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**Underpinnings for Prospective, Net Revenue  
Forecasting in Hog Finishing: Characterizing the Joint  
Distribution of Corn, Soybean Meal and Lean Hogs  
Time Series**

by

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**Underpinnings for Prospective, Net Revenue Forecasting  
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## **Underpinnings for Prospective, Net Revenue Forecasting in Hog Finishing:**

### **Characterizing the Joint Distribution of Corn, Soybean Meal and Lean Hogs Time Series**

#### **Abstract**

*This research focuses on developing a biannual net revenue forecasting model for hog producers based on Monte Carlo simulation of the joint distribution of hog, corn and soybean meal price series. The relative forecasting power of historical volatility, implied volatility and GARCH-based volatility is examined. Consistent with recent research, the performance of these three methods is both commodity and horizon specific, which means there is no single best predictor. However, implied volatility often performs well. Thus, implied volatility is used to forecast variance. Historical covariance is introduced to capture the co-movement of the three price series. Our forecasting model performs well out of sample; most of the realized net revenues fall in 95 percent prediction interval. Based on this forecasting model and the assumption of a utility function, we compare our prospective evaluation with retrospective evaluation of risk management strategies. Though prospective evaluation is not significantly superior to retrospective evaluation for this particular dataset, it is useful because all the market information has been incorporated in this model and because it did protect producers from adverse price movements.*

**Keywords:** *Hogs, Net Revenue, Volatility, Monte Carlo Simulation, Prospective Evaluation, Retrospective Evaluation, Time Series, Forecast, Joint Distribution, Risk Management.*

#### **Introduction**

Hog producers face many uncertainties: the cost of feeds, the efficiency of hog growth, the price of feeder and live hogs, weather shocks, and so on. These sources of uncertainty create substantial financial risk for an individual producer. During recent years the structure of the hog production industry has experienced profound changes with rapid emergence of larger operations that have gained substantial efficiencies by exploiting economies of size. However, during the recent downturn in the hog cycle, these large, efficient operations lost money – many of them for the first time ever.

Hence, all hog producers are motivated to understand and perhaps protect against downside risk. Methods currently available to hog producers to limit downside risk include the use of forward, futures, options and/or marketing contracts. Currently, it is estimated that about 75 percent of all producers are involved in some type of marketing price contract; however, only 18 percent of producers are involved in contracts that substantial limit price risk.

To choose the marketing strategy that helps achieve profit and risk objectives, hog producers must understand the nature of the underlying risk and the ability of various strategies

to alter this risk. The present study develops a prospective net revenue forecasting model that allows hog producers to evaluate various risk management strategies.

### Model of Producers' Net Revenue

Feed costs and live hog prices dominate the profitability of hog production. Feed costs accounted for about 40 percent of total costs each year from 1992 to 1999 in US, while live hog prices had an obvious impact on firm profitability. Hence, we restrict our attention to two key feed ingredients (corn and soybean meal) and live hog prices and define net revenue for a hog producer as:

$$(1) \quad \tilde{R}_T = \sum_{t=0}^T (\tilde{P}_{t,1} Q_{t,1} - \tilde{P}_{t,2} Q_{t,2} - \tilde{P}_{t,3} Q_{t,3}),$$

where  $\tilde{R}_T$  is the net revenue from hog finishing activities over the time period 0 to T net of corn and soybean meal costs;  $\tilde{P}_{t,1}$  is the price received for finished live hogs at time t;  $\tilde{P}_{t,2}$  and  $\tilde{P}_{t,3}$  are the prices paid for corn and soybean meal at time t, respectively;  $Q_{t,1}$  is the number of hogs sold at time t;  $Q_{t,2}$  and  $Q_{t,3}$  are the quantity of corn and soybean meal purchased at time t, respectively; and  $\sim$  indicates a random variable.

The purpose of this study is not to help hog producers figure out the optimal production plans nor the best marketing time. Rather, it is to help them manage price risks they face. So we assume all the quantity variables are exogenous. Namely, we assume the quantities of inputs and outputs are perfectly known in advance and internally consistent with common hog production technology and the timing of purchases and sales are predetermined by the producer.

To forecast the distribution of net revenue for hog producers at a future date, we have to project the joint distribution of the three price series. According to the market efficiency hypothesis, the observed futures price of a commodity is the unbiased forecast of the commodity spot price at the futures expiration date, i.e.

$$(2) \quad F_{0,i} = E(P_{T,i}),$$

where  $F_{0,i}$  is the observed futures price for a futures contract expired at time T and  $P_{T,i}$  is the spot price at time T. Though there remains some controversy about employing such an assumption<sup>1</sup>, this hypothesis is widely accepted in academic and empirical studies. We use the observed futures price as the point estimation for the spot price and formulate our forecasting of spot price movement based on this hypothesis.

