

# NCCC - 134

APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

**The Role of the Bid-Ask Spread in  
a Dynamic – Time -Varying Optimal Hedging Model**

by

Michael S. Haigh

Suggested citation format:

Haigh, M. S. 2001. “The Role of the Bid-Ask Spread in a Dynamic – Time-Varying Optimal Hedging Model.” Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. St. Louis, MO. [<http://www.farmdoc.uiuc.edu/nccc134>].

**The Role of the Bid-Ask Spread in a Dynamic – Time-Varying Optimal Hedging Model.**

**Michael S. Haigh\***

Paper presented at the NCR-134 Conference on Applied Commodity Price  
Analysis, Forecasting, and Market Risk Management  
St.Louis, Missouri, April 23 – 24, 2001

Copyright 2001 by M.S.Haigh. All rights reserved. Readers may make  
verbatim copies of this document for non-commercial purposes by any means,  
provided that this copyright notice appears on all such copies.

\* Assistant Professor ([mshaigh@tamu.edu](mailto:mshaigh@tamu.edu)), Department of Agricultural Economics, Texas  
A&M University

## **The Role of the Bid-Ask Spread in a Dynamic – Time-Varying Optimal Hedging Model.**

*This paper presents a manageable and effective way of nesting two popular, yet distinct approaches to obtain optimal hedging ratios – time-series econometrics (GARCH) and dynamic programming (DP). The nested DP-GARCH model is then compared to a DP-GARCH model that accounts for variability in the bid-ask spread often unobserved (and hence ignored) in most studies. Results from an empirical application using data from an importantly traded commodity – sugar – suggest that a DP-GARCH model that incorporates the bid-ask spread still outperforms more traditional models. Moreover, the hedging ratios are far less volatile, and statistically different, than those recommended by the traditional GARCH methods that ignore the spread.*

**Key Words:** Bid – Ask Spread, Multi-Period Hedging, Dynamic Programming, GARCH.

### **1. Introduction**

The determination of the optimal hedging ratio has been the focus of much theoretical and empirical research since Johnson (1960) and Stein's (1961) seminal works on the subject matter. From an empirical and theoretical standpoint, one major thrust in the literature has been devoted to investigating models wherein there is a multi-period hedging horizon and the optimal hedging ratio (OHR) may be updated each period. This research has relied largely on applications of dynamic programming (DP), with empirical examples coming from Anderson and Danthine (1983); Karp (1983); Martinez and Zering (1992); Mathews and Holthausen (1991), Lence et al. (1993); and Vukina and Anderson (1993). These studies while being intuitively appealing (but operationally fairly complicated) have found that the multi-period hedging models have not resulted in significant improvements over static models from a risk management perspective.

Another major area of research in the empirical optimal hedging literature has been the use of time-series econometrics to model conditional variance and covariance dynamics for cash and futures prices. The non-constancy of the variance of price changes for commodities has received considerable attention since the issue was re-addressed over 35 years ago by Samuelson (1965). However, it wasn't until the important contributions of Engle (1982) and Bollerslev (1986) in developing ARCH/GARCH models that the estimation of non-constant hedging ratios really became popular.<sup>1</sup> Researchers employing these time-varying OHRs have claimed that significant gains could be achieved from a risk management standpoint relative to more traditional, basic econometric techniques. Examples include Yeh and Gannon (2000), Haigh and Holt (2000), Gagnon et. al (1998), Kroner and Sultan (1993), and Baillie and Myers (1991).

Like the DP approach, the time-series approaches allow for updating, but unlike the DP approach, the time series approaches have thus far, surprisingly, been limited to determining only sequentially updated one-period-ahead optimal hedging ratios. A recent study by Myers and Hanson (1996) did discuss feasible ways of using various models to estimate OHRs in a dynamic (DP) framework (including the ARCH/GARCH approaches), but unfortunately did not present specific details nor empirical applications.

The first objective of this paper is to therefore present an operationally tractable way of uniting the GARCH time series model approach and the DP approach for the first time. In an empirical application, using sugar cash and futures prices, a theoretically consistent yet realistic risk management model is developed which allows the representative trader to adjust the optimal hedge ratio originally set in place at the start of the hedging horizon. To make the results directly comparable to other related hedging studies it is assumed that the trader's objective is to minimize the variability of terminal wealth, and by employing DP recursive relations, the OHRs are thereby derived. These optimal hedging ratios are compared to hedge ratios developed using more basic approaches.

The second objective of the paper is to investigate whether the hedging ratios, calculated from the competing models, are in fact statistically different from one another. To this end the delta method (which relies on a first order approximation) is employed to derive standard errors and hence confidence intervals around the time-varying hedging ratios. This exercise is of interest because even though there has been a considerable amount of research on optimal hedging there has, to date, been no study that has assessed whether competing hedge ratios are different from one another in a statistical as well as an economic sense.

The third objective of the paper is to investigate the role of the bid – ask spread on the optimal hedging strategies. As highlighted by Campbell, Lo and Mackinley (1997) rather than simply one price for the futures price of an asset– there are in fact three relevant to a trader: a bid price, an asking price *and* the transaction price. However, given that neither the bid nor the ask are usually reported by most open outcry markets, much of the research to date has been dedicated to establishing an accurate way of estimating the bid - ask spread (e.g., Roll (1984), and Smith and Whaley (1994)). As such, only a limited number of research papers have evaluated the importance of the bid-ask spread in trading activities, simply because bid – ask data is not usually available in the market that is being studied. Results from studies that have employed available bid – ask data have tended to suggest that ignoring the role of the spread can indeed be costly. To cite just one example, Bae, Chan, and Cheung (1998) demonstrated that failing to consider bid - ask spreads would lead to false conclusions regarding the profitability of stock index futures arbitrage. It is the last objective of this paper to therefore introduce the role of the bid – ask spread on the optimal hedging models for the first known time.

In this study a brief overview of optimal hedging is presented, followed by the introduction of the nested DP-GARCH that ignores the role of the bid-ask spread (model I). Next the role of the bid-ask spread in the nested DP – GARCH approach is discussed and a modified DP-GARCH model (model II) is presented. The data used in the empirical analysis is discussed and the econometric estimation results are then presented, followed by a presentation of hedging results, complete with a description on the development of time-varying confidence bands around the optimal hedging ratios. Finally, the results and their implications are summarized in the conclusion.

## 2. Hedge Ratio Estimation

One basic concept in the hedging literature is the notion that traders may optimally select combinations of cash and futures positions to minimize portfolio risk. These combinations, typically expressed in terms of proportion of cash to futures positions for an asset, are commonly referred to as hedge ratios. One popular method of determining an optimal hedging strategy is to employ a minimum-variance (MV) framework, where it is assumed a merchandiser minimizes the variability of wealth associated with an expected sale. Such a framework has proven to be the benchmark in the hedging literature for several reasons. First, the MV hedge ratio is the optimal for exceptionally risk averse traders (Ederington, 1979; Kahl, 1983). The MV hedge ratio is also optimal when futures markets are unbiased. This is especially important, as such a phenomenon has been verified in several empirical studies (Baillie and Myers, 1991; Martin and Garcia, 1981). As such, MV methodology has been widely accepted and utilized in many previous studies partly because of the theoretical justification of finding unbiased markets and partly because the components of the minimum variance hedge ratio may be retrieved from variance and covariance estimates of the underlying cash and futures prices.

To illustrate, consider a simple one period wealth function:

$$(1) \quad W_t = (C_t - \overline{C_{t-1}}) + b_{t-1}(\overline{F_{t-1}} - F_t),$$

where  $W_t$  is comprised of both the return on purchasing the asset at time  $t - 1$  and selling it at time  $t$  and the returns from hedging the cost. Here  $C_t$  denotes the future cash price associated with commodity sale,  $\overline{C_{t-1}}$  is the known cash price associated with the initial purchase,  $\overline{F_{t-1}}$  is the known (short) futures price locked in at time  $t-1$ , and  $F_t$  is the (long) futures price obtained to offset the original futures transaction at time  $t$ . Also,  $b_{t-1}$  represents the OHR to be determined. The concept of the minimum-variance hedge method is to minimize the variance of the wealth function of the hedge portfolio. For the simple case illustrated in (1), variance of wealth,  $Var_{t-1}$ , may be written as

$$(2) \quad Var_{t-1}(W_t) = Var(C_t) + b_{t-1}^2 Var(F_t) - 2b_{t-1} Cov(C_t, F_t).$$

The first order condition for an extremum associated with (2) is, after simplifying:

$$(3) \quad \frac{\partial \text{Var}_{t-1}(W_t)}{\partial b_{t-1}} = b_{t-1} \text{Var}(F_t) - \text{Cov}(C_t, F_t) = 0.$$

Solving (3) for the OHR,  $b_{t-1}$ , yields:

$$(4) \quad b_{t-1} = \frac{\text{Cov}(C_t, F_t)}{\text{Var}(F_t)},$$

which is comprised of variance and covariance estimates for underlying cash and futures prices.

Many studies have used the rule in (4) to calculate the optimal hedge by simply regressing changes in cash prices on changes in futures prices with historical data. The resulting slope coefficient is then interpreted as an estimate of the OHR (Ederington, 1979; Kahl, 1983). This is because in the simple Ordinary Least Squares (OLS) regression model, the slope coefficient equals the term shown in (4). This particular form of the hedge ratio is commonly known as the MV hedge ratio and has been employed in many studies including those by Mathews and Holthousen (1991) and Lence et al. (1993).

Implicit in this traditional methodology of estimating hedge ratios is the assumption that the covariance matrix of cash and futures prices, and hence the hedge ratio, is constant through time. Fama (1965) observed, however, that variances and covariances of asset returns are not constant over time; that large changes in asset returns tend to be followed by other large changes; and that small changes tend to be followed by small changes. For this reason Engle's (1982) ARCH model—which captures many of “volatility clustering” features observed in the data—was originally applied to optimal hedging models (Cecchetti et al., 1988). Bollerslev (1986) subsequently proposed the GARCH model to circumvent problems associated with the long lags often needed to specify correctly an ARCH model. The result is that a large body of recent research has focused on utilizing the GARCH framework to compute time-varying (conditional) hedge portfolios. Several GARCH specifications have been proposed, arguably the most popular being the constant correlation parameterization. Indeed, Cecchetti et al. (1988), Baillie and Bollerslev (1990), Garcia et al. (1995), and Kroner and Sultan (1993) employed such a framework.

To estimate time-varying hedge ratios it is necessary to model jointly the first two moments of the cash and futures settlement prices (model I) and the first two moments of the cash, bid and ask prices (model II). Focusing on model I the constant correlation model is as follows:

$$(5) \quad \begin{aligned} \Delta P &= \mathbf{m} + \mathbf{e}_t \\ \mathbf{e}_t | \Omega_{t-1} &\sim N(0, H_t) \end{aligned}$$

where  $\Delta P_t = (C_t, F_t)^T$  is a (2 x 1) vector containing cash and futures prices, ( $T$  is a transpose operator);  $\mathbf{m}$  is a (2 x 1) mean vector of cash and futures prices (the intercept or drift terms), respectively;  $\mathbf{e}_t$  is a (2 x 1) vector of mean-zero, bivariate normally distributed cash and futures price innovations;  $\Omega_{t-1}$  is the information set available at time  $t - 1$ ; and  $H_t$ , where  $\text{vech}(H_t) = (h_{11,t}, h_{12,t}, h_{22,t})^T$ , is a (2 x 2) conditional covariance matrix. The constant correlation parameterization implies that the  $H_t$  matrix may be specified according to:

$$(6) \quad \begin{aligned} h_{ii} &= \mathbf{w}_i + \mathbf{a}_{ii} \mathbf{e}_{i,t-1}^2 + \mathbf{b}_{ii} h_{ii,t-1} \\ h_{ij} &= \mathbf{r}_{ij} (h_{ii} h_{jj})^{\frac{1}{2}} \quad i, j = 1(C_t), 2(F_t) \quad i \neq j \end{aligned}$$

where  $\mathbf{r}_{ij}$  denotes the  $ij^{\text{th}}$  constant conditional correlation. In general  $\mathbf{r}_{ij}$  can be time varying (hence the  $t$  subscript), but consideration simplifications arise in estimation and inference if it is assumed that  $\mathbf{r}_{ij}$  is constant for all  $t$ . Returning to the optimal hedging problem, it follows that, given the (time-varying) nature of the variance-covariance matrix  $H_t$ , the time-varying hedge ratio may be expressed as:

$$(7) \quad b_{t-1} = \frac{\text{Cov}(F_t, C_t | \Omega_{t-1})}{\text{Var}(F_t | \Omega_{t-1})} = \frac{h_{12,t}}{h_{22,t}}$$

where  $b_{t-1}$  is the OHR conditional on all available information at time  $t - 1$ , represented by  $\Omega_{t-1}$ .

Several papers have focused on time-varying hedging by using variants of the Engle-Bollerslev ARCH/GARCH approach. This body of research has focused on modeling the cash-futures price distribution and then using the results to estimate OHRs by relaxing the assumption that conditional variances (covariances) are time independent. For instance, Cecchetti et al. (1988) applied a bivariate ARCH model to financial futures prices, while Baillie and Myers (1991), Myers (1991), and Sephton (1993) applied bivariate GARCH models to commodity prices. Park and Switzer (1995), Tong (1996), and Yeh and Gannon (2000) compared GARCH-generated OHRs to OLS hedging strategies for stocks. Kroner and Sultan (1993) and Lin et al. (1994) used a bivariate GARCH framework in foreign currency hedging.

