Weighted Expected Utility Hedge Ratios

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Abstract
We derive a new hedge ratio based on weighted expected utility. Weighted expected utility is a generalization of expected utility that permits non-linear probability weights. Generally speaking weighted expected utility hedge ratios are less than minimum variance hedge ratios and larger than expected utility hedge ratios.

Keywords: Hedging, hedge ratio, weighted expected utility, Allais Paradox.

Introduction

Minimum variance hedge ratios dominate the hedging literature for a variety of reasons, they are easy to compute and widely understood by both academics economic agents. Much of the current research in hedging is directed towards finding superior methods for estimating minimum variance hedge ratios, see Cecchetti, Cumby and Figlewski, Myers and Thompson, and Castelino to name a few. Lence (1995) makes a compelling argument that this is not the proper direction for academic researchers; he finds the value of "better" minimum variance hedge ratios to be negligible and the differences between optimal hedge ratios and minimum variance hedge ratios to be large.

One potential problem with minimum variance hedge ratios is their inability to capture agent's aversion to catastrophic losses and gains. Minimum variance hedge ratios can be shown to be consistent with expected utility maximization under certain circumstances. However there are numerous problems with expected utility germane to risk management, most notably the Allais Paradox (1953). It is not uncommon for economic agents to weight "catastrophic" gains and losses differently than "normal" gains and losses. This type of behavior is not consistent with expected utility theory. Machina (1987) proposes using weighted expected utility to explain agent's behavior in the face of "extreme" gains and losses. Generally speaking, weighted expected utility allows agents to weight the probabilities of expected outcomes in addition to the outcomes themselves. Expected utility functions are linear in probabilities.

We propose a new type of hedge ratio based on weighted expected utility theory that does not depend on any idiosyncratic parameters to be computed. This new hedge ratio is superior because it is more general than hedge ratios based on maximizing expected utility or minimizing the variance of returns. The weighted expected utility hedge ratio accounts for deviations from expected utility that occur for small-probability events, as suggested by the Allais Paradox.

In this paper we will construct a hedging problem from a commodity producer's perspective. The intent is to keep the hedging situation as simple as possible. The basic problem faced by the hedger is modeled, followed by derivations of minimum variance hedge ratios and expected utility hedge ratios. An explanation of the Allais Paradox is included to motivate a brief explanation of weighted expected utility theory. In the next section we derive the weighted expected utility hedge ratio. In the final section we empirically compare the weighted expected utility result to the more traditional approaches. Instead of focusing on particular commodities the empirical comparison is based on generic market circumstances that could apply to any commodity.

Underlying Model

The producer of an agricultural commodity seeks to maximize profits. However at planting there is uncertainty about future prices so hedging decisions are made conditional on the information at planting time. The producer must decide how much to hedge, with futures contracts, given the amount they plant. The underlying model is governed by:

$$\pi = px - fy \tag{1}$$

$$E(\pi \mid I_0) = xu_p + yu_f \tag{2}$$

$$Var(p \mid I_0) = E(p - u_p)^2 = \sigma_p^2 \tag{3}$$

$$Var(f \mid I_0) = E(f - u_f)^2 = \sigma_f^2 \tag{4}$$

$$Cov(p, f \mid I_0) = E[(p - u_p)(f - u_f)] = \sigma_{pf} \tag{5}$$

$$Var(\pi \mid I_0) = x^2\sigma_p^2 + y^2\sigma_f^2 - 2xy\sigma_{pf} \tag{6}$$

where p is the return from the spot market, x is the amount at risk, f is the return from the futures market and y is the amount hedged. The agent plants at time 0 and observes information set I_0. The only relevant price the producer observes at time 0 is f_0 the futures price today. The conditional expected return in the spot market is u_p and the expected return from the futures market is u_f. The overall expected return to the producer is (2) and the variance of this return is (6)

Derivation of Minimum Variance Hedge Ratio

Due to the uncertainty in spot and futures market returns the producer chooses the optimal amount to hedge, y. The minimum variance hedge ratio (MVHR) is determined by minimizing the variance of the producer's returns with respect to y, the amount hedged. The MVHR is calculated by differentiating (6) and solving for $\frac{y}{x}$,

$$\frac{\partial}{\partial y} Var(\pi \mid I_0) = 2y\sigma_f^2 - 2x\sigma_{pf} \tag{7}$$

$$2y\sigma_f^2 - 2x\sigma_{pf} = 0 \rightarrow \frac{y}{x} = MVHR = \frac{\sigma_{pf}}{\sigma_f^2} \tag{8}$$

Hence the MVHR is the ratio of the covariance of the spot and futures returns and the variance of the futures returns.