The covariance matrix of the joint distribution of the three price series is another key for forecasting the distribution of net revenue for hog producers at a future date. Three widely used volatility forecasts for the variance terms in the covariance matrix are: historical volatility, implied volatility and GARCH-based volatility. Their relative forecasting power has been compared in the fields of stock indices (Canina and Figlewski; Lamoureux and Lastrapes) and currency indices (Amin and Ng). Also there has been increasing interest in applying hybrid forecasting methods (Diebold and Lopez) including an application of hybrid approaches to forecasting volatility of individual agricultural commodities (Manfredo, Luethold and Irwin).

However, applications to forecasting the joint distribution of agricultural commodities are limited (e.g. Baillie and Myers). Given that variance and covariance among these series are critical for forecasting net revenue, we explore several approaches to forecasting the variance structure of the three cash time series. Historical estimates are used for forecasting covariance terms. The relative forecasting power of implied volatility, historical volatility and GARCH-based volatility is examined based on Manfredo et al.'s (1999) framework. Our results show that implied volatility usually outperforms the other two forecasting methods. Then we generate a forecast for the distribution of net revenue based on the joint distribution of the three price series by using implied volatility, historical covariance and the assumption of unbiased futures prices.

While retrospective (ex post) evaluation is the most prevalent method for assessing the efficacy of risk management strategies in the hog sector (e.g., Zanini and Garcia; Lawrence and Vontalge), interest in prospective (ex ante) assessment techniques is growing in other areas of commodity agriculture (e.g., Schnitkey and Miranda). We conduct prospective assessments of risk management strategies based on our net revenue forecasting model and explore the effect of one typical risk management strategy on a producer's net revenue over a six-month period.

## Data

To compare the forecasting power of alternative volatility forecasting methods for hog, corn and soybean meal price series, we need to calculate the cash return series for these commodities. To calculate the implied volatility for these three return series, we also need the futures price, options data and the risk-free interest rate. The continuously compounded rate of return is used to a great extent when futures and options are being priced. So we define the rate of return as:

$$(3) \quad R_{t,i} = \ln(P_{t,i}) - \ln(P_{t-1,i}),$$

where  $R_{t,i}$  is the weekly rate of return of the price series at time  $t$  for commodity  $i$ ;  $\ln$  is the natural logarithm;  $P_{t,i}$  is the Wednesday price of commodity  $i$  at week  $t$ ; and  $P_{t-1,i}$  is the Wednesday price of this commodity at week  $t-1$ . Since many hog producers market hogs once each week or once every several weeks, it is reasonable to use weekly price rather than daily price in this forecasting model. When the Wednesday price is not available, the Tuesday price is used as an alternative.

The cash and futures prices and options data for these three commodities are from 1990 to 1999. There are 522 observations in the sample. The hog cash price (\$/lb) is the price of Indiana-Ohio plant delivered US 1-2 51%-52.9% live hogs. The corn cash price (\$/bu) is the price of Toledo No.2 yellow corn. The soybean meal price (\$/ton) is the price of Illinois 48 percent soybean meal. We take the average of the reported low and high price of each commodity each Wednesday as the Wednesday price used in our model for all of the three commodities. The risk-free interest rate we use in our analysis is the 6-month t-bill rate from the Federal Reserve. We also utilize the Wednesday settlement futures and options prices for hog contracts from Chicago Mercantile Exchange<sup>ii</sup> and those for corn and soybean meal contracts from Chicago Board of Trade.

## Evaluation of Volatility Forecasting Methods

Manfredo, Leuthold and Irwin (1999) examined the relative forecasting power of several forecasting methods including implied volatility, historical volatility and GARCH volatility for fed cattle, feeder cattle and corn. Using similar evaluation methods, we test the relative forecasting power of historical, implied and GARCH-based volatility for the price returns of hog, corn and soybean meal. It is common practice in the volatility forecasting literature to constrain the mean return of a series to be zero when developing volatility forecasts. We will do this for all the three forecasting methods and the realized volatility.

Historical volatility can be derived from the observed historical return series. It is defined as:

$$(4) \quad \hat{\sigma}_{t,1,i} = \sqrt{\frac{1}{T} \sum_{m=0}^{T-1} R_{t-m,i}^2},$$

where  $\hat{\sigma}_{t,1,i}$  is the next period (weekly) volatility forecast for commodity  $i$ ;  $T$  is the number of past observations of the return series from the beginning of our dataset, the first week of 1990, up to the start of the forecasting period; and  $R_{t,i}^2$  is the square of realized return defined as in (3) at time  $t$  for commodity  $i$ . To forecast the volatility for a horizon longer than one period, we multiply the  $t+1$  forecast by the square root of the horizon,  $h$ , i.e.

