allowing volatility to vary over time produces slightly higher price estimates compared with the “static volatility” approach. Given that derivative prices rise in volatility (vega is always positive for a long position in either a European put or call), this result suggests that pricing models that ignore the time-dependence nature of volatility will consistently underestimate the derivative’s value. This is consistent with Hull and White and Myers and Hanson, who show that a standard Black-Scholes pricing model – one that assumes constant volatility – underprices options that are out of the money. More importantly, their result is not unique to financial options as Myers and Hanson show that pricing models for options on agricultural futures that incorporate time-varying volatility provide better estimates (as measured by root mean square error) than do constant-volatility alternatives. Indeed, with an estimated volatility in model (4) of 2% and a fitted value of 7.8% for model (6), the marked difference in equilibrium derivative values is not surprising. Finally, comparing model (2) with (1) and model (4) with (3) shows

the marginal impact of allowing for a jump-diffusion process. Consistent with the equity-options (Merton; Naik and Lee) and foreign-exchange options literatures (Bates 1996), the results in tables 4 and 5 show that small probabilities of extreme fundamental events cause higher derivative price estimates. This is an important result given the nature of weather risks facing agricultural producers. Particularly in California's Central Valley, the band of normal fluctuation for both rain and temperature is quite small, but infrequent frosts or storms are the largest cause of economic damage.

Although this study shows that weather derivative price estimates depend critically on a variety of modeling assumptions, it also demonstrates that it is possible to write down a relatively simple model that captures the complexities of pricing weather derivatives while providing useful price information. Without a benchmark against which we can compare the outcome our preferred model it is impossible to state that our results are entirely accurate, but the same can also be said of the Black-Scholes model. Instead, our approach can fulfill the same role as that model – as an agreed standard on the basis of which traders will confidently take positions.

Conclusions

If weather derivatives are to achieve sufficiently liquidity that they may become widely used revenue risk-management tools for specialty crop growers, then finding better pricing models is a necessary step. Defining “better” as a more accurate representation of the true value of the claim on a CDD index, an improved pricing model must take into account both the complexity of the underlying weather process and the fact that weather is a non-traded asset. In doing so, this study conducts a series of statistical tests in order to find the appropriate form for the stochastic process governing a temperature index and then uses the parameters of this process to define and simulate a Monte Carlo derivative pricing model. As a case-study, this study uses weather, yield and pricing data for nectarine growers in the Fresno, CA area. The creation of such a model is important to these growers as they lack many of the risk management tools that growers of traditional crops take for granted.

Previous research into similar types of processes (electricity, stock prices, exchange rates) have found the usual geometric Brownian motion assumption to be inadequate. Specification tests among several alternative weather processes find that the preferred model for a growing-season CDD index is a mean-reverting, geometric Brownian motion process with first-order autoregressive errors and a log-normally distributed jump term. Given the complexity of this process and the fact that weather is not tradable, the study derives and estimates an equilibrium weather derivative pricing model using Monte Carlo simulation approach.

The equilibrium simulation finds that misspecifying the underlying weather process can result in significant underpricing of derivatives based on a cumulative CDD index. Specifically, allowing for mean reversion, time-varying volatility and the fact that weather processes consist of discrete jump-diffusion rather than continuous diffusion processes each lead to higher weather derivative prices. Of these three factors, mean-reversion of the weather index emerges as the single most important factor,

while time-varying volatility and jump-diffusion are second and third, respectively. While more accurate pricing models may exist, the simple yet comprehensive nature of our model will, hopefully, contribute to the development of a practical model that can serve as an industry benchmark for over the counter weather products.

As this study concerns only the nature of the underlying weather process for a single region, opportunities for future research that build on this foundation are clear. First, it would be of considerable practical value to use the weather derivative prices consistent with the preferred underlying process in simulated revenue-risk management programs for California tree fruit growers to determine their potential effectiveness. Second, issues of basis risk and micro-climate variation can be considered by estimating similar processes for adjacent weather stations and determining their correlation with the Fresno series. Third, research is also needed into derivatives for other key weather variables – precipitation, heating degree days during the winter season, or derivatives specifically for catastrophic frost risks such as that which hit the California orange industry in the winter of 1998. Fourth, it is always the case that other researchers could extend our model by considering more complex error structures, such as GARCH, or higher order ARCH processes. Finally, as we suggest above, comparing weather derivative prices under risk-neutral and equilibrium pricing solution methods would be an important contribution to this emerging field of study.

Reference List

Alaton, P., B. Djehiche, and D. Stillberger. “On Modelling and Pricing Weather Derivatives.” Working paper, Department of Mathematics. KTH, Stockholm, SE. 2001.

Ball, C. A. and W. N. Torous. “On Jumps in Common Stock Prices and Their Impact on Call Option Pricing.” *Journal of Finance* 40(March 1985): 155-173.