The MVHR is widely used in practice and widely studied by academics. However, one must not lose sight of the fact that it is based on variance minimization, which is not necessarily a suitable objective. A hedge ratio based on profit maximization or utility maximization would be more consistent with economic theory and superior to the MVHR.

Derivation of Expected Utility Hedge Ratio

The expected utility hedge ratio (EUHR) is superior to the MVHR on theoretical grounds. The decision to minimize the variance of expected returns is essentially arbitrary. A more appropriate objective, in terms of economic theory, is the maximization of utility. In order to calculate the EUHR the agent hedges in order to maximize expected utility. A popular functional form is the negative exponential utility function given by (9),

$$\text{(9)} \qquad U(\pi) = k_0 - k_1 e^{-\rho\pi}$$

where k_o, k_1, and r are constants. It can be shown that the second order Taylor series expansion of (9) is equivalent to a mean variance utility function, $V(\pi)$ --

$$\text{(10)} \qquad V(\pi) = E(\pi) - \frac{\rho}{2} Var(\pi)$$

where

$$\text{(11)} \qquad E(\pi) = xu_p - yu_f$$

$$\text{(12)} \qquad Var(\pi) = x^2\sigma_p^2 + y^2\sigma_f^2 - 2xy\sigma_{pf}$$

By maximizing expected utility we can derive the EUHR:

$$\text{(13)} \qquad \frac{\partial V}{\partial y} = -u_f - \rho y\sigma_f^2 + \rho x\sigma_{pf} = 0$$

$$\text{(14)} \qquad EUHR = \frac{y}{x} = \frac{\sigma_{pf}}{\sigma_f^2} - \frac{u_f}{x\rho\sigma_f^2} = MVHR - \frac{u_f}{x\rho\sigma_f^2}$$

The EUHR is superior to the MVHR insofar as it is consistent with economic theory. However, the EUHR has some undesirable qualities. First of all it depends on x, the amount at risk. Second, and more importantly, it depends on, ρ, the coefficient of absolute risk aversion. We are left with a theoretically superior hedge ratio that is agent dependent. Furthermore we do not directly observe ρ, making any calculation of EUHR suspect. The EUHR is equivalent to the MVHR if agents are infinitely risk averse, $\rho \to \infty$, or futures markets are unbiased, $u_f \to 0$.

The advantages of the EUHR are also mitigated by weakness in expected utility theory itself. Although economists have gotten a lot of mileage out of expected utility theory it does have some glaring weaknesses. Violations of expected utility were first widely recognized as a result of work by Allais (1953). The Allais paradox is based on empirical observations that imply agents weigh *both* expected outcomes and the probabilities associated with expected outcomes. The latter is a clear violation of expected utility theory which requires expected utility functions be linear in probabilities.

Below is an example of the Allais Paradox (Mas-Colell et al., 1995, p. 179):

> Consider a 1x3 payoff vector; 2.5 million dollars, 0.5 million dollars, and zero dollars. Individuals face two sets of probabilities, with each element of a probability vector associated with a corresponding element in the payoff vector. Individuals are asked to state which probability vector they prefer, given the payoff vectors, within each probability set:
>
> Set 1: p1 = (0,1,0) and p2 = (.10,.89,.01)
>
> Set 2: p3 = (0,.11,.89) and p4 = (.10,0,.90)
>
> If someone prefers p1 to p2 then expected utility theory dictates they also prefer p3 to p4. However it is often found that people prefer p4 to p3.

The Allais paradox exists, in all likelihood, because agents place weights on the probabilities of expected outcomes. The weighting of probabilities is intuitively linked to individual's attitudes towards risk. It seems plausible that people weight *catastrophic* (low probability) losses or gains (as in the Allais paradox) differently than they weight *normal* losses or gains. What constitutes *catastrophic* and *normal* is difficult to characterize mathematically. Because this type of behavior is so prevalent researchers have attempted to develop new utility theories that account for the Allais paradox.