$$(5) \quad \hat{\sigma}_{t,h,i} = \hat{\sigma}_{t,1,i} \sqrt{h},$$

Implied volatility is the annualized volatility derived from the futures price and options premium<sup>iii</sup>. Because it is the forecasted volatility based on all the currently available market information, it is widely believed to be superior to other alternatives. Implied volatility for commodity return series was first suggested by Black (1976). In his paper, the option pricing model for a commodity is:

$$(6) \quad C = e^{-rT} [F_0 N(d1) - XN(d2)],$$

$$(7) \quad P = e^{-rT} [XN(-d2) - F_0 N(-d1)],$$

$$(8) \quad d1 = \frac{\ln(F_0 / X) + IV^2 T / 2}{IV \sqrt{T}},$$

$$(9) \quad d2 = d1 - IV \sqrt{T},$$

where  $C$  and  $P$  are the option premium for a call and a put option respectively;  $r$  is the risk-free interest rate;  $T$  is the time to expiration of the option;  $F_0$  is the futures price;  $X$  is the strike price of the option;  $N(\cdot)$  is the cumulative probability distribution function for a standard normal distribution;  $IV$  is the implied volatility in annualized form. Numerical methods are needed to solve equations (6)-(9) to get the implied volatility. Since we use weekly price of the three commodities, the volatility forecast for an  $h$ -week horizon is defined as:

$$(10) \quad \hat{\sigma}_{t,h,i} = IV_{t,i} \frac{\sqrt{h}}{\sqrt{52}},$$

where  $IV_{t,i}$  is the implied volatility at time  $t$  for commodity  $i$ . All the options are written on the futures contracts. So the implied volatility is an exact measure of the volatility of futures price.

However, as in Manfredo et al.'s paper, we use it as a proxy for the volatility of the cash price series<sup>iv</sup>.

GARCH-based volatility is another popular way to forecast the volatility of returns. Bollerslev (1986) first suggested GARCH (Generalized Autoregressive Conditional Heteroskedasticity model). Since then, there have been a large number of studies on financial data based on the GARCH model. The GARCH (1,1) model with a normal distribution has been the most frequently used model in financial data analysis; such a model is often favored over GARCH models with different orders, different distributional assumptions and other extensions. However, since we observe seasonality for commodity return volatilities, we need to add in terms that can capture seasonality. Monthly dummy variables and a Fourier expansion proposed by Roberts (2000) are two available choices. The problem of using monthly dummy variables is that it causes a big jump of the volatility forecasting from the end of one month to the beginning of the following month. A Fourier expansion can cure this problem by smoothing the seasonality of volatility. So our GARCH (1,1) model with Fourier expansion is defined as:

$$(11) \quad \sigma_{t,i}^2 = \alpha_0 + \alpha_1 R_{t-1,i}^2 + \beta_1 \sigma_{t-1,i}^2 + \sum_{m=1}^M [\phi_m \sin(2\pi m \tau) + \varphi_m \cos(2\pi m \tau)],$$

where  $\sigma_{t,i}^2$  is the conditional variance at time  $t$  for commodity  $i$ ;  $R_{t-1,i}$  is the return of commodity  $i$  defined as (3);  $\sigma_{t-1,i}^2$  is the variance at time  $t-1$  for commodity  $i$ ;  $\tau$  ( $0 \leq \tau \leq 1$ ) is the time of year of the observation. For example,  $\tau$  is  $2/52$  for the observation of the 2<sup>nd</sup> week of a year.

$\sum_{m=1}^M [\phi_m \sin(2\pi m \tau) + \varphi_m \cos(2\pi m \tau)]$  is the Fourier expansion term.  $\alpha_0$ ,  $\alpha_1$ ,  $\beta_1$ ,  $\phi_m$ ,  $\varphi_m$  are parameters estimated by maximum likelihood method. By fitting equation (11) to our dataset,  $M$  is determined by likelihood ratio test. Our results show  $M=1$  best captures the seasonality of volatility for all the three commodities. So our model is simplified as:

$$(12) \quad \sigma_{t,i}^2 = \alpha_0 + \alpha_1 R_{t-1,i}^2 + \beta_1 \sigma_{t-1,i}^2 + \phi_1 \sin(2\pi \tau) + \varphi_1 \cos(2\pi \tau),$$

This is actually a GARCH (1,1) model with two independent variables in the variance equation. As we did in estimating the historical volatility, the data used are from the beginning of our dataset, the first week of 1990, up to the start of the forecasting period. Following the method of Kroner, Kneafsey and Claessens (1994), we can get our volatility forecast for  $h$  horizon by the square root of the sum of the  $h$  conditional variances.

To evaluate the three volatility-forecasting methods, we define the realized (ex post) volatility as:

$$(13) \quad \sigma_{t,h,i} = \sqrt{\sum_{j=1}^h R_{t+j,i}^2},$$

where  $\sigma_{t,h,i}$  is the realized volatility from time  $t$  to  $t+h$  for commodity  $i$ ;  $R_{t,i}$  is the return of commodity  $i$  defined in (3).

The beginning of our forecasting period is the first Wednesday of 1992. We put the 105 observations of 1990 and 1991 as the base to generate initial forecasts. We choose six different horizons, which are  $h=1, 2, 4, 8, 12,$  and  $26$  weeks. To avoid the problem of forecasting error autocorrelation caused by overlap of horizons, we choose  $1, 2, 4, 8, 12, 26$  weeks before the expiration date of selected options as the starting date of a forecasting period. The specific

selection rules are shown in Table 1. In Manfredo's study, they selected two non-overlapping periods. One advantage of our selection rule is we make the sample size as large as possible.