Bates, D. S. “Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options.” *Review of Financial Studies* 9(Spring 1996): 69-107.

Black, F. and M. Scholes. “The Pricing of Options and Corporate Liabilities.” *Journal of Political Economy*. 81(1973): 637-659.

Bollerslev, T. “Generalized Autoregressive Conditional Heteroscedasticity.” *Journal of Econometrics* 31(1986): 307-327.

Boyle, P. “Options: A Monte Carlo Approach.” *Journal of Financial Economics* 4(1977): 323-338.

California Department of Food and Agriculture. *California Agricultural Statistics*. Sacramento, CA. 2001.

Cao, M. and J. Wei. "Pricing Weather Derivatives: An Equilibrium Approach." Working paper, The Rotman Graduate School of Management, The University of Toronto, Toronto, Ont. October 1999.

Cox, J. and S. A. Ross. "The Valuation of Options for Alternative Stochastic Processes." *Journal of Finance and Economics* 3(1976): 145-166.

Cox, J., J. E. Ingersoll, and S. A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica* 53(1985): 385-407.

Dischel, R.. "The Fledgling Weather Market Takes Off," *Applied Derivatives Trading Focus*. (<http://www.adtrading.com>). 1998a.

Dischel, B. "Black-Scholes Won't Do." Weather Risk Special Report. Energy and Power Risk Management. Risk Publications. October, 1998b.

Dixit, A. K. and R. S. Pindyck. *Investment Under Uncertainty*. Princeton, N.J.: Princeton University Press, 1994.

Hull, J. C. and A. White. "The Pricing of Options on Assets with Stochastic Volatilities." *Journal of Finance* 42(1987): 281 - 300.

Jarrow, R. A. and E. R. Rosenfeld. "Jump Risks and the Intertemporal Capital Asset Pricing Model." *Journal of Business* 57(1984): 337-351.

Jorion, P. "On Jump Processes in the Foreign Exchange and Stock Markets." *Review of Financial Studies* 1(Winter 1989): 427-445.

Lucas, R. E. "Asset Prices in an Exchange Economy." *Econometrica*. 46(1978): 1429-1445.

Merton, R. C. "Option Pricing when Underlying Stock Returns are Discontinuous." *Journal of Financial Economics* 3(1976): 125-144.

Myers, R. J. and S. D. Hanson. "Pricing Commodity Options When the Underlying Futures Price Exhibits Time-Varying Volatility." *American Journal of Agricultural Economics* 75(1993): 121-130.

Naik, V. and M. Lee. "General Equilibrium Pricing of Options on the Market Portfolio with Discontinuous Returns." *Review of Financial Studies* 39(Winter 1990): 493-521.

Pirrong, C. and Jermakyan, M. "Valuing Power and Weather Derivatives on a Mesh Using Finite Difference Methods." Working Paper, Olin School of Business, Washington University. St. Louis, Missouri. June 1999.

Turvey, C. "A Pricing Model for Degree-Day Weather Options." forthcoming in the *Journal of Risk*. 2001.

U.S. National Climatic Data Center. National Oceanic and Atmospheric Administration. Washington, D.C. 2001.

Table 1. Summary of Fresno Weather Data by Half-Decade

| Decade | N | Mean CDD | Std. Deviation | Minimum CDD Index | Maximum CDD Index |
|--------------------|----------|-----------------|-----------------------|--------------------------|--------------------------|
| 1970 - 1974 | 460 | 387.14 | 314.41 | 0.00 | 1145.00 |
| 1975 - 1979 | 460 | 369.73 | 302.33 | 0.00 | 1145.00 |
| 1980 - 1984 | 460 | 423.86 | 366.48 | 0.00 | 1343.50 |
| 1985 - 1989 | 460 | 443.95 | 355.07 | 0.00 | 1302.50 |
| 1990 - 1994 | 460 | 382.41 | 319.09 | 0.00 | 1193.50 |
| 1995 - 1999 | 460 | 354.42 | 317.76 | 0.00 | 1157.00 |

Table 2. Likelihood Ratio Specification Tests for CDD Index

| Models¹ | <i>q</i> | Critical P² | Estimated P² |
|-------------------------------------|-----------------|--------------------------------------|---------------------------------------|
| (1) GBM-J vs. GBM | 3 | 7.815 | 7,021.46 |
| (2) MRGBM vs. GBM | 1 | 3.840 | 9.94 |
| (3) MRGBM-J vs. MRGBM | 3 | 7.815 | 7,204.24 |
| (4) MRGBM-J vs. GBM-J | 1 | 3.840 | 192.72 |
| (5) GBM-J-ARCH vs. MRGBM-J | 1 | 3.840 | 4.40 |
| (6) MRGBM-J-ARCH vs. MRGBM-J | 1 | 3.840 | 50.42 |

¹ A 5% level of significance is used for all critical values. The likelihood ratio chi-square statistic is calculated as: $LR = 2 (LLF_U - LLF_R)$ where LLF is the log-likelihood function value and q restrictions.