### 3. Combining DP Hedging Models with GARCH Time-Series Techniques

Consider first a trader who wishes to optimize an objective function dependent upon wealth at a future terminal date, and can update the futures position between the original time

period and the terminal date to incorporate new information. Assume that the trader starts with an initial amount of wealth, which is invested in the commodity for resale and a later date. Also the hedging decision for each purchase is made several periods prior to the terminal date and the hedged portfolio can then be updated (modified) at each period up to that time. Such a model builds upon the relatively restrictive dynamic model presented by Mathews and Holthausen (1991) and Vukina and Anderson (1993) by allowing the variances and covariances to be updated at each period using modern time series techniques. Therefore in this scenario, the relatively straightforward DP model will be combined with a bivariate GARCH model, which when used in previous hedging strategies (e.g., Baillie and Myers (1991), Myers (1991), and Sephton (1993)) has been shown to significantly outperform more traditional hedging strategies.

The DP – GARCH framework using just cash and futures settlement prices (model I) assumes that the price at which the commodity trader purchases the commodity in the future is uncertain, and is consequently stochastic. The objective of the trader is therefore to pick the optimal number of futures contracts to be locked into at each period for the trader to optimize an objective function that is dependent on the total wealth at the future sale date. Therefore, the trader is permitted to update the estimates of variances and covariances that are used to generate the optimal hedged portfolio between the time that the initial hedge is placed and when the commodity is sold in the cash market.

For simplicity, we follow Anderson and Vukina (1993) by considering a three-period problem,  $t - 2$ ,  $t - 1$ , and  $t$  which is the delivery date. Three periods before the sale of the commodity, after the cash position has been established, the trader decides on the initial futures position that covers the trading period between period three and period two,  $b_{(S)t-2}$  (the (S) indicates the use of just the futures settlement prices). The futures position then evolves in the sense that the quantity hedged in the next period,  $b_{(S)t-1}$ , may be different than the quantity hedged in  $t - 2$ . The following period,  $t$ , the trader closes out all outstanding futures positions, and sells the cash commodity, and collects the proceeds. Hence from the perspective of the current period,  $t - 2$ , we can define wealth at the terminal date  $t$ ,  $W_t$ , as:

$$(8) \quad W_t = (1+r)(-\overline{C_{t-2}}) + (1+r)(\overline{F_{t-2}} - F_{t-1})b_{(S)t-2} + (F_{t-1} - F_t)b_{(S)t-1} + C_t.$$

Variable  $\overline{C_{t-2}}$  is the initial (known) price at which the exogenously determined cash commodity is purchased; and  $C_t$  is the stochastic cash price at which the commodity must be sold at the end of the three periods.  $\overline{F_{t-2}}$  is the (known) futures price available at the initial time period that the decision is made;  $F_{t-1}$  and  $F_t$  are the stochastic futures prices in the respective periods;  $r$  is the one-period risk-free interest rate;  $b_{(S)t-2}$  and  $b_{(S)t-1}$  are the hedging ratios that capture the quantity of futures sold (bought if negative). The terminal monetary wealth  $W_t$  reflects the fact

that the trader's futures account is marked to market, meaning that all profits and losses related to the futures positions are realized each period.

Given the wide spread acceptance of the MV hedge ratio, and given that we wish to compare the results in this paper with previous related papers, it is the objective of this paper to first derive a dynamic version of the myopic MV hedge ratio. Introducing time-varying variances and covariances and then employing DP techniques to solve for the optimal hedging ratio does precisely this. So to remain consistent with the optimal hedging literature, we suggest that at each decision date, the trader first decides the quantity to be hedged in order to minimize the variance of terminal wealth, given the cash position. The trader's objective at the initial time period,  $t - 2$ , is to calculate the hedge ratio,  $b_{(S)t-2}$  that minimizes the variability of terminal wealth:

$$(9) \quad \underset{t-2}{\text{MinVar}}(W_t).$$

The general solution to this problem is obtained through backward induction in a manner similar to that employed by Mathews and Holthausen (1991) and Anderson and Danthine (1983). Therefore, in order to find the hedge ratio that would be used at time  $t - 2$ ,  $b_{(S)t-2}$ , the trader must estimate the hedge ratio that would be employed the following week,  $t - 1$ . Following Mathews and Holthausen (1991), we work backwards, so the trader estimates the hedge ratio that would be used the week prior to the cash sale,  $b_{(S)t-1}$ , in order to minimize the variability of wealth. The conditional variance of the wealth (and suppressing conditioning information notation for ease of reading) associated with that week is therefore:

$$(10) \quad \begin{aligned} \text{Var}_{t-1}(W_t) &= \text{Var}_{t-1}[(1+r)(-\overline{C_{t-2}}) + (1+r)(\overline{F_{t-2}} - \overline{F_{t-1}})\overline{b_{(S)t-2}} + (\overline{F_{t-1}} - \overline{F_t})\overline{b_{(S)t-1}} + C_t] \\ &= b_{(S)t-1}^2 \text{Var}_{t-1}(F_t) + \text{Var}_{t-1}(C_t) - 2b_{(S)t-1} \text{Cov}_{t-1}(C_t, F_t) \end{aligned}$$

After obtaining the first order condition for an extremum, and then solving for the optimal hedging ratio we are left with precisely the same hedge ratio presented in equation (7). Therefore, the hedge ratio is simply comprised of the one step ahead forecast of the cash and futures settlement price covariance divided by the one step ahead forecast of the futures settlement price variance. Substituting the expression for  $b_{(S)t-1}$  into the wealth expression (10), we can find the variance of wealth at the initial trade date.<sup>2</sup> Therefore, in the initial period (two periods prior to the eventual cash sale) the trader minimizes the variability of terminal wealth relevant at that date:

$$\begin{aligned}
(11) \quad \text{Var}_{t-2}(W_2) &= \text{Var}_{t-2}(1+r)(-\overline{C_{t-2}}) + (1+r)(\overline{F_{t-2}} - F_{t-1})b_{(s)t-2} + (F_{t-1} - F_t)b_{(s)t-1} + C_t \\
&= (1+r)^2 b_{(s)t-2}^2 \text{Var}_{t-2}(F_{t-1}) + b_{(s)t-1}^2 \text{Var}_{t-2}(F_{t-1}) + b_{(s)t-1}^2 \text{Var}_{t-2}(F_t) + \text{Var}_{t-2}(F_t) - \\
&\quad 2(1+r)b_{(s)t-2}b_{(s)t-1} \text{Var}_{t-2}(F_{t-1}) + 2(1+r)b_{(s)t-2}b_{(s)t-1} \text{Cov}_{t-2}(F_{t-1}, F_t) - \\
&\quad 2(1+r)b_{(s)t-2} \text{Cov}_{t-2}(F_{t-1}, C_t) - 2b_{(s)t-1}^2 \text{Cov}_{t-2}(F_{t-1}, F_t) + 2b_{(s)t-1} \text{Cov}_{t-2}(F_{t-1}, C_t) \\
&\quad - 2b_{(s)t-1} \text{Cov}_{t-2}(F_t, C_t).
\end{aligned}$$

The variance at time period  $t - 2$  is a function of several variances and covariances; the hedge ratio that is used at time  $t - 2$ ,  $b_{(s)t-2}$ , (the operational hedge ratio); and the expected hedge ratio to be used the next time period,  $b_{(s)t-1}$  (the forecasted ratio). Taking the first order condition of equation (11) and solving for the optimal first-period futures position (the OHR) gives:

$$(12) \quad b_{(s)t-2} = \frac{\text{Cov}_{t-2}(F_{t-1}, C_t)}{(1+r)\text{Var}_{t-2}(F_{t-1})} + \frac{b_{(s)t-1}}{(1+r)} \left[ 1 - \frac{\text{Cov}_{t-2}(F_t, F_{t-1})}{\text{Var}_{t-1}(F_{t-1})} \right].$$

This hedge ratio might be viewed as the sum of an inter-subperiod hedge ratio plus the discounted next period hedge ratio weighted by a small positive weight if  $\text{Cov}_{t-2}(F_t, F_{t-1}) < \text{Var}_{t-2}(F_{t-1})$ . However, if the futures market in question can be shown to have little or no systematic bias, in the sense of Martin and Garcia (1981), then all terms remaining after the first term on the right hand side of each optimal hedging ratio disappear. This is so because if a futures market can be shown to be unbiased (the price series is represented by a martingale) then the hedge ratio at each time period can be shown to be independent of all other hedge ratios.<sup>3</sup> This implies that if we have unbiased futures markets, the optimal hedging ratios developed within the DP framework differ because of the discount rate, and by the timing of the forecasts of volatility, but are independent of any future hedge ratios.

#### 4. Bid-Ask Spreads and the DP-GARCH model

According to Campbell, Lo and Mackinley (1997) there are three futures prices relevant to the trader, a bid price, an asking price and the transaction price, not just the settlement price often used in empirical research. Therefore even though the bid and ask price represent prices related to the same commodity, they may not be perfectly correlated and should not be treated as such. Recent research by Gagnon, Lypney and McCurdy (GLM) (1998) employed a *trivariate* GARCH system allowing for time-varying covariability between related prices (that were not perfectly correlated) in a portfolio. They discovered that significant gains in hedging performance may be enjoyed by modeling the prices jointly in a portfolio compared to individual strategies. Moreover, further gains may be achieved simply because significantly fewer futures contracts would be recommended in the portfolio approach thus reducing commission charges on the futures contracts. It is an empirical question as to whether the 'portfolio' of bid, ask and cash prices yields similar results to that achieved by GLM (1998), and so it is to this that we now turn our attention to.

In introducing the bid and ask prices into the analysis we continue to concentrate on the three trade hedging model. It is assumed, as it was in model I, that in period  $t - 2$  the trader decides on the initial futures position,  $b_{(BA)t-1}$  (the (BA) indicates that the hedging ratios have been developed using the bid and ask prices). However, now instead of going short at the settlement price recorded by the exchange, the trader must go short at the bid price which can in some instances be much lower than the settlement price, particularly in thinly to moderately traded markets (see data section). The futures position then evolves, (like the case presented for model I) in the sense that the quantity hedged in period  $t - 1$ ,  $b_{(BA)t-1}$ , may be different than the quantity hedged in  $t - 2$ . The next period,  $t$ , the trader closes out all outstanding futures positions, by offsetting the futures position by going long at the asking price rather than the settlement price (model I). The trader would then sell the cash commodity, and collects the proceeds (if any) from the hedge. Hence from the perspective of the current period,  $t - 2$ , we can define wealth at the terminal date  $t$ ,  $W_t$ , as:

$$(13) \quad W_t = (1+r)(-\overline{C_{t-2}}) + (1+r)(\overline{B_{t-2}} - A_{t-1})b_{(BA)t-2} + (B_{t-1} - A_t)b_{(BA)t-1} + C_t.$$

All variables are as previously defined except  $\overline{B_{t-2}}$  which is the futures bid price available at the initial time period that the decision is made.  $B_{t-1}$  and  $A_t$  are the stochastic futures bid and ask prices in the respective periods; and  $b_{(BA)t-2}$  and  $b_{(BA)t-1}$  are the hedging ratios that capture the quantity of futures sold (bought if negative). Again, we suggest that at each decision date, the trader decides upon the quantity to be hedged in order to minimize the variance of terminal wealth, given the cash position. We maintain this methodology even though a mean-variance methodology (see Haigh and Holt, 2000 for example) may be more appropriate in the case of the bid-ask model because the expected return is likely to be negative, and so may alter the hedge ratio slightly.<sup>4</sup> The solution is once again obtained through backward induction so in order to find the hedge ratio that would be used at time  $t - 2$ ,  $b_{(BA)t-2}$ , the trader must estimate the hedge ratio that would be employed the following week. The conditional variance of the wealth associated with that period is:

$$(14) \quad \begin{aligned} Var_{t-1}(W_t) &= Var_{t-1}[(1+r)(-\overline{C_{t-2}}) + (1+r)(\overline{B_{t-2}} - A_{t-1})b_{(BA)t-2} + (B_{t-1} - A_t)b_{(BA)t-1} + C_t] \\ &= b_{(BA)t-1}^2 Var_{t-1}(A_t) + Var_{t-1}(C_t) - 2b_{(BA)t-1} Cov_1(C_2, A_2) \end{aligned}$$

After obtaining the first order condition for an extremum, and then solving for the optimal hedging ratio we are left with:  $b_{(BA)t-1} = \frac{Cov_1(C_2, A_2)}{Var_1(A_2)}$ , which is composed of the one step ahead forecast of the cash and futures asking price covariance divided by the forecasted variance of the futures asking price.