**Table 1. The Selected Options and the Corresponding Sample Sizes for Each Forecasting Horizon for Hog, Corn and Soybean Meal<sup>V</sup>.**

Forecasting Horizon	Hog		Corn		Soybean Meal	
	Option Selected	Sample size	Option Selected	Sample Size	Option Selected	Sample Size
h=1	Feb, Apr, Jun, Jul, Aug, Oct, Dec	56	Mar, May, Jul, Sep, Dec	40	Jan, Mar, May, Jul, Aug, Sep, Oct, Dec	64
h=2	Feb, Apr, Jun, Jul, Aug, Oct, Dec	56	Mar, May, Jul, Sep, Dec	40	Jan, Mar, May, Jul, Aug, Sep, Oct, Dec	64
h=4	Feb, Apr, Jun, Jul, Aug, Oct, Dec	56	Mar, May, Jul, Sep, Dec	40	Jan, Mar, May, Jul, Aug, Sep, Oct, Dec	64
h=8	Feb, Apr, Jun, Aug, Oct, Dec	47	Mar, May, Jul, Sep, Dec	39	Mar, May, Jul, Sep, Dec	40
h=12	Apr, Aug, Dec	24	Mar, Jul, Dec	23	Mar, Jul, Dec	23
h=26	Jun, Dec	15	Mar, Sep	15	Mar, Sep	15

All the volatility forecasts for each horizon are ranked according to the size of the mean squared prediction error (MSPE); the Diebold and Mariano (1995) test to decide whether the MSPE of two methods are significantly different is used. This is a nonparametric test that creates a new variable  $d_j$  when the sample size is  $m$ , with  $j=1, 2, \dots, m$ .  $d_j$  equals 1 when the squared prediction of one method exceeds that of another, and zero otherwise. The test statistic is constructed as:

$$(14) \quad S = (m/4)^{-1/2} \left[ \sum_{h=1}^m d_j - (m/2) \right] \sim N(0,1),$$

When  $S$  exceeds the critical value, one forecasting method is superior to another. Table 2 through 4 shows the empirical results of comparison of historical volatility, implied volatility and GARCH-based volatility for hog, corn and soybean meal. Forecasting methods are ranked according to the size of MSPE and the Diebold and Mariano nonparametric test has been carried out for each pair of forecasting methods.

**Table 2. MSPE of Hog Volatility<sup>1</sup>**

	Rank	1	2	3
h=1	Forecast	IV	Hist	GARCH
	MSPE	0.87	0.89	0.95
h=2	Forecast	IV	GARCH	Hist
	MSPE	0.93	1.01	1.03
h=4	Forecast	IV	Hist	GARCH
	MSPE	1.71*	1.92	2.13
h=8	Forecast	IV	GARCH	Hist
	MSPE	5.45	6.84	7.09
h=12	Forecast	GARCH	IV	Hist
	MSPE	4.36	4.48	5.19
h=26	Forecast	GARCH	Hist	IV
	MSPE	20.45	20.47	24.19

<sup>1</sup> All MSPE are multiplied by 1,000.

# Rank 1 method is significantly superior to Rank 2 method.

\* Rank 1 method is significantly superior to Rank 3 method.

\*\* Rank 2 method is significantly superior to Rank 3 method.

**Table 3. MSPE of Corn Volatility<sup>1</sup>**

	Rank	1	2	3
H=1	Forecast	Hist	IV	GARCH
	MSPE	0.36*	0.44	0.69
H=2	Forecast	Hist	IV	GARCH
	MSPE	0.59	0.60	0.66
H=4	Forecast	IV	GARCH	Hist
	MSPE	0.60	0.64	0.73
H=8	Forecast	IV	GARCH	Hist
	MSPE	0.86*	1.13**	1.49
h=12	Forecast	IV	Hist	GARCH
	MSPE	1.55*	1.79	2.06
h=26	Forecast	IV	Hist	GARCH
	MSPE	1.46*#	2.46	3.08

<sup>1</sup> All MSPE are multiplied by 1,000.

# Rank 1 method is significantly superior to Rank 2 method.

\* Rank 1 method is significantly superior to Rank 3 method.

\*\* Rank 2 method is significantly superior to Rank 3 method.

**Table 4. MSPE of Soybean Meal Volatility<sup>1</sup>**

	Rank	1	2	3
h=1	Forecast	Hist	IV	GARCH
	MSPE	0.37*	0.47	0.53
h=2	Forecast	Hist	IV	GARCH
	MSPE	0.40*	0.51	0.61
h=4	Forecast	IV	Hist	GARCH
	MSPE	0.76*	0.97	1.05
h=8	Forecast	IV	GARCH	Hist
	MSPE	1.98	2.41	2.42
h=12	Forecast	IV	Hist	GARCH
	MSPE	1.60*	1.73**	2.86
h=26	Forecast	IV	Hist	GARCH
	MSPE	3.13*	3.43**	4.71

<sup>1</sup> All MSPE are multiplied by 1,000.