Weighted Expected Utility Theory

The obvious problem with expected utility theory is the linear probability assumption. The obvious solution to the linear probability assumption is to weight the probabilities. Summarizing research done by Chew and Waller (1986) and Hess and Holthausen (1990) a weighted expected utility function has the following form:

$$(15) \qquad V(x, pr) = \frac{\sum_i U(x_i) pr_i}{\sum_j W(x_i) pr_j}$$

Where the numerator is a valuation function, analogous to an expected utility function, and the denominator is a weighting function. If W(.) is constant then weighted expected utility is equivalent to expected utility. Hence expected utility is a special case of weighted expected utility. It can be shown (Machina, 1987) that a weighted expected utility function with a properly specified weighting function consistently explains behavior observed in Allais Paradox type experiments.

When deriving the EUHR we used negative exponential utility, equation (12). In order to construct a weighted expected utility function we can use two negative exponential utility functions, one a valuation function and the other a weighting function. Just as in the expected utility case we can approximate each of the negative exponential utility functions with mean-variance utility functions.

Weighted Expected Utility Hedge Ratios

The weighted expected utility hedge ratio (WEUHR) is derived in a manner similar to the derivation of EUHR. For both the valuation and weighting functions we use mean-variance approximations of negative exponential utility. The agents choose a hedge ratio that maximizes their weighted mean-variance expected utility:

$$\text{(16)} \qquad V(\pi) = \frac{aE(\pi) - b\,\mathrm{var}(\pi)}{cE(\pi) - d\,\mathrm{var}(\pi)} = \frac{a(xu_p + yu_f) - b(x^2\sigma_p^2 + y^2\sigma_f^2 - 2xy\sigma_{pf})}{c(xu_p + yu_f) - d(x^2\sigma_p^2 + y^2\sigma_f^2 - 2xy\sigma_{pf})}$$

The numerator is the valuation function and the denominator is the weighting function. In this case both the valuation and weighting functions are mean-variance utilities that are approximations of negative exponential utility functions. The agents weight expected returns by a and c, and the variance of returns by b and d. By taking the first derivative of (16) and setting it equal to zero we can solve for the hedge ratio $\frac{y}{x}$. For this derivation some of the intermediate steps will be suppressed, we begin with:

$$\text{(17)} \qquad \frac{\partial V}{\partial y} = 0 \rightarrow (ad - bc)(x^2\sigma_p^2 + y^2\sigma_f^2) - 2xy\sigma_{pf}u_f + 2(ad - bc)\sigma_f^2(xu_p - yu_f)y$$
$$- 2x(ad - bc)(xu_p - yu_f)\sigma_{pf} = 0$$

Notice we can cancel all of the (ad-bc) terms, if $(ad - bc) \neq 0$, thus eliminating all of the idiosyncratic parameters. Equation (17) can then be simplified further yielding,

$$\text{(18)} \qquad y^2(-\sigma_f^2 u_f) + y(2\sigma_f^2 u_p x) + (\sigma_p^2 u_f x^2 - 2\sigma_{pf} u_p x^2) = 0$$

Equation (18) is a quadratic formula in y, the amount hedged. We can solve (18) and recover the WEUHR as follows,

$$\text{(19)} \qquad y = x\frac{u_p}{u_f} \pm x\sqrt{\frac{s_p^2}{s_f^2} + \left(\frac{u_p}{u_f}\right)\left(\frac{u_p}{u_f} - 2\frac{s_{pf}}{s_f^2}\right)} = x\frac{u_p}{u_f} \pm x\sqrt{\frac{s_p^2}{s_f^2} + \left(\frac{u_p}{u_f}\right)\left(\frac{u_p}{u_f} - 2MVHR\right)}$$

$$\text{(20)} \qquad \frac{y}{x} = WEUHR = \frac{u_p}{u_f} \pm \sqrt{\frac{s_p^2}{s_f^2} + \left(\frac{u_p}{u_f}\right)\left(\frac{u_p}{u_f} - 2MVHR\right)}.$$

Where (20) is the WEUHR. Although the WEUHR looks complicated it is a function of moments that are easily estimated. The WEUHR is a function of the means and variances of spot and futures returns and the MVHR. Unlike the EUHR, the WEUHR is the same for everyone, there are no agent specific variables confounding the calculation. More importantly the WEUHR is based on weighted expected utility, which is a superior generalization of expected utility. Not only is the WEUHR theoretically superior to the EUHR and the MVHR, it is as easy to calculate as the MVHR.