Substituting the expression for  $b_{(BA)t-1}$  into the wealth expression (13), we can find the variance of wealth at the initial trade date. Therefore, in the initial period the trader minimizes the variability of terminal wealth relevant at that date:

$$\begin{aligned}
(15) \quad \text{Var}_{t-1}(W_t) &= \text{Var}_{t-1}((1+r)(-\overline{C_{t-2}}) + (1+r)(\overline{B_{t-2}} - A_{t-1})b_{(BA)t-2} + (B_{t-1} - A_t)b_{(BA)t-1} + C_t) \\
&= (1+r)^2 b_{(BA)t-2}^2 \text{Var}_{t-2}(A_{t-1}) + b_{(BA)t-1}^2 \text{Var}_{t-2}(B_{t-1}) + b_{(BA)t-1}^2 \text{Var}_{t-2}(A_t) + \text{Var}_{t-2}(C_t) - \\
&\quad 2(1+r)b_{(BA)t-2}b_{(BA)t-1} \text{Cov}_{t-2}(A_{t-1}, B_{t-1}) + 2(1+r)b_{(BA)t-2}b_{(BA)t-1} \text{Cov}_{t-2}(A_{t-1}, A_t) - \\
&\quad 2(1+r)b_{(BA)t-2} \text{Cov}_{t-2}(A_t, C_t) - 2b_{(BA)t-1}^2 \text{Cov}_{t-2}(B_{t-1}, A_t) + 2b_{(BA)t-1} \text{Cov}_{t-2}(B_{t-1}, C_t) \\
&\quad - 2b_{t-1} \text{Cov}_{t-2}(A_t, C_t).
\end{aligned}$$

If the ask price can be shown to be unbiased then taking the first order condition for equation (11) and solving for the optimal first-period futures position leave us with:

$$(16) \quad b_{(BA)t-2} = \frac{\text{Cov}_{t-2}(A_t, P_t)}{(1+r)\text{Var}_{t-2}(A_{t-1})} + \frac{b_{(BA)t-1}}{(1+r)} \left[ \frac{\text{Cov}_{t-2}(B_{t-1}, A_{t-1})}{\text{Var}_{t-2}(A_{t-1})} - 1 \right].$$

In this case, the second term on the right hand side of equation (16) may not disappear simply because the bid prices and ask prices may exhibit different behaviors in the short run particularly in thinly to moderately traded markets (see data section for a more complete explanation). So only if  $\text{Cov}_{t-2}(B_{t-1}, A_{t-1}) = \text{Var}_{t-2}(A_{t-1})$  we are left with a weighted initial hedge ratio, (the first term on the right-hand side) where the next period hedge ratio,  $b_{(BA)t-1}$  does not effect the current period hedge ratio  $b_{(BA)t-2}$ . As this may not be the case every time period, we have reason to believe that the next periods hedge ratio will effect the current period hedge ratio.

To implement the DP-GARCH framework, regardless whether we focus on model I or model II, a specification must be chosen for the time-varying covariance matrices. Multi step ahead forecasts of relevant variances and covariances can then be derived from the underlying bivariate GARCH model (Model I) and trivariate GARCH model (Model II).<sup>5</sup> Based upon residual diagnostic tests (see econometric estimation results presented below), each series is specified as a simple martingale process, thereby satisfying the assumption of unbiased markets.<sup>6</sup> Because of the constant correlation structure (presented in equation 2) is, parsimonious in parameters, and is relatively easy to estimate, it is employed here. Estimating the hedge ratios therefore involves choosing an appropriate discount rate and estimating the relevant variances and covariance forecasts at each date. In the ensuing analysis this discount rate is set throughout at 10%, and we continue with the three-period hedging scenario as in the example.

## 5. Data

On November 27<sup>th</sup> 2000 the open outcry system used for most of LIFFE's commodity products was replaced by the electronic trading system, LIFFE CONNECT<sup>TM</sup>. Therefore trading in LIFFE's Cocoa, Coffee, Wheat, Barley, Potato and BIFFEX all ceased to be traded in an open outcry format after that date. As such, all bid/asks and transaction volumes are now easily available on a real time basis. Sugar, on the other hand, has always been traded on the electronic trading system, and so a more complete history of all closing bid/asks and transaction volumes are available. This unique LIFFE data set, which was made available through the order transit and trade registration system, therefore facilitates an accurate empirical research on the microstructure of futures markets.

Daily closing futures prices (bid, ask and settlement) for white sugar traded at LIFFE were collected from Bloomberg International covering the period 13<sup>th</sup> December 1995 – 12<sup>th</sup> January 2000. While bid-ask quotes are collected and recorded throughout the day (and are made publicly available) the final bid, ask and settlement price of the day was collected here based on the assumption that our representative trader makes updates and hedging decisions towards the close of trading once a week. Therefore weekly price data (214 observations) were constructed using Wednesday prices (which have the least holidays and do not suffer from the so called 'weekend effect' (French, (1980))). However, if closing bid-ask quotes based on Wednesday prices were not available, then a Thursday, or a Friday price was used.<sup>7</sup> The futures prices are for the nearby contract month which forms the first value for the continuous series, and runs until the last day of trading of the contract.

In addition to the closing bid, ask and settlement prices weekly London cash prices for sugar covering the same time period were collected from Datastream International. Figure 1 illustrates the four price series (closing bid, ask, settlement and cash prices over the five-year horizon). As can be seen from the main graph it is extremely difficult to distinguish between the related futures price series as the average bid-ask spread as a percentage of the settlement price is very small. However, as can be seen from the smaller graphical insert, the bid, ask, and settlement are quite different, and tend to move together over time, albeit by non-constant amounts. Such a phenomenon is not uncommon in moderately traded markets like the sugar market at LIFFE.<sup>8</sup> Figure 2 further illustrates this point by simply presenting time-series plot of the bid-ask spread over the time horizon. As can be seen, from the chart, the bid-ask spread varies by uniform amounts (the minimum tick size) and can on some occasions be as high as \$2 per tonne. The mean value of the spread over the time period is \$0.4557 while the modal value is \$0.2 or twice the size of the minimum tick value.

## 6. Econometric Estimation Results

Each price series was first examined for the existence of a unit root using Augmented Dickey Fuller (ADF) tests. Results indicated that all four series (futures bid, ask, settlement and cash) are nonstationary and when the same unit root testing procedures were applied to the

first differenced data, the ADF test statistics reject the null hypothesis of a unit root. Correspondingly, each series was first differenced in the econometric estimation.<sup>9</sup> Quasi-maximum likelihood estimates of model parameters are obtained here by using the BFGS (Broyden, Fletcher, Goldfarb and Shanno) algorithm.<sup>10</sup> In many cases the assumption of conditional normality cannot be maintained, and as an alternative to using a non-normal distribution is to obtain quasi-maximum likelihood estimates by using the log likelihood function from the conditional normal specification. Under fairly weak conditions, the resulting estimates are consistent even when the conditional distribution of the residuals is non-normal (Bollerslev and Wooldridge (1992)). Parameters estimators for both the bivariate and trivariate GARCH (1,1) models (model I and II respectively) are reported in Table 1. Point estimates of the GARCH parameters along with the robust standard errors indicate substantial evidence of conditional variance dynamics for each model. Interestingly, the GARCH parameter point estimates,  $b_i$ , in model II are considerably lower than that of model I suggesting perhaps that modeling the portfolio of prices reduces some of the variability in the model. Residual diagnostics and the Ljung-Box Q and  $Q^2$  test statistics for, respectively, standardized residuals and squares and cross products of (standardized) residuals for each model suggest that the models appear to do a reasonable job of explaining conditional mean and variance dynamics of the cash, futures settlement, and bid and ask prices.<sup>11</sup> Overall, the constant-correlation GARCH (1,1) models appear to do a good job of characterizing the essential features of the data, and are therefore potentially useful tools for examining dynamic time-varying hedging strategies.

## 7. Hedging Results

The key question in any hedging evaluation study is how well does the proposed model perform relative to other models? To answer this question, we first turn our attention to the DP-GARCH model that only utilizes the cash and futures settlement prices (model I). In particular an evaluation is made of its performance relative to other more standard models including a straightforward GARCH, OLS, Naïve and Unhedged model which do not employ any kind of recursive substitution. After examining the nested DP-GARCH model (model I) we will then turn our attention to the DP-GARCH model that incorporates the bid-ask prices (model II). Comparing these results will then enable a trader to evaluate whether there are advantages to modeling both the bid and the ask prices in a hedging strategy, and whether indeed, the standard GARCH, and the DP-GARCH (from model I) might be outperformed by this model.

### *Hedging Results: Model I*

The left-hand panel of table 2 presents sample average hedge ratios, along with standard deviations around the average and minimum and maximum hedge ratios, for each of the DP-GARCH ( $b_{(S)I-2}$  and  $b_{(S)I-1}$ ) and GARCH ( $b_{(S)GARCH}$ ) models. Also included are the average values for the OLS, naïve, and unhedged hedging strategies. As well, plots of the DP-GARCH, OLS ( $b_{(S)OLS}$ ) and Naïve ( $b_{(S)NAIVE}$ ) hedge ratios for the sample period along with confidence bands are reported in the upper two panels of Figure 3. Since analytical

expressions for hedging ratio standard errors may be impossible to obtain, an asymptotic approximation is applied here. In particular, we make use of the delta method, which amounts to a Taylor series approximation (Kendall and Stuart (1977)) for deriving standard errors around the hedge ratios.<sup>12</sup> The resulting time-varying confidence bands can then be calculated and can be seen surrounding the time-varying hedge ratios in figure 3. To illustrate, in weeks 190 – 200 the DP-GARCH hedge ratios at time  $t - 1$  and  $t - 2$  are larger than the naïve hedging ratio ( $b_{(S)NAIVE}$ ), but as the confidence band overlaps with the naïve ratio we could infer that they are statistically indistinguishable from one another.

For the GARCH hedge ratio ( $b_{(S)GARCH}$ ) it is assumed that once the hedge is in place it is not updated over the hedge horizon. Of course a weekly sampling frequency enables the trader relying on a myopic GARCH model to update the hedge ratio (but not use DP analysis); however, for comparison sake, this hedge is also left in place for the entire hedge period. Consequently, the average GARCH hedge ratio,  $b_{(S)GARCH}$ , is identical to the DP-GARCH hedge ratio developed at the start of trading. The OLS and Naïve hedging ratios,  $b_{(S)OLS}$  and  $b_{(S)NAIVE}$ , on the other hand do not change from week to week, as they are simply obtained from an OLS regression of the change in the futures settlement price on the change in the cash price, or set equal to 1 respectively. The unhedged hedge ratio is set equal to 0 for each and every week.

As illustrated in Table 2 and Figure 3 (Panels A and B), on average each estimated model calls for short hedging, as indicated by the positive signs associated with the OHRs. This outcome is as expected for a risk-averse trader anticipating making cash sales in the future. While results from the DP-GARCH portfolio show substantial variation in OHRs at each hedge horizon through time, there is relatively little variation among OHRs across hedge horizons. To illustrate, during the initial period,  $t - 2$ , the average hedge ratio for the trader is 0.7613; conversely the hedging ratio in the next period is 0.7627, indicating that on average the hedge ratio increases modestly. The fact that the hedge ratio increases over time is consistent with the findings of Anderson and Danthine (1983). If no variation occurred across hedge horizon, the DP-GARCH hedge ratios ( $b_{(S)t-2}$  and  $b_{(S)t-1}$ ) would be identical to the GARCH hedge ratios ( $b_{(S)GARCH}$ ), and there would be no incentive in combining the DP and GARCH approaches. Results reported in Table 2 also reveal for the DP-GARCH portfolio that, at most, 112.88% of the cash position would have been hedged by the sugar trader, with the least amount hedged being about 48.85% of the cash position.

Results reported in Table 2 and Figures 3 (panels A and B) also indicate that over the entire sample period that DP-GARCH hedge ratios ( $b_{(S)t-2}$  and  $b_{(S)t-1}$ ) may vary considerably. Indeed the standard error around the mean is 12.88%. Visual inspection of the hedge ratios illustrate that the DP-GARCH hedge ratios are quite erratic, sometimes recommending that about 50% of the hedged position be lifted, or locked into in just a matter of weeks. Such a

recommendation could imply that a trader might incur substantial transaction costs associated with updating the portfolio. The lower part of Table 2 reveals that the OLS hedge ratio ( $b_{(s),OLS}$ ) for the sugar trader is 0.7104, suggesting somewhat less hedging on average than either the DP-GARCH model or the static GARCH model. By adopting the OLS, naïve or unhedged approaches, the hedger would employ the same hedge ratio every week over the entire time frame, and so this approach shows no variability.

While sample and average hedge ratios are instructive, they tell us little about how the various models perform. To this end, this research extends the work of Mathews and Holthausen (1991) (who just studied the pattern of the hedge ratios) and calculate the average variance of total wealth (as defined in equation 3) for each model. These results, along with other descriptive statistics including the standard deviation of the wealth variance and minimum and maximum wealth variances are reported, for each model in Table 3. Also, Panels 3C, 3D and 3E present the time-varying percentage improvement of the DP-GARCH model over the OLS (Panel 3C), naïve (Panel 3D), and the unhedged model (Panel 3E). It is clear that on average the DP-GARCH approach outperforms all basic alternatives with the worst performing strategy being the unhedged approach (as one might expect). Interestingly, there are several times when the DP-GARCH model is outperformed by the OLS and naïve unhedged approaches. However, the negative percentage improvement figures associated with these time periods are not of large magnitudes suggesting that even if the DP-GARCH is beaten by simpler alternatives the trader would not be too heavily penalized.

According to Table 3 there appear to be some gains to using both the GARCH and the DP – GARCH approach relative to the more basic models in terms of average variance reductions. The performance of the GARCH model is of no surprise and appears to be very close (in terms of performance) to other papers that have evaluated its performance (e.g., Baillie and Myers, (1991)). What are the advantages to using the DP – GARCH model compared to the GARCH approach? According to Table 3 not much. In particular, the average performance of the DP – GARCH model over the static GARCH is just 0.2203%. This number would clearly be more significant to a trader hedging a large quantity of sugar, but it might also be of importance to know the variance around this improvement. Figure 4 provides this answer. While the average hedge ratios appear quite similar (panel A), and hence average improvement quite low, there are periods of time when the static model is beaten quite convincingly. For instance the DP-GARCH outperforms the static GARCH by approximately 9% at week 100. Indeed, visual inspection of the percentage improvement of the DP-GARCH model over the static GARCH and the hedge ratios with the confidence bands verify that there are indeed times when the market suddenly and abruptly turns and the trader following the static GARCH methodology may have lost out.

### *Hedging Results: Model II*

The key difference between model I and model II is the incorporation of the two futures price series – the bid and ask, instead of just using the futures settlement price. Consequently, the hedging ratios developed in this model are generated using a portfolio of prices, and such hedging ratios, when estimated in other research papers (see for example GLM (1998)) have tended to be much lower than those when just the cash and futures prices are estimated jointly.