# Rank 1 method is significantly superior to Rank 2 method.

\* Rank 1 method is significantly superior to Rank 3 method.

\*\* Rank 2 method is significantly superior to Rank 3 method.

From these tables, we confirm what Manfredo et al. found: the relative forecasting power of historical volatility, implied volatility and GARCH-based volatility vary across horizons and commodities. Implied volatility seems to be the best in most of the cases, though it is seldom significantly better than the second-ranked predictor. For corn and soybean meal, the near-term forecasting comparison (h=1 and 2) seems best. It outperforms GARCH-based approach significantly, but cannot be significantly distinguished from implied volatility. The mid-term (h=4 and 8) and long-term (h=12 and 26) comparison for corn and soybean meal shows the implied volatility is the best and that GARCH-based volatility still performs poorly. The results for hogs are different. For near-term and mid-term forecasting, implied volatility ranks first. For long-term forecasting, GARCH appears to perform better than implied volatility, though not significantly. The reason might be that far-from-maturity hog futures and options markets are very thin; so implied volatility does not fully incorporate all the market information. Also note that hog return volatility was the most difficult to forecast of the three commodities; i.e., it had the largest MSPE. In summary, implied volatility, while not dominant, seems to be the best of the three methods considered. Based on this conclusion, we use implied volatility to forecast variance of the three commodities in our net revenue forecasting model for hog producers.

### Net Revenue Forecasting

To forecast the net revenue for hog producers, we need to know the joint distribution of the three price series. A widely accepted model describing one commodity price movement is a random walk with drift model in the natural logarithm level:

$$(15) \quad \ln(P_{t,i}) = \ln(P_{t-1,i}) + \mu_i + \varepsilon_{t,i},$$

where  $\ln$  denotes the natural logarithm;  $P_{t,i}$  is the price of the commodity  $i$  at time  $t$ ;  $\mu_i$  is the drift; and  $\varepsilon_{t,i}$  is an innovation term at time  $t$  for commodity  $i$  that follows a normal distribution.

This is slightly different from the definition of return series in (3) because of the drift. The drift term denotes the intrinsic force driving the price movement. The innovation term is the random shock outside the intrinsic factor. Thus, the joint distribution for the natural logarithm level prices can be modeled as:

$$(16) \quad \ln(P_t) = \ln(P_{t-1}) + \mu + \varepsilon_t,$$

where  $P_t$  is a price vector at time  $t$ ;  $\mu$  is a vector for drifts;  $\varepsilon_t$  is an innovation vector at time  $t$ , i.e.,

$$(17) \quad P_t = \begin{bmatrix} P_{t,1} \\ P_{t,2} \\ P_{t,3} \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}, \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{t,1} \\ \varepsilon_{t,2} \\ \varepsilon_{t,3} \end{bmatrix} \sim N(0, \Sigma) \text{ and } \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{bmatrix},$$

where the subscript 1, 2, and 3 denote hog, corn, soybean meal, respectively, and  $\Sigma$  is the symmetric covariance matrix. The variance terms in the matrix are weekly variance, which are derived by dividing the square of corresponding implied volatilities by 52. The covariance terms are derived from historical covariance:

$$(18) \quad \hat{\sigma}_{t,i,j} = \frac{1}{T} \sum_{m=0}^{T-1} R_{t-m,i} R_{t-m,j},$$

where  $\hat{\sigma}_{t,i,j}$  is the forecasted weekly covariance for commodity  $i$  and  $j$  starting from time  $t$ ;  $T$  is the number of past observation of the return series from the beginning of our dataset, the first week of 1990, up to the start of the forecasting period; and  $R_{t,i}$  is the realized return defined as in (3) at time  $t$  for commodity  $i$ ,  $j \neq i$ . Note that historical covariance is calculated based on the constraint that the mean of return series is zero, which is not realistic. But since our data show the mean of return is not significantly different from zero for all the three commodities, equation (18) can be viewed as a good proxy to the real covariance.

When we have all the variance and covariances term available, a Monte Carlo simulation can be done by repeatedly generating random numbers following the multivariate normal distribution.