Table 3. Fresno Weather / Yield Functions: OLS Estimates

| Variable | Nectarine | | Raisin Grape | | Peach | |
|----------------------|-----------|---------|--------------|---------|----------|---------|
| | Estimate | t-ratio | Estimate | t-ratio | Estimate | t-ratio |
| Const. | -5.025* | -2.009 | -7.609 | -0.534 | 13.456* | 9.610 |
| <i>t</i> | -0.044 | -1.549 | 0.175* | 2.326 | 0.059 | 1.146 |
| w_1 | 27.471* | 2.956 | 0.034 | 1.349 | N.A. | N.A. |
| w_1^2 | -11.028* | -2.628 | -0.002 | -1.457 | N.A. | N.A. |
| w_2 | N.A. | N.A. | N.A. | N.A. | -5.880 | -1.887 |
| w_2^2 | N.A. | N.A. | N.A. | N.A. | 0.658 | 1.608 |
| $w_1 w_2$ | N.A. | N.A. | N.A. | N.A. | 0.005 | 1.885 |
| R² | 0.618 | | 0.475 | | 0.470 | |

¹ In the above table, *t* is a linear time-trend variable, w_1 is the value of the CDD index at the end of the sample period, and w_2 is the cumulative precipitation recorded at the Fresno weather station over the sample period (scaled by a factor of 10^3 for presentation purposes). Yield is defined as tons per acre. N.A. indicates that the variable was not statistically significant in preliminary estimation so was excluded from the final model.

Table 4. Weather Derivative Monte Carlo Price Estimates: Nectarine Model

| Model ¹ | Derivative Price | | | | | |
|--------------------|-------------------------------|------------|---------------------------|------------|----------------------|-----------------------|
| | Positive Market Price of Risk | | Zero Market Price of Risk | | t-ratio ² | Implied Price of Risk |
| Mean | Std. Dev. | Mean | Std. Dev. | | | |
| (1) GBM | \$8,599.81 | \$1,594.62 | \$8,266.96 | \$1,511.73 | 4.79 | 6.80% |
| (2) GBM-J | \$8,925.90 | \$2,472.33 | \$8,580.22 | \$2,353.49 | 3.20 | 6.81% |
| (3) MRGBM | \$9,760.26 | \$1,207.83 | \$9,366.77 | \$1,147.19 | 7.47 | 7.09% |
| (4) MRGBM-J | \$11,565.38 | \$1,499.31 | \$11,090.82 | \$1,437.87 | 7.22 | 7.22% |
| (5) GBM-J-ARCH | \$9,341.03 | \$666.25 | \$8,967.51 | \$631.52 | 12.87 | 7.03% |
| (6) MRGBM-J-ARCH | \$15,250.83 | \$567.01 | \$14,670.46 | \$563.32 | 22.96 | 6.68% |

¹ In this table, N.A. = not applicable, GBM = Geometric Brownian Motion, MRGBM = Mean Reverting Geometric Brownian Motion, GBM-J = Geometric Brownian Motion with Log-Normal Jump, MRGBM-J = Mean Reverting GBM with Log-Normal Jump, GBM-J-ARCH = GBM with Log-Normal Jump and Autoregressive Conditional Heteroskedastic error term and MRGBM-J-ARCH = Mean Reverting GBM with Log-Normal Jump and ARCH. The base scenario uses the weather process parameter estimates in table 3, a long-run expected price of \$6.55 per carton and a nominal interest rate of 5%. All simulations use 1,000 replications of the normal and Poisson deviates.

² The null hypothesis is that the derivative price estimate under a positive market price of risk is the same as with a zero market price of risk.

Table 5. Tests of Weather Process Models

| Models: | Test of: | t-ratio: | df: | Implication: |
|------------------------|--------------------|-----------------|------------|---|
| (5) v (6) ¹ | Mean Reversion | 213.615 | 1998 | No MR understates true derivative price |
| (4) v (2) | Mean Reversion | 28.867 | 1998 | No MR understates true derivative price |
| (5) v (2) | ARCH error process | 5.127 | 1998 | No ARCH understates true derivative price |
| (4) v (6) | ARCH error process | 72.706 | 1998 | No ARCH understates true derivative price |
| (4) v (3) | Jump-Diffusion | 29.649 | 1998 | No JUMP understates true derivative price |
| (2) v (1) | Jump-Diffusion | 3.505 | 1998 | No JUMP understates true derivative price |

¹ In each case, the null hypothesis is that the derivative price calculated under the first model is equal to the price calculated under the assumptions of the second. At a 5% level of significance, the critical t-ratio in each case is 1.96.