Results presented on the right hand side of Table 2 verify that not only are the average hedging ratios lower, their variability is also lower. For instance, at time  $t - 2$ , the average optimal hedging ratio associated with the DP-GARCH model that uses bid and ask prices, ( $b_{(BA)t-2}$ ) is 0.7476 compared to the average ratio recommended in model I ( $b_{(S)t-2}$ ) of 0.7613. A similar pattern emerges in  $t - 1$  whereby the average optimal hedging ratio ( $b_{(BA)t-1}$ ) is 0.7490 compared to the hedge ratio of 0.7627 recommended by model I. These results seem to verify the findings of GLM (1998) that hedging ratios estimated in a portfolio tend to be lower than those estimated in a bivariate setting. Interestingly, the closer to the cash sale date, the greater the hedging ratio tends to get, which is the same general result as that in Anderson and Danthine's Appendix A2, that the hedge ratio (the futures positions) grow over time. The other important observation is the fact that the standard errors around the average hedging ratios are much lower in model II compared to model I. In particular the standard error associated with  $b_{(BA)t-2}$  and  $b_{(BA)t-1}$  in model II are 0.0459 and 0.0420 respectively, whereas the corresponding standard errors in model I are 0.1288 and 0.1287, about three times as large. Comparing the optimal hedging ratios for model I and model II, complete with their confidence bands in Figure 5 illustrates this fact visually. The hedge ratios from the DP – GARCH framework in model II are far more stable but do experience time-variation. The hedging ratios from the DP – GARCH framework in model I are much more volatile.

Comparing the time-pattern and statistical difference (if any) of the hedging ratios from different models is interesting in itself the more important question from an economic standpoint is how do they hedging ratios compare in terms of reducing variability for the trader. Therefore, as was done for model I we evaluate the usefulness of adopting the DP-GARCH approach using the bid-ask and cash prices (model II) over all alternatives described thus far, including the DP-GARCH approach that utilizes only the cash and futures settlement price (model I). If the true portfolio relevant to the trader really does incorporate the bid-ask prices (which should be more readily available with the electronic platform), then the system to be estimated should involve three price series rather than two.

Table 4 presents the descriptive variance statistics for all the models and how they compare to the bid-ask DP-GARCH approach. The results are quite striking. Model II (DP-GARCH using  $b_{(BA)t-2}$  and  $b_{(BA)t-1}$ ) appears to outperform, in terms of reduced variability, model I (DP-GARCH using  $b_{(S)t-2}$  and  $b_{(S)t-1}$ ) by 1.833% which, on the surface does not seem like a dramatic improvement. However, as described previously, averages can be deceiving and so the time-series plot of the improvements are presented in Figure 5, panel C. Clearly, when the

model I hedge ratios are extremely volatile, the trader would lose out by following that approach, if the true model was indeed the model incorporating the bid and the ask prices (model II).

It is clear however from table 4 that the DP-GARCH model that utilizes both  $b_{(BA)t-2}$  and  $b_{(BA)t-1}$  outperforms all the ‘basic’ alternatives, beating the unhedged model by 52.105%. The OLS model performs better when evaluated against model II (compared to model I) because as can be seen from Figure 6, the OLS hedging ratios are closer to the DP-GARCH hedge ratios, simply because the DP-GARCH ratios are less volatile as they were estimated in a portfolio setting. Unlike the case of model I, while the percentage improvement over the OLS approach is lower, the DP-GARCH model is never beaten by the simpler alternative. The same results are obtained for the naïve and unhedged approach.

While the DP-GARCH approach using  $b_{(BA)t-2}$  and  $b_{(BA)t-1}$  performs the best out of the simpler alternatives it does not seem to significantly outperform the static DP-GARCH approach (that just utilizes  $b_{(BA)t-2}$ ). Indeed the percentage reduction from using the dynamic model over the static approach is 0.099%. Again, this result is not particularly surprising given the finding that the optimal hedge ratios do not vary as much as the DP-GARCH model presented in model I. However as mentioned previously, traders are probably more interested in the distribution of the variability of the improvement over time. To this end, Panels 7A and 7B provide more evidence on this. Firstly, panel 7A illustrates that in general the hedge ratios generated from the dynamic DP-GARCH ( $b_{(BA)t-2}$  and  $b_{(BA)t-1}$ ) are not, in general, statistically different from the hedge ratio employed both periods in the static version ( $b_{(BA)t-2}$ ). That is, the confidence bands overlap for most of the period of time analyzed. One might suspect therefore that if there is no statistical difference between the hedging ratios then there might not be any improvement, from an economic sense. Panel 7B illustrates that when the percentage improvement is very low, (e.g., week 40 to week 100) the optimal hedge ratios are statistically indistinguishable. However, when the market suddenly and abruptly turns, (see the graphical inset in Panel 7A), the performance of the dynamic approach improves. This may lead us to a simple conclusion. The dynamic hedging ratios are forward looking (they are forecasts) and so if the new hedge ratio ( $b_{(BA)t-1}$ ) is not different from the older hedge ratio ( $b_{(BA)t-2}$ ) from a statistical sense, then it is unlikely that any economic reward will be yielded. Consequently, the trader would save on transaction costs from updating the old hedge ratio. The same conclusion, and advice could be given to the trader considering using the DP – GARCH model over the OLS, naïve and unhedged strategies. Interestingly, compared to model I, as can be seen from panels 5C, 6C, 6D, 6E and 7B, the DP-GARCH model that optimally uses both the bid and the ask prices is never outperformed by any other approach. This was not the case in model I.

## 8. Conclusions

In this paper several questions were addressed simultaneously. Are there any advantages to combining two related but distinct hedging strategies – dynamic programming (DP) and time series econometrics (GARCH) models? Are the generated optimal hedging models from this approach statistically different from more standard approaches? Are there any economic gains to be enjoyed by combining the approaches, and are there certain times when a trader following a more basic strategy loses out? What is the role of the bid-ask spread on the optimal hedging strategy, and are there any gains to accounting for this in the optimal hedging strategy?

On all counts the results here are encouraging. First, given the ability to forecast volatilities and finding unbiased commodity markets enables us to derive a simple rule for developing the optimal DP-GARCH hedge ratios using cash and futures settlement price data. Second, using the well-known delta method, time-varying confidence bands were obtained such that a trader could distinguish whether or not optimal hedging ratios are statistically different from other models. It was also discovered that while the DP-GARCH model does slightly outperform the static GARCH approach on average, results verify that a trader that follows the static GARCH approach would lose out when the market suddenly and abruptly turns. Incorporating bid-ask prices into the traders portfolio resulted in hedge ratios lower than those recommended in the two price equation portfolio (using just settlement and cash prices), suggesting lower transaction costs, with much less volatility associated with the hedging ratios. Interestingly, while the gains from following the DP-GARCH approach after accounting for the bid-ask spread over the static approach are small, on average there are times that the trader would lose out when the market suddenly and abruptly turns, just like in the more basic model that ignores the spread. The more sophisticated models like DP-GARCH or static GARCH that ignore the bid-ask spread are outperformed by more stable hedging strategies like the OLS, when evaluated in the bid-ask environment. This result is also consistent with previous research whereby ignoring a natural portfolio results in far more volatile hedge ratios, which may induce risk rather than reduce it. This element of the research shows that if the forecasts of the DP-GARCH model incorporating the bid-ask spread suggest a statistically significantly different hedging ratio compared to the hedge ratio employed the previous period, then the trader should update the portfolio. As a result some gains may be achieved in terms of risk reduction. Alternatively, in more stable periods, the trader should continue with a static GARCH model that uses all three prices relevant to the trader, and enjoy potentially lower transaction costs.

Incorporating bid-ask prices into the trader's optimal hedging model may be more important in markets that have volatile bid-ask spreads. While the empirical application has focused on a moderately traded commodity in terms of volume (sugar), it would certainly be of interest to extend the same analysis to a more heavily traded commodity (with less volatile bid-ask spreads) or to a less heavily traded commodity (with more volatile spread behavior). Migration to a fully electronic trading system which generally provides more detailed pricing

information (bid, ask, transaction and trading volume) will enable research to be conducted on other commodities and time frames and enable further study on the microstructure of futures markets. These and other related issues remain, however, as important topics for future research.

## References

- Anderson, R.W., and J.P. Danthine, "The Time Pattern of Hedging and the Volatility of Futures Prices." *Review of Economic Studies* 50, 249-266, (April 1983).
- Bae, K., Chan.K, and Cheung, Y, "The Profitability of Index Futures Arbitrage: Evidence from Bid-Ask Quotes." *The Journal of Futures Markets*, 18, 743 – 763. (1998).
- Baillie, R.T., and R.J. Myers, "Bivariate GARCH Estimation of the Optimal Commodity Futures Hedge." *Journal of Applied Econometrics* 6, 109-124, (April-June 1991).
- Bollerslev, T., "Generalized Autoregressive Conditional Heteroskedacity." *Journal of Econometrics* 31, 301-327. (April 1986).
- Bollerslev, T., and J.M. Wooldridge, "Quasi-maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances." *Econometric Reviews* 11, 143 – 172. (1992).
- Campbell, J.Y., A.W. Lo, and A.C. Mackinley, "The Econometrics of Financial Markets." Princeton, New Jersey: Princeton University Press, (1997).
- Cecchetti, S.G., R.E. Cumby., and S. Figlewski, "Estimation of the Optimal Futures Hedge." *Review of Economics and Statistics* 70, 623-630, (November 1988).
- Dickey, D.A., and W. Fuller, "Likelihood Ratio Tests for Autoregressive Time Series with a Unit Root." *Econometrica* 49, 1057-1072, (June 1981).
- Ederington, L.H., "The Hedging Performance of the New Futures Markets." *Journal of Finance* 34, 157-170, (March 1979).
- Engle, R.F., "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation." *Econometrica* 50, 987-1008, (July 1982).
- Fama,E.F., "The Behaviour of Stock Market Prices." *Journal of Business* 38, 34 – 105. (January 1965).
- French, K., "Stock Returns and the Weekend Effect", *Journal of Financial Economics* 8, 55 – 69, (1980).
- Gagnon, L., G.J. Lypny, and T.H. McCurdy, "Hedging Foreign Currency Portfolios," *Journal of Empirical Finance* 5, 197-220, (September 1998).
- Garcia, P., J.-S. Roh, and R.M. Leuthold, "Simultaneously Determined, Time-Varying Hedge Ratios in the Soybean Complex." *Applied Economics* 27, 1127-1134, (December 1995).
- Haigh, M.S. and M.T. Holt, "Hedging Multiple Price Uncertainty in International Grain Trade." *American Journal of Agricultural Economics* 82, 881-896, (November 2000).
- Johnson, L.L., "The Theory of Hedging and Speculation in Commodity Futures." *Rev. Econ. Stud.* 27, 139 – 151, (June 1960).
- Kahl, K.H., "Determination of the Recommended Hedging Ratio." *American Journal of Agricultural Economics* 65, 603-605, (August 1983).
- Karp, L.S., "Dynamic Hedging with Uncertain Production." *International Economic Review* 29, 621-637, (November 1988).
- Kendall, M.G., and A. Stuart. "The Advanced Theory of Statistics – Volume 1: Distribution Theory (4<sup>th</sup> Edition). New York: Macmillan. (1977).
- Kroner, K.F., and J. Sultan, "Time Varying Distributions and Dynamic Hedging with Foreign Currency Futures." *Journal of Financial and Quantitative Analysis* 28, 535-551, (December 1993).

- Lence, S., K. Kimle, and M. Hayenga, "A Dynamic Minimum Variance Hedge." *American Journal of Agricultural Economics* 76, 1063-1071, (November 1993).
- Lin, J.W., M. Najand, and K. Yung, "Hedging with Currency Futures: OLS vs GARCH." *Journal of Multinational Financial Management* 4, 45-67, (1994).
- Martin, L., and P. Garcia, "The Price-Forecasting Performance of Futures Markets for Live Cattle and Hogs: A Disaggregated Analysis." *American Journal of Agricultural Economics* 63, 209-215, (May 1981).
- Martinez, S., and K.D. Zering, "Optimal Dynamic Hedging Decisions for Grain Producers." *American Journal of Agricultural Economics* 74, 879-888, (November 1992).
- Mathews, K.H., "Minimum Risk Hedge Ratios." Ph.D thesis, North Carolina State University. (1989).
- Mathews, K.H., and D.M. Holthausen, Jr., "A Simple Multiperiod Minimum Risk Hedge Model." *American Journal of Agricultural Economics* 73, 1020-1026 (November 1991).
- Myers, R.J., and S.D. Hanson, "Optimal Dynamic Hedging in Unbiased Futures Markets." *American Journal of Agricultural Economics* 78, 13-20, (February 1996).
- Myers, R.J., "Estimating Time-Varying Optimal Hedge Ratios on Futures Markets." *Journal of Futures Markets* 11, 39-53, (February 1991).
- Park, T.H., and L.N. Switzer, "Bivariate GARCH Estimation of the Optimal Hedge Ratios for Stock Index Futures: A Note." *Journal of Futures Markets* 15, 61-67, (February 1995).
- Press, W.H., B.P Flannery, S.A. Teukolsky and W.T. Vettering. Numerical recipes in C. New York: McGraw-Hill. (1988).
- Roll, R. "A Simple Implicit Measure of the Effective Bid – Ask Spread in an Efficient Market." *The Journal of Finance*, 23, 1127 – 1139. (1984).
- Samuelson, P.A., "Proof that Properly Anticipated Prices Fluctuate Randomly." *Industrial Management Review*, 6, 41 – 49, (1965).
- Sephton, P.S., "Optimal Hedge Ratios at the Winnipeg Commodity Exchange." *Canadian Journal of Economics* 26, 175-193, (February 1993).
- Smith, T., and R.E. Whaley. "Estimating the Effective Bid – Ask Spread from Time and Sales Data." *The Journal of Futures Markets*, 14, 437 – 456, (1994).
- Stein, J.L., "The Simultaneous Determination of Spot and Futures Prices." *Amer. Econ. Rev* 51, 1012 –1025, (December 1961)
- Tong, W.H.S. "An Examination of Dynamic Hedging." *Journal of International Money and Finance* 15, 19-35, (February 1996).
- Vukina, T. and J.L. Anderson, "A State-Space Forecasting Approach to Optimal Intertemporal Cross-Hedging." *American Journal of Agricultural Economics* 75, 416-424, (May 1993).
- Yeh, S.C., and G.L. Gannon, "Comparing Trading Performance of the Constant and Dynamic Hedge Models: A Note." *Review of Quantitative Finance and Accounting* 14, 155-160, (March 2000).