Below is a summary of the advantages and disadvantages of the three hedge ratios discussed in this paper.

Table 1: Comparison of Hedge Ratios

Hedge Ratio	Advantages	Disadvantages
MVHR	familiar, easy to calculate	not based on utility theory
EUHR	based on expected utility	agent dependent, suffers from Allais paradox type problems
WEUHR	based on a generalization of expected utility, easy to calculate	unfamiliar see below

There are a couple of potential disadvantages to the WEUHR. One potential problem is the possibility of imaginary numbers arising from the square root portion of the WEUHR. At first glance it is not obvious that the value under the square root is necessarily positive. However this turns out not to be a problem, in the Appendix is a proof that the value under the square root has to be positive. Another problem is the decision of which root to use, the positive or negative. It can be shown that the negative root maximizes utility for an agent long in the spot market, the case studied in this paper, and the positive root maximizes utility for an agent short in the spot market. Consider equation (16) when a=1, d=-1, and b=c=0:

$$V(\pi) = \frac{xu_p - yu_f}{x^2\sigma_p^2 + y^2\sigma_f^2 - 2xy\sigma_{pf}} \tag{21}$$

assume:

$$\frac{u_p}{u_f} \geq 0$$

$$y = x\frac{u_p}{u_f} \rightarrow V(\pi) = 0 \tag{22}$$

let

$$y' = x\frac{u_p}{u_f} + e, e > 0 \rightarrow V(\pi) < 0 \tag{23}$$

Equation (23) is the WEUHR with a positive root, which implies the hedge ratio using the negative root yields a higher utility than the hedge ratio using the positive root.

Comparative Statics

The three hedge ratios are interrelated. Both the EUHR and the WEUHR are functions of the MVHR. Furthermore the three hedge ratios are, at least partially, functions of the distributional moments of spot and futures markets returns. To determine how the moments effect the hedge ratios we differentiate each hedge ratio with respect to u_p, u_f, σ_p^2, σ_f^2, and σ_{pf}. In the table below we summarize the signs of the derivatives, we assume that the ratio of the expected returns and the covariance are positive. These assumptions allow us to sign the derivatives in a realistic setting. In general it is not possible to sign some of the derivatives.

Table 2: Comparative Static Results

Moment	MVHR	EUHR	WEUHR
u_p	0	0	+
u_f	0	-	-
σ_p^2	0	0	-
σ_f^2	-	-	-
σ_{pf}	+	+	+
MVHR	1	+	+

There is no true divergence between the three ratios, whenever they share a common variable the sign of the derivative is the same. The WEUHR is a function of more variables, notably; it depends on the expected returns in the spot and futures markets and the variance of spot returns. These variables have no direct effect on the MVHR or the EUHR. It is interesting to note that WEUHR is increasing in u_p and decreasing in σ_p^2.

Empirical Comparison of WEUHR, EUHR, and MVHR

The WEUHR has obvious theoretical value but it remains to be seen how it compares to more traditional hedge ratios. In this section we will compare WEUHR to EUHR and MVHR. Instead of looking at specific markets we will present generic hedging circumstances based on market conditions in which agents are likely to find themselves. The agent observes historical returns, variances, and covariance. To simplify the approach we assume the ratio of the expected spot market returns to futures market returns, denoted by m, can take on three values, 1, 1.1, and .9. There will be three tables each corresponding to a different m. The ratio of the spot return variance to the futures market return variance, denoted v in the tables, can take on values from .85 to 1.5 in .05 increments. The covariance and MVHR can range from .85 to 1.05 in .05 increments. For ease of comparison we will normalize the variance of the futures market returns to 1. By doing this normalization the relevant MVHR equals the covariance of returns from the spot and futures markets. This greatly simplifies the number of cases we need to examine. We will compare the WEUHR to both the MVHR and the EUHR. For the EUHR we assume a coefficient of absolute risk aversion of 2. This empirical comparison is designed to give readers a basic understanding of how WEUHR compares, generally, to MVHR and EUHR. Because the WEUHR is new its basic properties must be explored to determine if it is a valuable hedging tool. The "na" entries in the tables correspond to cases that are not feasible, an indirect result of the normalization procedure. The covariance between two variables must be less than or equal to the square root of the product of the variable's variances. The cases labeled "na" consist of variance-covariance combinations that violate this constraint. Across the top of the tables are the MVHR and the corresponding EUHR, down the side of the table is the variance ratio v. Within the table is the WEUHR for each combination of MVHR, EUHR, and variance ratio.