To convert the natural logarithm level price into the original price level for each price series and to incorporate unbiased point estimation by futures prices, we go backward from time  $t$  to zero according to (15). Adding all the equations up yields:

$$(19) \quad \ln(P_{T1,i}) = \ln(P_{0,i}) + T1 \mu_i + \sum_{n=0}^{T1} \varepsilon_{t-n,i},$$

where  $P_{0,i}$  is the spot price of commodity  $i$  at time zero, the beginning of the forecasting period;  $P_{T1,i}$  is the cash price of commodity  $i$  at the expiration date of the nearby futures; and  $T1$  is the time periods from the beginning of forecasting period to the expiration date of the nearby futures. From (19), we get

$$(20) \quad P_{T1,i} = \exp[\ln(P_{0,i}) + T1 \mu_i + \sum_{n=0}^{T1} \varepsilon_{t-n,i}],$$

Recall in equation (2) we assume the nearby futures price observed at time zero is an unbiased forecast for the spot price at the futures expiration date, thus we get

$$(21) \quad F_{0,T1,i} = E(P_{T1,i}) = E\{\exp[\ln(P_{0,i}) + T1\mu1_i + \sum_{n=0}^{T1} \varepsilon_{T1-n,i}]\},$$

where  $F_{0,T1,i}$  is the observed nearby futures price at time zero for commodity  $i$ ; the nearby futures expires on time  $T1$ ; and  $E(\cdot)$  is expectation. Because the innovation term is assumed to be distributed as  $\varepsilon_{t,i} \sim N(0, \sigma^2)$ , we generate the psuedo-random numbers via a Monte Carlo simulation. Thus we can solve for  $\mu1_i$ . When we know what  $\mu1_i$  is, we can use it and the same random numbers to generate the expected spot price for each period from time zero up to  $T1$ . Following the same rule, the price movement between two futures expiration date can be derived. For example, the price movement from the nearby futures expiration date to the 1<sup>st</sup> deferred futures expiration date can be derived based on equation (21):

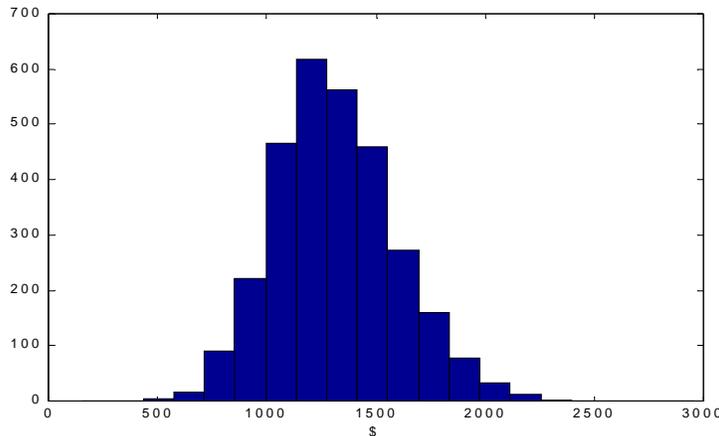
$$(22) \quad F_{0,T2,i} = E\{\exp[\ln(F_{0,T1,i}) + (T2-T1)\mu2_i + \sum_{n=0}^{T1} \varepsilon_{T2-n,i}]\},$$

where  $F_{0,T2,i}$  is the observed futures price at time zero for commodity  $i$ ; the futures contract expires at time  $T2$ ;  $\mu2_i$  is the drift term from  $T1$  to  $T2$ .

Therefore, when we want to forecast the net revenue for the next  $T$  period and have generated  $N$  groups of random numbers, we would have  $N$  groups of price series. Each group contains three price series, thus we can calculate the net revenue by equation (1) for each of the  $N$  groups. The forecasting mean and standard deviation can be derived across the  $N$  net revenue forecasts.

We choose the 1<sup>st</sup> week of January and July of each year from 1991 to 1999 as the starting point of a 26 week forecasting period and forecast the net revenue for a hypothetical Indiana-Ohio hog producer who sells one hog and buy enough feed for finishing one hog each week. The sample size is 18 and the number of Monte Carlo simulation trials is 3,000 per period. To keep the simulation process simple, we choose July and December futures and options to calculate implied volatility. One important limiting assumption in our practice is that there is no basis risk. As an example of illustration, the forecasted mean, standard deviation and value at risk starting from the first week of 1999 is shown in Figure 1 and Table 5.

**Figure 1. The Distribution of Forecasted Net Revenue for 26 week Period Starting from the First Week of 1999.**



**Table 5. The Forecasted Mean, Standard Deviation and Value at Risk for 26 Week Period Starting from the First Week of 1999.**

Realized Net Revenue (\$)	Forecasted Net Revenue (\$)	Standard Deviation (\$)	95% Prediction Interval	Value at Risk
1,426	1,318	280	(821, 1926)	5%, 901 20%, 1,083 35%, 1,194 50%, 1,294 65%, 1,408 80%, 1,541 95%, 1,805

We can see from the distribution that it is slightly skewed to the right. The realized net revenue falls in the 95 percent prediction interval and the interval one standard deviation away from the mean. Summarizing all the 18 cases, in 10 out of the 18 cases (55.6 percent of the cases), the realized net revenues fall in the interval one standard deviation from the mean. In 16 out of the 18 cases (88.9 percent of cases), the realized net revenues fall in the 95 percent prediction interval. In the two cases that are not within the 95 percent interval, the realized net revenue is lower than the lower bound of the 95 percent prediction intervals. These two cases occurred in the second half of 1997 and 1998; hog prices experienced a dramatic, unexpected drop in both cases. In half of the 18 cases our model overestimates the realized net revenue. On average, our model overestimates by 5.2 percent. If we remove the two outliers, our model overestimates by only 0.3 percent.