**Table 1. Estimation of Model I (using settlement prices) and Model II (bid and ask prices).**

<b>Model I</b>					
Cash (C)			Settlement (S)		
Parameter	Coefficient		Parameter	Coefficient	
$m_c$	-0.7410 (0.5581)		$m_s$	-0.7906 (0.4456)	
$w_c$	2.8746 (1.7739)		$w_s$	4.6252 (3.6181)	
$a_c$	0.1015 (0.0402)		$a_s$	0.1582 (0.0891)	
$b_c$	0.8494 (0.0348)		$b_s$	0.7493 (0.1303)	
<b>Correlation Parameter</b>					
	Parameter			Coefficient	
	$r_{cs}$			0.6774 (0.0458)	
<b>Log Likelihood: -972.58</b>					
<b>Model II</b>					
Cash (C)		Bid (B)		Ask (A)	
Parameter	Coefficient	Parameter	Coefficient	Parameter	Coefficient
$m_c$	-1.1703 (2.700)	$m_B$	-0.8596 (0.197)	$m_A$	-0.8367 (0.2047)
$w_c$	54.559 (15.691)	$w_B$	46.855 (6.023)	$w_A$	45.026 (0.098)
$a_c$	0.1456 (0.002)	$a_B$	0.1270 (0.014)	$a_A$	0.1512 (0.001)
$b_c$	0.5715 (0.003)	$b_B$	0.5227 (0.047)	$b_A$	0.5363 (0.001)
<b>Correlation Parameters</b>					
	Parameter		Coefficient		
	$r_{CB}$		0.6600 (0.008)		
	$r_{CA}$		0.6549 (0.014)		
	$r_{BA}$		0.9987 (0.001)		
<b>Log Likelihood: -1011.99</b>					

Robust standard errors are in parenthesis.

**Table 2. Descriptive Statistics for Hedge Ratios for Risk Minimizing Static and Dynamic Objectives, Three-Week Hedging Horizon.**

	<b>Model I</b>		<b>Model II</b>	
Hedge Model	$b_{(S)t-2}$	$b_{(S)t-1}$	$b_{BA(t-2)}$	$b_{(BA)t-1}$
DP-GARCH				
Avg.	0.7613	0.7627	0.7476	0.7490
SE	0.1288	0.1287	0.0459	0.0420
Min	0.4885	0.4895	0.5518	0.5663
Max	1.1288	1.1310	0.8870	0.8852
GARCH	$b_{(S)GARCH}$		$b_{(BA)GARCH}$	
Avg.	0.7613		0.7476	
SE	0.1288		0.0459	
Min	0.4885		0.5518	
Max	1.1288		0.8870	
OLS	$b_{(S)OLS}$		$b_{(BA)OLS}$	
	0.7104		0.7104	
Naive	$b_{(S)NAIVE}$		$b_{(BA)NAIVE}$	
	1		1	
Unhedged	$b_{(S)Unhedged}$		$b_{(BA)Unhedged}$	
	0		0	

**Note:** The annualized discount rate,  $r$ , is 0.10. Avg. denotes sample average, SE is the corresponding standard error of the average of the hedge ratios, Min is the sample minimum and Max is the sample maximum. The GARCH hedge ratio for both model I ( $b_{(S)GARCH}$ ) and model II ( $b_{(BA)GARCH}$ ) represents the average hedge ratio that would be used by the trader over the entire trading period. It is equal to the hedge ratio used at  $t - 2$  by the DP – GARCH user as it is assumed that the simple GARCH user uses weekly data to form the hedge ratio to be applied at  $t - 2$  and left in place until the commodity is purchased at the end of the trading horizon. The OLS,  $b_{(S)OLS}$  and  $b_{(BA)OLS}$  and Naïve,  $b_{(S)NAIVE}$  and  $b_{(BA)NAIVE}$  hedge ratios and, used each week are not, like the DP-GARCH and GARCH counterparts updated each week.

**Table 3. Descriptive Variance Statistics for Static and Dynamic Objectives for Model I.**

DP-GARCH using $b_{(s)t-2}$ and $b_{(s)t-1}$	
Avg.	133.21
SE	50.960
Min	62.219
Max	337.382
GARCH – Static using just $b_{(s)t-2}$	
Avg.	133.84
SE	51.127
Min	62.346
Max	337.49
OLS – Static using $b_{(s)OLS}$	
Avg.	137.93
SE	53.436
Min	62.565
Max	631.07
Naïve – Static using $b_{(s)NAIVE}$	
Avg.	142.41
SE	60.900
Min	65.803
Max	468.28
Unhedged – Static using $b_{(s)UNHEDGED} = 0$ .	
Avg.	231.67
SE	84.900
Min	104.01
Max	464.70
% variance reduction from using the DP – GARCH model relative to:	
GARCH – Static using just $b_{(s)t-2}$	0.2203%
OLS – Static using $b_{(s)OLS}$	2.8404%
Naïve – Static using $b_{(s)NAIVE}$	5.1180%
Unhedged – Static using $b_{(s)UNHEDGED} = 0$ .	42.274%

The annualized discount rate,  $r$ , is 0.10. Avg. denotes sample average, SE denotes the corresponding standard deviation. Min is the sample minimum and Max is the sample maximum. Results are based on 212 weekly hedging periods. The DP-GARCH model uses the optimal hedging ratios generated from the cash and futures settlement prices. The GARCH-Static model only employs the hedging ratio developed in  $t - 2$ , and does not optimally update. The OLS model uses the hedging ratio developed from a simple regression of the futures settlement price on the cash price. The Naïve and Unhedged ratios are set equal to 1 and 0 respectively.

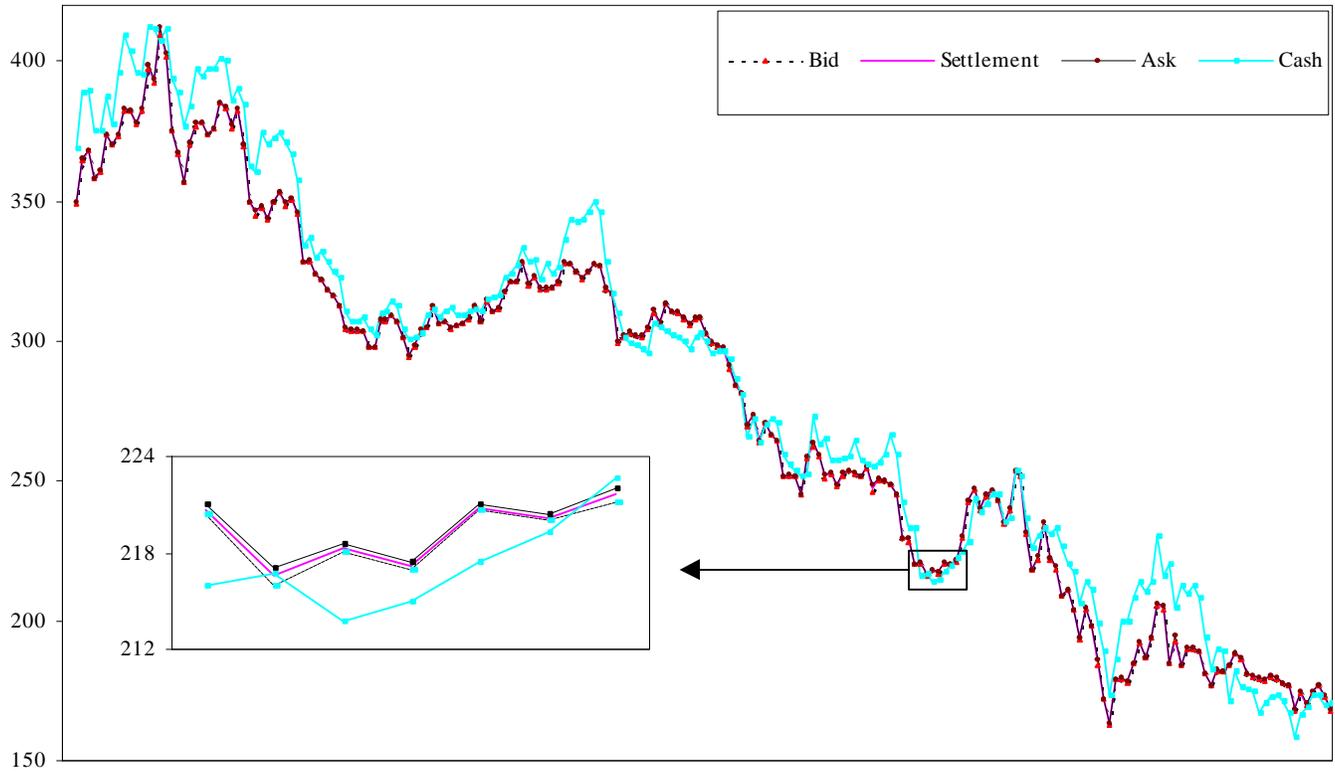
**Table 4. Descriptive Variance Statistics for Static and Dynamic Objectives for Model II.**

DP-GARCH using $b_{(BA)_{t-2}}$ and $b_{(BA)_{t-1}}$	
Avg.	237.75
SE	42.855
Min	148.56
Max	515.46
GARCH – Static using just $b_{(BA)_{t-2}}$	
Avg.	238.013
SE	43.155
Min	148.960
Max	517.810
DP-GARCH using $b_{(s)_{t-2}}$ and $b_{(s)_{t-1}}$	
Avg.	242.131
SE	42.6720
Min	150.823
Max	524.361
GARCH – Static using just $b_{(s)_{t-2}}$	
Avg.	242.850
SE	43.103
Min	150.621
Max	526.082
OLS – Static using $b_{(BA)_{OLS}}$	
Avg.	239.079
SE	43.1437
Min	151.824
Max	527.319
Naïve – Static using $b_{(BA)_{NAIVE}}$	
Avg.	269.2831
SE	57.0637
Min	160.308
Max	656.380
Unhedged – Static using $b_{(BA)_{UNHEDGED}} = 0$ .	
Avg.	494.356
SE	58.0823
Min	359.168
Max	822.060
% variance reduction from using the DP – GARCH model relative to:	
GARCH – Static using just $b_{(BA)_{t-2}}$	0.099
GARCH – Dynamic using $b_{(s)_{t-2}}$ and $b_{(s)_{t-1}}$	1.833
GARCH – Static using just $b_{(s)_{t-2}}$	2.095
OLS – Static using $b_{(BA)_{OLS}}$	0.5667
Naïve – Static using $b_{(BA)_{NAIVE}}$	11.297
Unhedged	52.105

The DP-GARCH model uses hedging ratios generated from cash, bid and ask prices. The GARCH-Static employs the cash, bid and ask prices but only the ratio developed in  $t - 2$ , and does not update. The DP-GARCH is the bid-ask model that employs hedging ratios developed from model I. The GARCH static uses the bid-ask model, but employs the ratio from model I. The OLS model uses the hedging ratio developed from a simple regression of the futures settlement price on the cash price. The Naïve and Unhedged ratios are set equal to 1 and 0 respectively.