Table 3: Comparison of WEUHR to MVHR & EUHR when m = 1.

MVHR	**0.70**	**0.75**	**0.80**	**0.85**	**0.90**	**0.95**	**1.00**	**1.05**	**1.10**	**1.15**
EUHR	**0.20**	**0.25**	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**	**0.55**	**0.60**	**0.65**
V										
0.70	0.45	0.55	0.68	1.00	na	na	na	na	na	na
0.75	0.41	0.50	0.61	0.78	na	na	na	na	na	na
0.80	0.37	0.45	0.55	0.68	1.00	na	na	na	na	na
0.85	0.33	0.41	0.50	0.61	0.78	na	na	na	na	na
0.90	0.29	0.37	0.45	0.55	0.68	na	na	na	na	na
0.95	0.26	0.33	0.41	0.50	0.61	0.78	na	na	na	na
1.00	0.23	0.29	0.37	0.45	0.55	0.68	1.00	na	na	na
1.05	0.19	0.26	0.33	0.41	0.50	0.61	0.78	na	na	na
1.10	0.16	0.23	0.29	0.37	0.45	0.55	0.68	1.00	na	na
1.15	0.13	0.19	0.26	0.33	0.41	0.50	0.61	0.78	na	na
1.20	0.11	0.16	0.23	0.29	0.37	0.45	0.55	0.68	1.00	na
1.25	0.08	0.13	0.19	0.26	0.33	0.41	0.50	0.61	0.78	na
1.30	0.05	0.11	0.16	0.23	0.29	0.37	0.45	0.55	0.68	1.00
1.35	0.03	0.08	0.13	0.19	0.26	0.33	0.41	0.50	0.61	0.78
1.40	0.00	0.05	0.11	0.16	0.23	0.29	0.37	0.45	0.55	0.68
1.45	-0.02	0.03	0.08	0.13	0.19	0.26	0.33	0.41	0.50	0.61
1.50	-0.05	0.00	0.05	0.11	0.16	0.23	0.29	0.37	0.45	0.55

Below the MVHR and EUHR values are the various WEUHR which depends on the mean ratio and the variance ratio. Suppose the MVHR is .95 which corresponds to a EUHR of .45, and that the variance ratio is 1, given these three values we get a WEUHR of .68. So, for each pair of MVHR and EUHR there are multiple WEUHRs that correspond to different variance ratios. A base scenario would be when the MVHR and the ratio of the variances both equal 1, in this case the WEUHR also equals 1. The WEUHR is always less than or equal to the MVHR. In fact it is always lower except in the base scenario. For all of the other cases the WEUHR is much lower than the corresponding MVHR. This suggests that when agents use the MVHR they hedge too much because the MVHR ignores the benefits of higher returns. The WEUHR is higher than the EUHR for low to moderate variance ratios and higher for larger variance ratios. This suggests that agents are under hedging when the variance ratio is less than or equal to 1.15 and over hedging when the variance ratio is greater than 1.15.

Table 4: Comparison of WEUHR to MVHR & EUHR when m = 1.1.