We also validate that the use of historical covariance provides better forecasting than does setting covariance equal to zero. For all of the 18 cases, imposing zero covariance inflates the standard deviation of net revenue forecast. The minimum degree of inflation for the standard deviation is 1.1 percent; the maximum is 13.3 percent; and the average is 2.9 percent. Thus, by using historical covariance, we provide a more accurate forecast.

### **Prospective vs. Retrospective Evaluation**

In the hog industry, the prevalent way of evaluating risk management strategies is to use retrospective evaluation in which the historical performance of several strategies are examined over a sufficiently long time period. For example, Lawrence and Vontalge (2000) used this method to evaluate several risk management tools and packer contracts and found none of them can consistently outperform the benchmark cash strategy in terms of net returns (though, for pure risk reduction purposes, several tools and contracts are preferred). This is consistent with the efficient market hypothesis.

Retrospective evaluation methods do not take advantage of all the available information contained in futures price and option premiums and potentially suffer the fate that undoubtedly lead to the ubiquitous disclaimers in investment commercials: Past performance does not guarantee future results. Prospective evaluation methods, such as the one proposed in our study, fully utilize all the market information and, thus, may provide more informed forecasts for hog producers choosing among risk management strategies.

To make a comparison between retrospective and prospective evaluation, we create a sequence of 18 non-overlapping 26-week price-window packer contracts from 1991 to 1999<sup>vi</sup>. The producer receives the IN-OH cash price less \$1/cwt so long as the IN-OH price is within the price window. The floor of the price window equals the average of the next four hog futures prices minus \$4/cwt and the ceiling is the average of the next four hog futures prices plus \$4/cwt. Following Garcia, Adam and Hauser (1994) we assume a simple mean-variance utility function for the risk-averse producer is:

$$(23) \quad E[U(E(\tilde{R}), \sigma)] = E(\tilde{R}) - \frac{1}{2} A \sigma^2,$$

where E denotes the expectation; U is the utility;  $\tilde{R}$  is the net revenue;  $\sigma$  is the standard deviation of the net revenue and A is the risk aversion coefficient. We assume the hog producer is moderately risk averse and assign A to be 0.01.

For retrospective evaluation, at the beginning of each forecasting period the producer observes the past performance of the window contract vs. the cash strategy and calculates the mean and variance of the net revenue for all the past 6-month periods under both strategies. If the utility calculated by (23) under the window contract is higher than under the cash strategy, then he/she chooses the contract, otherwise, he/she stays with the cash strategy. For prospective evaluation, we forecast the mean and variance of the net revenue under both window contract and cash strategy and the hog producer makes his/her choice based on the utility function (23). After choosing the strategy following the retrospective and prospective rules respectively, we examine whether employing each rule yields the higher realized net revenue than would simply buying and selling under a cash-only strategy every period. The comparison of retrospective and prospective rules by using cash strategy for all periods as the benchmark is shown in Table 6.

**Table 6. Summary of Returns of Alternative Risk Management Strategies, 1991-1999.**

Rule	Average Gain Over Benchmark	Volatility Reduction Over Benchmark	Times Choosing Right Strategy
Retrospective	3.4%	10.5%	7 out of 18
Prospective	3.9%	14.1%	8 out of 18

It can be seen from Table 6 that the prospective evaluation is slightly better than retrospective evaluation in terms of revenue enhancement and volatility reduction. Since the sample size is small, whether these advantages are systematic would require further investigation. Both the retrospective and prospective evaluations outperform the pure cash strategy by more than 3 percent each period on average. This is mostly because under both of them, the producer made the more profitable marketing decision before entering the periods with dramatic hog price drops. For example, both strategies guided producers to choose the window contract during the second half of 1998, in which the realized net revenue under the window contract is 58 percent higher than under the cash strategy. In summary, the prospective method performs marginally better than retrospective method. Furthermore, because prospective method incorporates all the market information and the retrospective method does not, the former is preferred on informational efficiency grounds.

## Conclusion

This research focuses on developing a net revenue forecasting model for hog producers based on the joint distribution of hog, corn and soybean meal price series. To calibrate the means of the expected joint distribution, we assume the efficient market hypothesis holds and, hence, use the futures prices to estimate future spot prices. To calibrate future volatility, we examine the relative forecasting power of three frequently used forecasting methods: historical volatility, implied volatility and GARCH-based volatility. Consistent with recent research, the performance of these three methods is both commodity and horizon specific. However, implied volatility performs well for most of the cases. Thus, we use implied volatility to forecast future price variance. Historical covariance is introduced to capture the co-movement of the three price series. We examine the out-of-sample forecasting performance of our forecasting model for a 26-week time horizon using data from 1991 to 1999. We find that realized net revenues fall within the 95 percent forecasting interval 16 of 18 time periods (88.9 percent) and that, on average, forecasted net revenues exceeded realized net revenue by 5.2 percent. Based on this forecasting model, we compare our prospective evaluation of cash strategy and a window contract with retrospective evaluations of the same two strategies. Use of the prospective evaluation model yields realized net revenues over the 1991-1999 period that are marginally better than retrospective method and is generally preferred because it incorporates all market information.