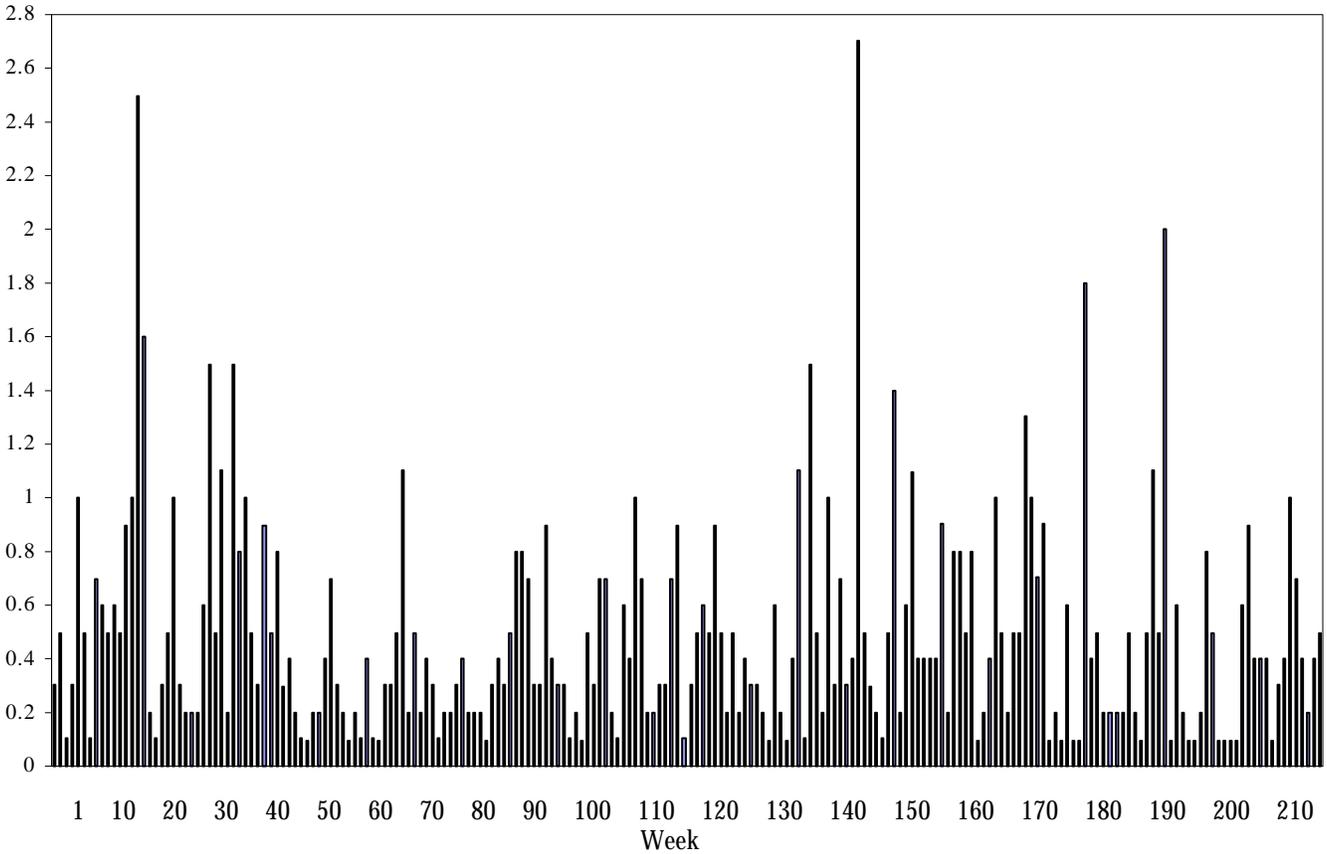
**Figure 1. Weekly Bid, Settlement, Ask and Cash Prices: December 1995 – January 2000.**

Dollars per tonne



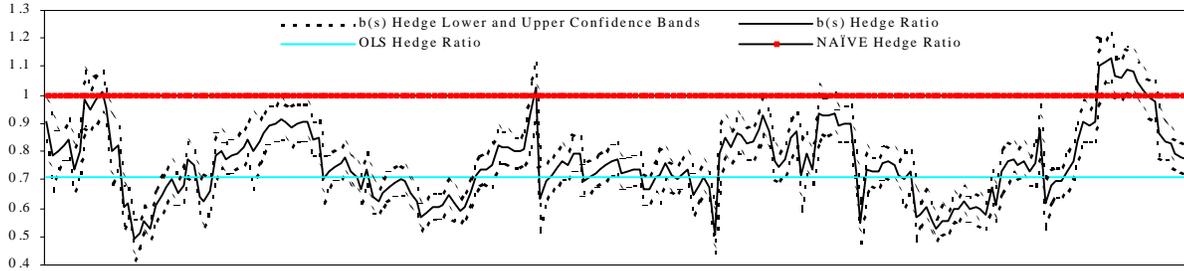
**Figure 2. Weekly Wednesday Closing Bid-Ask Spread: December 1995 – January 2000.**

Dollars per tonne

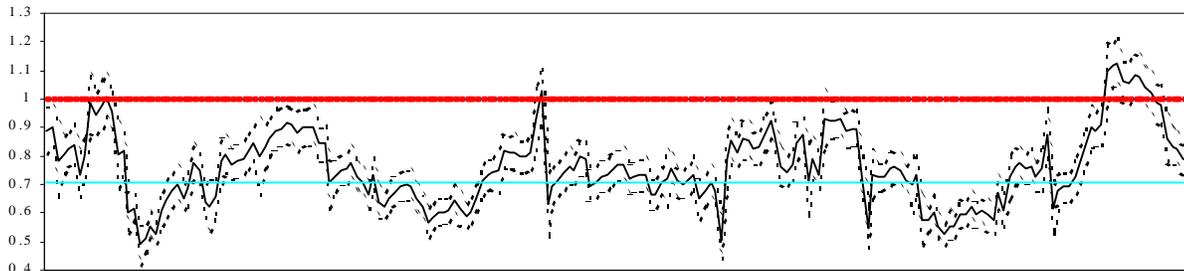


**Figure 3. Model I (DP-GARCH model using  $b(s)$  hedge ratios) with confidence bands (Panel 3A and 3B) at  $t-1$ ,  $t-2$  respectively, and percentage improvement over the OLS, Naïve and unhedged models (Panels 3C – 3E).**

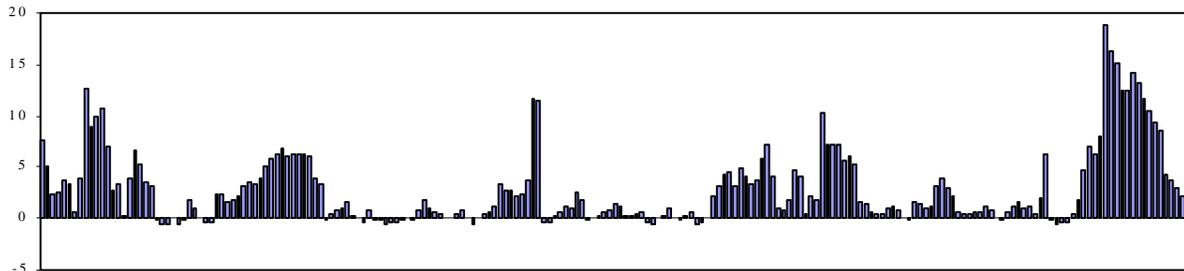
Panel 3A. Hedge Ratios at  $t-1$ .



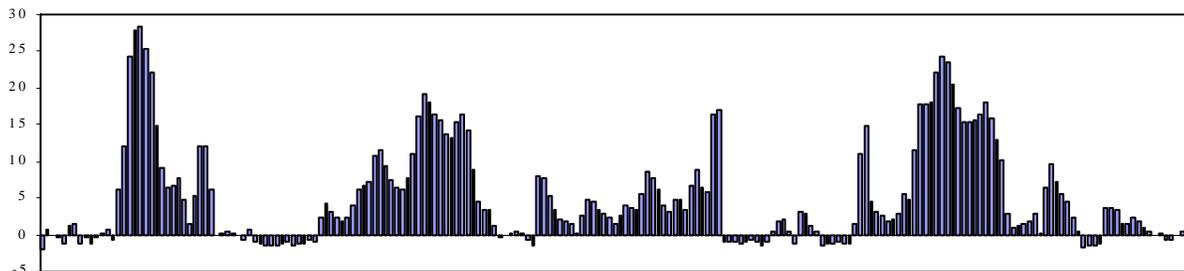
Panel 3B. Hedge Ratios at  $t-2$ .



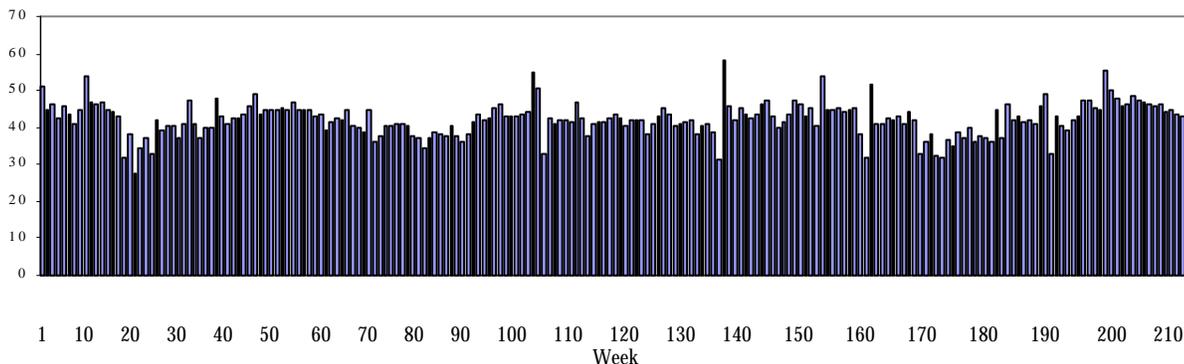
Panel 3C. Percentage Improvement of Model I over the OLS Model



Panel 3D. Percentage Improvement of Model I over the Naive Model



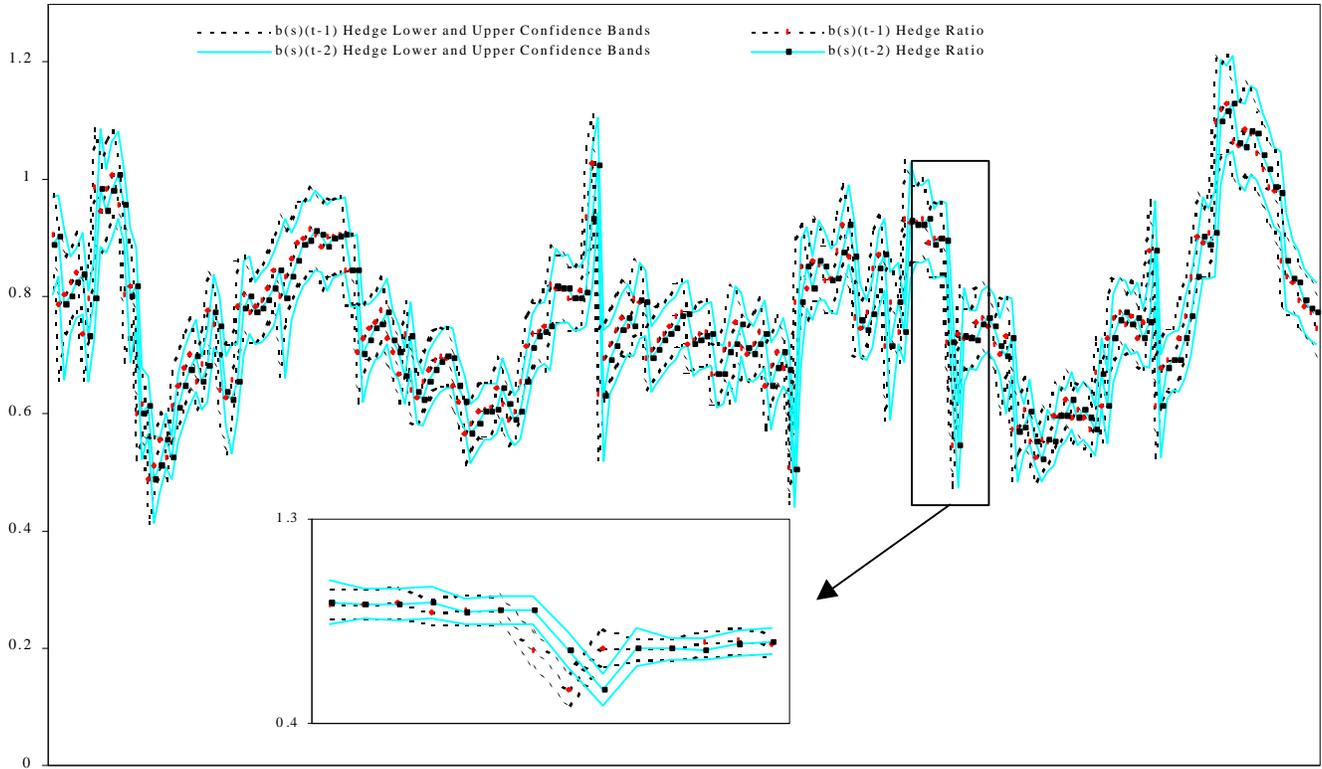
Panel 3E. Percentage Improvement of Model I over the Unhedged Model



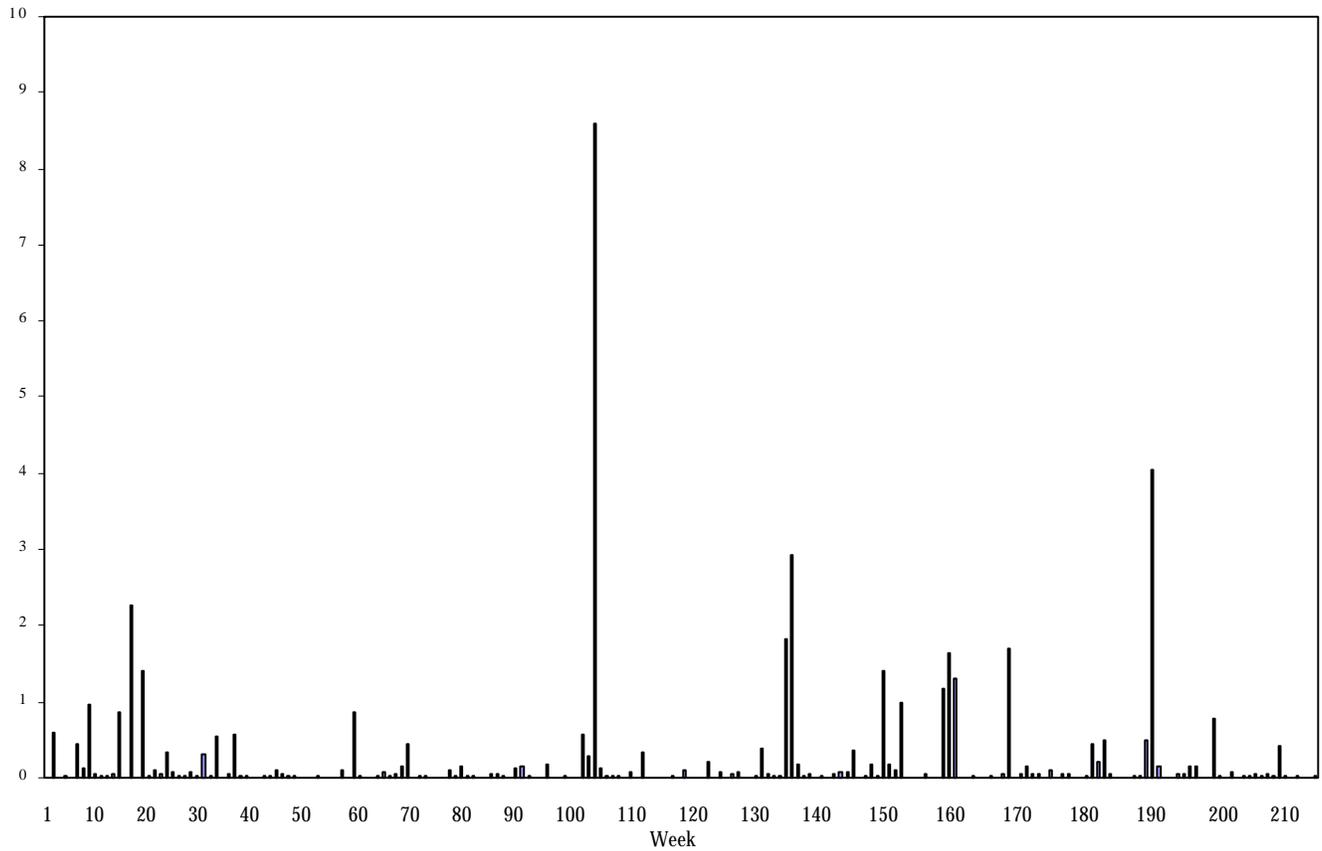
Note: The  $b_{(s)}$  hedge ratios refer to the optimal hedging ratios calculated using the futures settlement price data.