MVHR	**0.70**	**0.75**	**0.80**	**0.85**	**0.90**	**0.95**	**1.00**	**1.05**	**1.10**	**1.15**
EUHR	**0.20**	**0.25**	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**	**0.55**	**0.60**	**0.65**
v										
0.70	0.49	0.59	0.71	0.90	na	na	na	na	na	na
0.75	0.45	0.54	0.65	0.80	na	na	na	na	na	na
0.80	0.41	0.50	0.60	0.73	0.93	na	na	na	na	na
0.85	0.38	0.46	0.55	0.66	0.82	na	na	na	na	na
0.90	0.35	0.42	0.51	0.61	0.74	0.96	na	na	na	na
0.95	0.31	0.39	0.47	0.56	0.68	0.84	na	na	na	na
1.00	0.28	0.35	0.43	0.52	0.62	0.75	1.00	na	na	na
1.05	0.25	0.32	0.39	0.48	0.57	0.69	0.86	na	na	na
1.10	0.22	0.29	0.36	0.44	0.53	0.63	0.77	1.10	na	na
1.15	0.19	0.26	0.33	0.40	0.48	0.58	0.70	0.88	na	na
1.20	0.17	0.23	0.29	0.37	0.44	0.53	0.64	0.78	na	na
1.25	0.14	0.20	0.26	0.33	0.41	0.49	0.59	0.71	0.90	na
1.30	0.12	0.17	0.23	0.30	0.37	0.45	0.54	0.65	0.80	na
1.35	0.09	0.15	0.21	0.27	0.34	0.41	0.50	0.60	0.73	0.93
1.40	0.07	0.12	0.18	0.24	0.31	0.38	0.46	0.55	0.66	0.82
1.45	0.04	0.10	0.15	0.21	0.28	0.35	0.42	0.51	0.61	0.74
1.50	0.02	0.07	0.13	0.18	0.25	0.31	0.39	0.47	0.56	0.68

A slight increase in m, implying that expected returns are slightly higher in the spot market compared to the futures market has a small effect on the results Table 4). All of the WEUHR are higher than in the previous case suggesting that when returns are higher in the spot market agents should hedge more. Also, there are more cases that have a higher WEUHR than MVHR especially when the variance ratio is relatively low. This suggests that when spot market volatility is substantially lower than futures market volatility there are instances when agents should hedge even more than the MVHR suggests to enhance returns. Again, the WEUHR are higher than the EUHR for low to moderate variance ratios and lower for higher variance ratios. The magnitude of the difference between the WEUHR and the EUHR tend to be large.

Table 5: Comparison of WEUHR to MVHR & EUHR when m = .9.

MVHR	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15
EUHR	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
v										
0.70	0.40	0.50	0.64	na	na	na	na	na	na	na
0.75	0.35	0.44	0.55	0.73	na	na	na	na	na	na
0.80	0.31	0.39	0.49	0.62	na	na	na	na	na	na
0.85	0.27	0.34	0.43	0.54	0.70	na	na	na	na	na
0.90	0.23	0.30	0.38	0.48	0.60	0.90	na	na	na	na
0.95	0.19	0.26	0.33	0.42	0.53	0.68	na	na	na	na
1.00	0.16	0.22	0.29	0.37	0.46	0.58	0.80	na	na	na
1.05	0.13	0.19	0.25	0.33	0.41	0.51	0.66	na	na	na
1.10	0.09	0.15	0.21	0.28	0.36	0.45	0.57	0.76	na	na
1.15	0.06	0.12	0.18	0.24	0.32	0.40	0.50	0.64	na	na
1.20	0.03	0.09	0.15	0.21	0.28	0.35	0.44	0.55	0.73	na
1.25	0.01	0.06	0.11	0.17	0.24	0.31	0.39	0.49	0.62	na
1.30	-0.02	0.03	0.08	0.14	0.20	0.27	0.34	0.43	0.54	0.70
1.35	-0.05	0.00	0.05	0.11	0.17	0.23	0.30	0.38	0.48	0.60
1.40	-0.07	-0.03	0.02	0.08	0.13	0.19	0.26	0.33	0.42	0.53
1.45	-0.10	-0.05	-0.01	0.05	0.10	0.16	0.22	0.29	0.37	0.46
1.50	-0.12	-0.08	-0.03	0.02	0.07	0.13	0.19	0.25	0.33	0.41

When m is slightly below 1 (Table 5), returns in the spot market are less than the returns in the futures market, the WEUHR are lower than when m is greater than or equal to 1. The WEUHR are always less than the corresponding MVHR. This strongly suggests that agents that use the MVHR are over-hedging by a large amount. The relationship between the WEUHR and the EUHR for Table 3 is essentially the same as in Table 1 and Table 2.

The primary conclusion of this analysis is that WEUHRs differ substantially from MVHRs and EUHRs. In some cases the ratios are similar but these cases are the exceptions. The WEUHR tend to be lower than MVHR for all of the potential market conditions. When the variance ratio, v, is low to moderate the WEUHR tends to be larger than the EUHR and smaller otherwise.