To extend our model, we need to relax certain assumptions and enlarge the sample size. First, basis risk must be built in the forecasting model if the model is to be useful to producers. Second, a distribution other than lognormal needs to be considered because, as is common in commodity studies, the underlying distribution exhibited excess kurtosis. The  $t$  distribution, which features fatter tails, may be an alternative to the normal distribution. Third, we can use GARCH-based forecast or a hybrid forecast of GARCH and implied volatility to improve the long-term volatility forecasting for hog returns. Fourth, we compare prospective and retrospective evaluation for only one risk management strategy. More strategies need to be evaluated to examine the performance of our model.

## Cited Literature

- Amin, K., and Ng, V., 1997. "Inferring Future Volatility from the Information in Implied Volatility in Eurodollar Options: A New Approach." *The Review of Financial Studies*, 10, 333-367.
- Baillie, R., and Myers, R., 1991. "Bivariate GARCH Estimation of the Optimal Commodity Futures Hedge." *Journal of Applied Econometrics*, 6, 109-124.
- Black, F., 1976. "The Pricing of Commodity Contracts." *Journal of Financial Economics*, 3, 167-179.
- Canina, L., and Figlewski, S., 1993. "The Informational Content of Implied Volatility." *The Review of Financial Studies*, 6, 659-681.
- Diebold, Francis X. and Jose A. Lopez. 1998. "Forecast Evaluation and Combination," in G.S. Maddala and C.R. Rao (eds.), *Handbook of Statistics*, Amsterdam: North-Holland, 241-268.

- Garcia, P.G., Adam, B.D., and Hauser, R.J., 1994. "The Use of Mean-Variance for Commodity Futures and Options Hedging Decisions." *Journal of Agricultural and Resource Economics*, 19, 32-45.
- Kroner, K., Kneafsey K. and Claessens, S., 1994. "Forecasting Volatility in Commodity Markets." *Journal of Forecasting*, 14, 77-95.
- Lamoureux, C., and Lastrapes, W., 1993. "Forecasting Stock-return Variance: Toward an Understanding of Stochastic Implied Volatility." *The Review of Financial Studies*, 6, 293-326.
- Lawrence, J. and Vontalge, A., 2000. "Managing Hog Price Risk: Futures, Options, and Packer Contracts," Iowa State University Report.
- Manfredo, M., Leuthold, R. and Irwin, S., 1999. "Forecasting Cash Price Volatility of Fed Cattle, Feeder Cattle, and Corn: Time Series, Implied Volatility, and Composite Approaches." *Working Paper*.
- Roberts, M., 2000. "Curing Commodity Kurtosis?" Unpublished paper, North Carolina State University.
- Schnitkey, G. and Miranda, M., 1998. "AgRisk: Computer Decision Support for Agricultural Risk Management." AAEE Conference, 1998
- Zanini, F. C. and P. Garcia. 1995. "Did Producer Opportunities in the Live Hog Contract Decline?" University of Illinois Working Paper number 97-05, Champaign, IL.

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<sup>i</sup> Please refer to Houthakker (1957), Telser (1958), Gray (1961), Dusak (1973) and Chang (1985) for further information.

<sup>ii</sup> Beginning in 1997 the traditional live hog futures was replaced by lean hog futures. We convert the lean hog price to a live hog price by using a price conversion factor of 0.74.

<sup>iii</sup> The futures price and option premium used to calculate IV may not have been determined at the same exact time, which can affect the quality of the IV estimate

<sup>iv</sup> Cash and futures price are highly correlated and usually move in the same direction and by a similar amount. The Black's option pricing model uses a European option. But the futures options for hog, corn and soybean meal are American type, which can be exercised before the expiration date. This causes a slightly upward-biased estimate for the true implied volatility for an American option. Shastri and Tandon (1986) stated the bias is trivial for a short-term at-the-money option. Also, because the at-the-money option is the most actively traded option among options with different strike prices, we use at-the-money put and call options to calculate implied volatility and take the average of the two as our forecasted implied volatility. When the futures price is not the same as an option strike price, the just-out-of-the-money option is used as an alternative.

<sup>v</sup> We have 90-91 data as initial value and eight years (92-99) data to do our comparison. So normally, the sample size equals the number of options selected each year times 8. However, for some of the  $h > 4$  horizons, we need either 1991 or 2000 data to calculate the historical or realized volatility which is unusable or unavailable in our study. Hence, we have one less observation for some cases.

<sup>vi</sup> A price window contract establishes a price band for the duration of the contract. If the agreed-upon market price falls within that price band, the producer received the market price; if the market price is outside the price band, an alternative pricing method is used. For the purposes of this study, we assumed the floor (ceiling) price is paid to the producer if market price is below (above) the floor (ceiling) price with no ledger account.