**Figure 4. Model I hedge ratios at  $t - 1$ ,  $t - 2$  with confidence bands (Panel 5A) and percentage improvement over the static version of model I. (Panel 5B).**

Panel 4A. Hedge ratios and confidence bands at  $t - 1$  and  $t - 2$



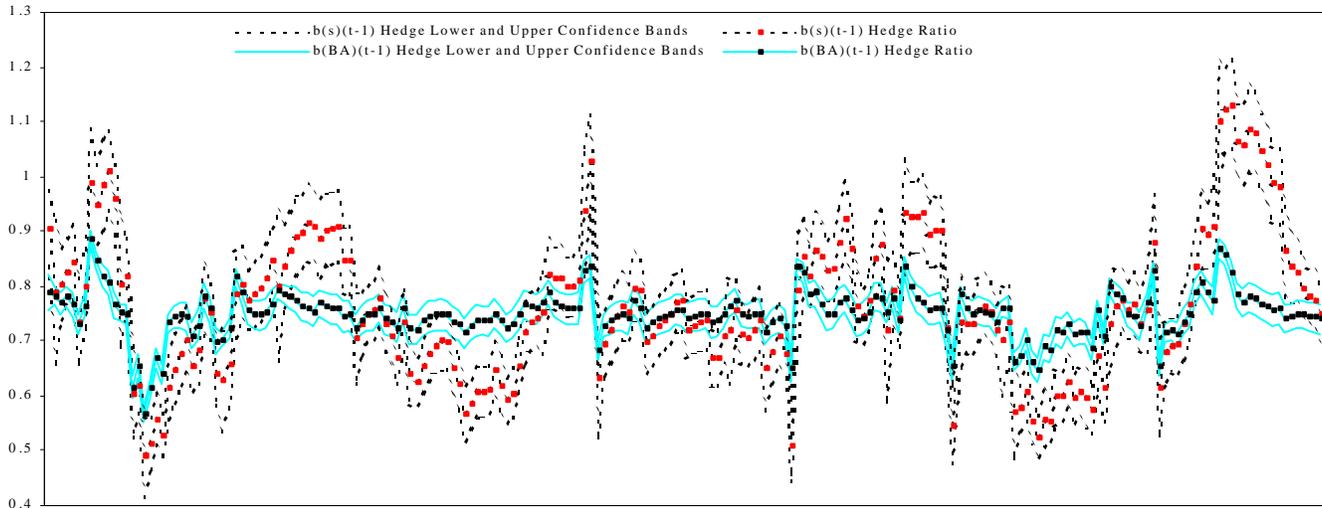
Panel 4B. Percentage Improvement of model I over the static version of model I.



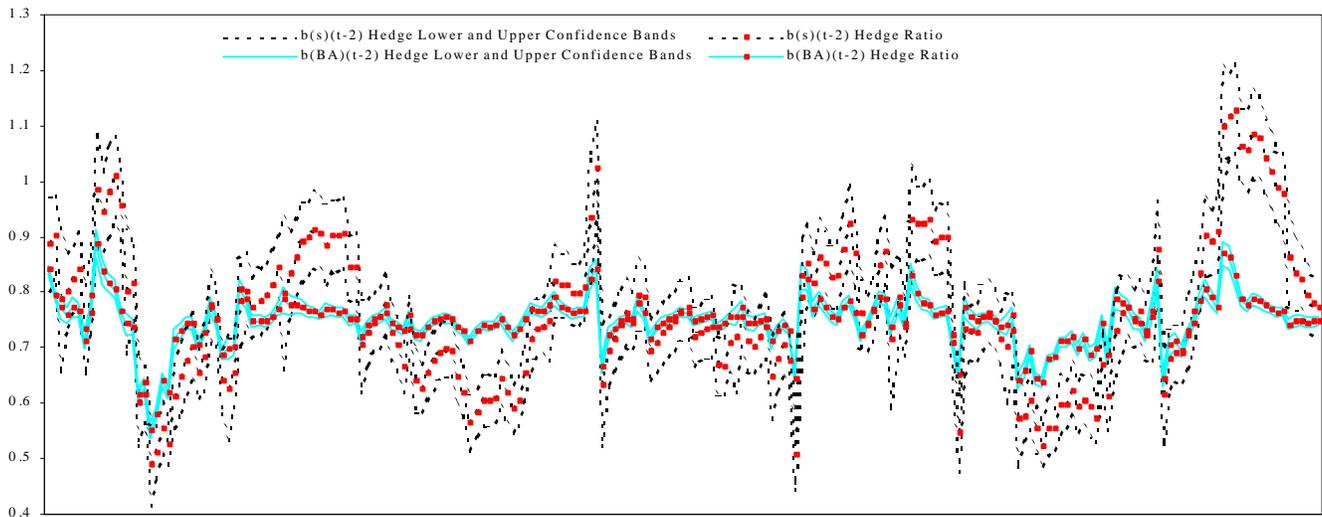
Note: Model I is a dynamic model in that it utilizes the optimal hedges established at  $t - 2$  and then updates by using the optimal hedge ratio at  $t - 1$ . The static version of the model utilizes the optimal hedge established at  $t - 2$  in both periods.

**Figure 5. Model I and Model II hedge ratios at  $t - 1$ ,  $t - 2$  with confidence bands (Panel 5A and 5B) and percentage improvement of Model II over Model I (Panel 5C).**

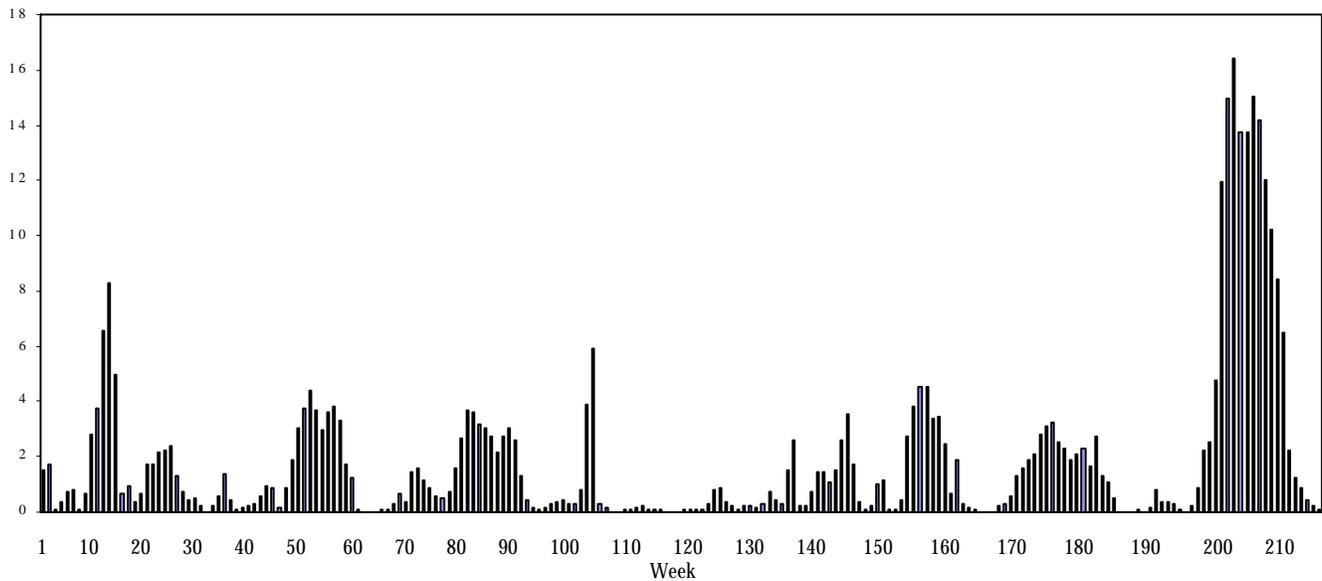
Panel 5A. Hedge ratios and confidence bands at  $t - 1$ .



Panel 5B. Hedge ratios and confidence bands at  $t - 2$ .

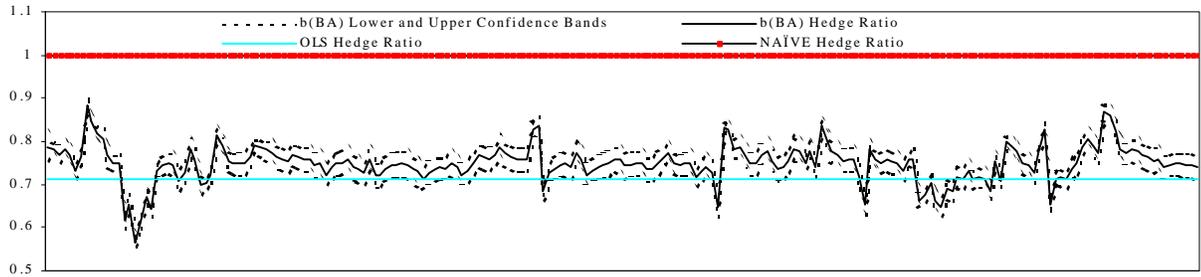


Panel 5C. Percentage improvement of Model II over Model I.

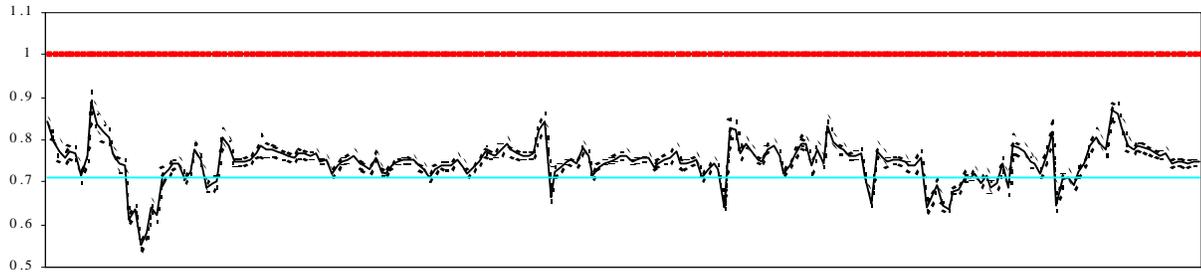


**Figure 6. Model II (DP-GARCH model using b(BA) hedge ratios) with confidence bands (Panel 4A and 4B) at  $t - 1$ ,  $t - 2$  respectively, and percentage improvement over the OLS, Naïve and unhedged Models (Panels 4C – 4E).**

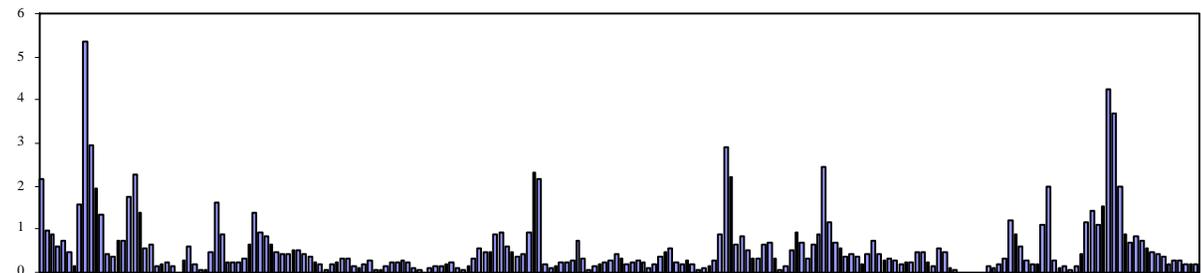
Panel 6A. Hedge ratios at  $t - 1$ .



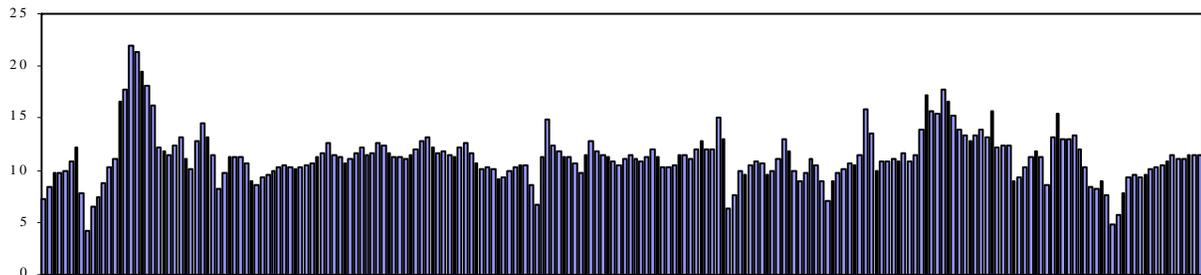
Panel 6B. Hedge ratios at  $t - 2$ .