Conclusions

Recent research in hedging tends to focus on finding superior methods of estimating MVHR. Lence (1995) argues that this may not be the appropriate direction. In this paper we have re-examined the objective of hedging. The objective of minimizing variance of returns is not consistent with economic theory except when certain tenuous assumptions are made. This has been recognized for some time but hedge ratios based on expected utility are not generally appealing for reasons grounded in theory and practice. We have demonstrated that weighted expected utility can be used to derive hedge ratios that overcome the problems with expected utility hedge ratios. Specifically, weighted expected utility and by extension WEUHR account for Allais Paradox type problems that are relevant to hedging. Furthermore, WEUHR are not agent dependent and hence just as easy to estimate as MVHR. At the very least WEUHR are worthy of further study to determine their applicability.

Our empirical comparison suggests that WEUHR are plausible although quite different from MVHR and EUHR. As expected WEUHR tend to be lower than MVHR because MVHR ignore the benefits of returns. The relationship between WEUHR and EUHR is more complicated, when the ratio of the spot return variance is relatively low the WEUHR are larger than the EUHR, when the ratio is relatively high the WEUHR is smaller than the EUHR.

Appendix

In this Appendix we prove that the discriminant of the WEUHR cannot be negative.

$$WEUHR = \frac{u_p}{u_f} \pm \sqrt{\frac{s_p^2}{s_f^2} + (\frac{u_p}{u_f})(\frac{u_p}{u_f} - 2\frac{s_{pf}}{s_f^2})}$$

The portion under the square root will be negative if:

$$\frac{s_p^2}{s_f^2} + (\frac{u_p}{u_f})(\frac{u_p}{u_f} - 2\frac{s_{pf}}{s_f^2}) < 0, i.e.$$

$$(24) \qquad \frac{s_p^2}{s_f^2} + (\frac{u_p}{u_f})^2 < 2\frac{s_{pf}}{s_f^2}\frac{u_p}{u_f}$$

If (24) holds than the roots will be imaginary. However u_p/u_f can be positive or negative. This leaves us with two cases to consider.

$$case(1): \frac{u_p}{u_f} > 0 \rightarrow \frac{1}{2}\frac{u_f}{u_p}[\frac{s_p^2}{s_f^2} + (\frac{u_p}{u_f})^2] < \frac{s_{pf}}{s_f^2}$$

$$case(2): \frac{u_p}{u_f} < 0 \rightarrow \frac{1}{2}\frac{u_f}{u_p}[\frac{s_p^2}{s_2^f} + (\frac{u_p}{u_f})^2] > \frac{s_{pf}}{s_f^2}$$

We now examine the two cases:

$$case(1) \rightarrow \frac{1}{2}[s_p^2\frac{u_f}{u_p} + s_f^2\frac{u_p}{u_f}] < s_{pf} \rightarrow \frac{1}{2}[s_p^2\frac{u_f}{u_p} + s_f^2\frac{u_p}{u_f}] < rs_p s_f$$

Where ρ is the correlation coefficient. It also follows that:

$$(25) \qquad \frac{1}{2}[\frac{\sigma_p}{\sigma_f}\frac{u_f}{u_p} + \frac{\sigma_f}{\sigma_p}\frac{u_p}{u_f}] < \rho$$

$$1 \geq \rho \geq -1$$

So if (25) is true than we will be left with the square root of a negative number suggesting a WEUHR with imaginary components. But (25) is the average of a positive number and it's inverse, which must be at least 1, hence (25) must be greater than or equal to 1. But the largest value ρ can take on is 1, hence (25) can never be true.

$$case(2) \rightarrow \frac{1}{2}[\sigma_p^2 \frac{u_f}{u_p} + \sigma_f^2 \frac{u_p}{u_f}] > \sigma_{pf} \rightarrow \frac{1}{2}[\sigma_p^2 \frac{u_f}{u_p} + \sigma_f^2 \frac{u_p}{u_f}] > \rho\sigma_p\sigma_f$$

$$(26) \qquad \frac{1}{2}[\frac{\sigma_p}{\sigma_f}\frac{u_f}{u_p} + \frac{\sigma_f}{\sigma_p}\frac{u_p}{u_f}] > \rho$$
$$1 \geq \rho \geq -1$$

So if (26) is true than we will be left with the square root of a negative number suggesting a WEUHR with imaginary components. But (26) is the average of a negative number and it's inverse, which must be less than -1, hence (26) must be less than or equal to -1. But the smallest value ρ can take on is -1, hence (26) can never be true.

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