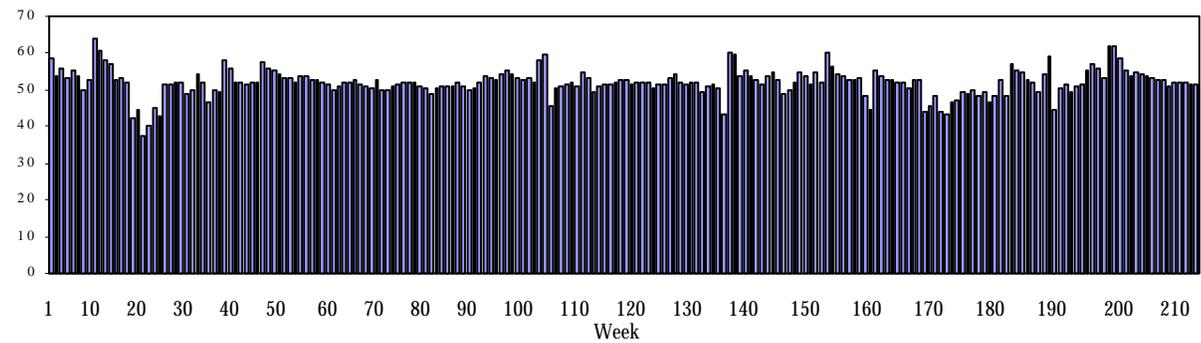
Panel 6C. Percentage Improvement of Model II over the OLS Model



Panel 6D. Percentage Improvement of Model II over the Naive Model

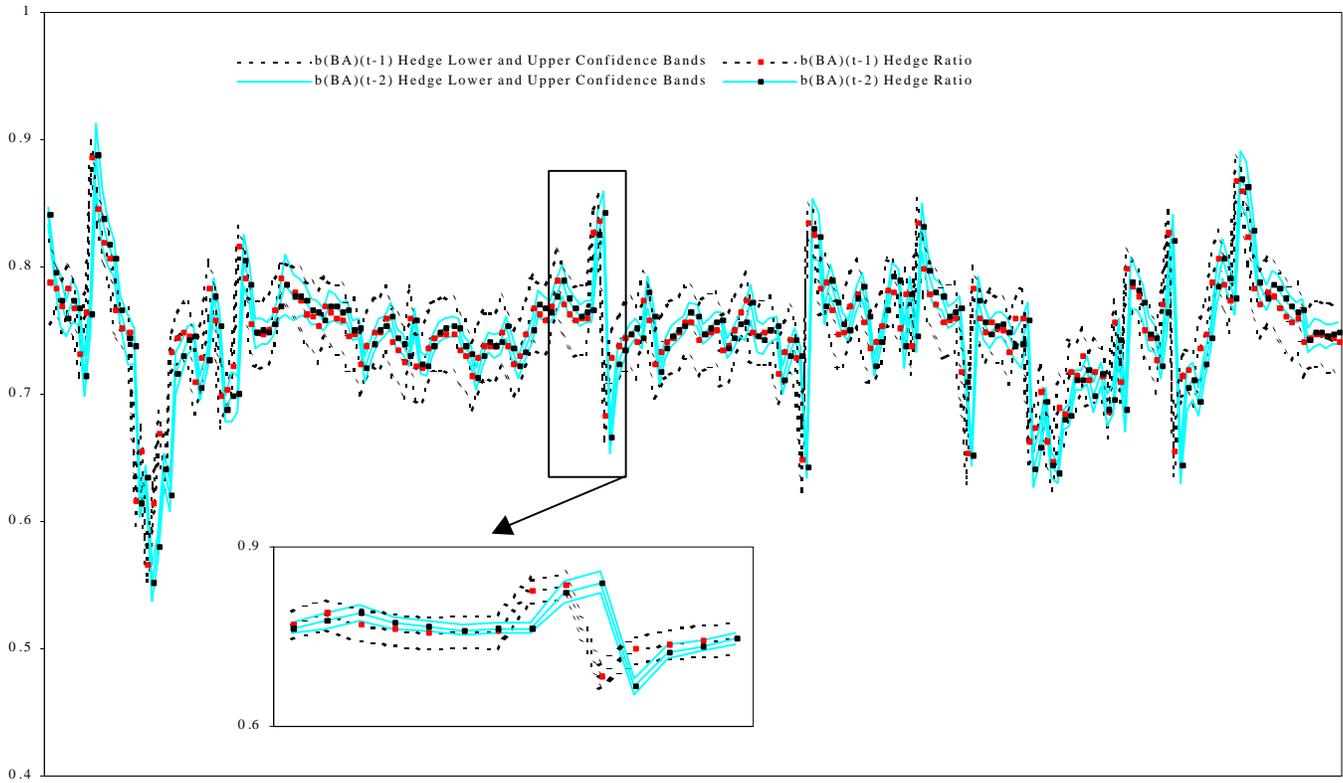


Panel 6E. Percentage Improvement of Model II over the Unhedged Model

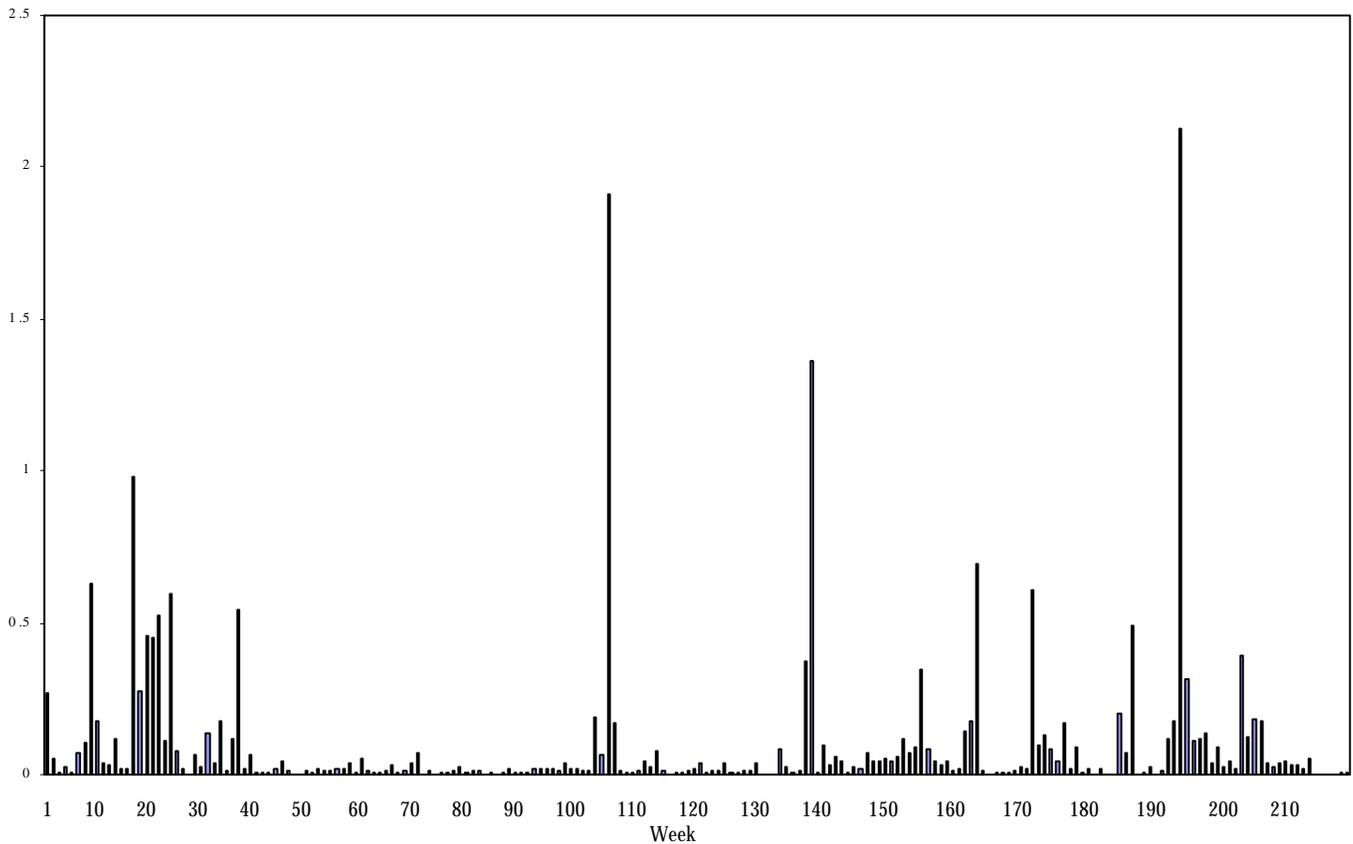


**Figure 7. Model II hedge ratios at  $t - 1$ ,  $t - 2$  with confidence bands (Panel 7A) and percentage improvement over the static version of model II. (Panel 7B).**

Panel 7A. Hedge Ratios and confidence bands at  $t - 1$  and  $t - 2$



Panel 7B. Percentage Improvement of Model II over the static version of the model.



Note: Model II utilizes the optimal hedges established at  $t - 2$  and then updates by using the optimal hedge ratio at  $t - 1$ . The Static model II utilizes the optimal hedge established at  $t - 2$  in both periods.

## Endnotes

1. Since then, this branch of the hedging literature adopted the term dynamic because these studies relax the assumption of constant conditional variances and covariances. Importantly however, when contrasted with the DP approach, these hedge ratios are not really dynamic, they are simply time-varying.
2. To simplify the model, we follow Mathews (1989) and Mathews and Holthausen (1991) by assuming that the hedger knows  $b_{(s)t-1}$  at the initial trade date. If we did not make this assumption, then  $b_{(s)t-1}$  would be stochastic and additional variance and covariance terms would be involved. This may not however be a very restrictive assumption, as estimates of the variances and covariances were based on historical relationships are relatively easy to forecast using GARCH models (see footnote 5).
3. For example, we can represent the futures price at time period  $t$  as  $F_t = F_{t-1} + u_t$  with a corresponding variance expressed as:  $Var_t(F_t) = E_t(u_t)^2$ . The futures price at period  $t - 1$  can then be expressed as:  $F_{t-1} = F_{t-2} + u_{t-1}$ , and because  $F_t = F_{t-2} + u_{t-1} + u_{t-2}$ , we have  $Cov_{t-2}(F_{t-1}, F_t) = E_{t-2}(u_{t-1}, u_{t-1} + u_{t-2})^2 = E_{t-2}(u_{t-1})^2$ . Therefore,  $Var_{t-2}(F_{t-1}) = Cov_{t-2}(F_{t-1}, F_{t-2})$ , and so the hedge ratio in (12), for example, collapses to  $b_{(s)t-2} = \frac{Cov_{t-2}(F_{t-1}, C_t)}{(1+r)Var_{t-2}(F_{t-1})}$ , and so the optimal hedge ratio to be employed at the initial period is simply the ratio of the forecasted covariances of  $F_{t-1}$  and  $C_t$ , divided by the discount rate and the variance of the next periods futures price,  $F_{t-1}$ .
4. However, we continue with the assumption (as do other papers that have employed the MV methodology) that the trader is extremely risk averse. Consequently, the speculative component of the hedging ratio – even if it is expected to be negative, is likely to be quite insignificant and hence the two hedging approaches should be extremely similar.
5. It can be shown (full details are available from the author upon request) that the M-step ahead conditional variance for the GARCH (1,1) model is simply  $s_{t,t+M}^2 = M\bar{s}^2 + [s_{t+1}^2 - \bar{s}^2] \frac{1 - (\mathbf{a} + \mathbf{b})^M}{1 - (\mathbf{a} + \mathbf{b})}$ , where  $\bar{s}^2 = \frac{\mathbf{w}}{(1 - \mathbf{a} - \mathbf{b})}$ , is the steady state volatility of the GARCH process.
6. Baillie and Myers (1991) also undertook their analysis using a similar framework, thus employing a relatively parsimonious model. In this paper, the structure for the time-series generating process is based on the usual residual diagnostic results (Ljung-Box  $Q$  and  $Q^2$  tests for white noise).
7. A total of seven Thursday prices and five Friday prices were used.

8. For example, total volume for the year 2000 for the sugar contract was 907,399, which ranked third in terms of total volume behind cocoa (1,636,322) and coffee (1,470,980). The BIFFEX freight futures contract was the most thinly traded commodity in year 2000 totaling just 3,244 contracts. In contrast however, the NYBOT #11 Sugar contract traded approximately 5,917,303 contracts in 2000, approximately 6.5 times the number of sugar contracts traded at LIFFE.
  
9. These results, like all results excluded to conserve space are available from the author upon request.
  
10. This algorithm is described in Press, et. al (1988).
  
11. These results, like all results excluded to conserve space are available from the author upon request.
  
12. In particular, each of the hedging ratios  $b_{(i=5 \text{ or } BA)}$  can be expressed as function  $b_i(\mathbf{q})$  of a parameter vector  $\mathbf{q} = (\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_p)'$ . If the covariance matrix of  $\hat{\mathbf{q}}$  is C and J is the gradient of  $b_i(\mathbf{q})$ , then approximately:  $b_i(\mathbf{q}) \doteq b_i(\mathbf{q}_0) + (\mathbf{q} - \mathbf{q}_0)' J$ , where  $\mathbf{q}_0 = E(\mathbf{q})$ , so  $Var(b_i(\hat{\mathbf{q}})) \doteq J' C J$