

Hedging with Futures and Options: A Demand Systems Approach

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Practitioner's Abstract: *The optimal hedging portfolio is shown to include both futures and options under a variety of circumstances when the marginal cost of hedging is non-zero. Futures and options are treated as substitute goods, and properties of the resulting hedging demand system are explained. The overall optimal hedge ratio is shown to increase when the marginal cost of trading options is reduced. The overall optimal hedge ratio is shown to decrease when the marginal cost of trading futures is decreased. The implication is that hedging demand can be stimulated by reducing the perceived cost of trading options, by educating hedgers about options and by initiating programs like the Dairy Options Pilot Program. The demand systems approach is applied to estimate optimal hedge ratios for dairy producers hedging corn inputs in five regions of Pennsylvania.*

Keywords: Hedging, Options, Futures

Introduction

The correspondence between optimal hedging behavior and the demand for hedging has not been transparent in the agricultural marketing literature. Authors such as Castelino (1989, 1992); Paroush and Wolf (1989); Moser and Helms (1990); Park and Antonovitz (1990); Lapan, Moschini, and Hanson (1991); Castelino, Francis, and Wolf (1991); Briys, Crouhy, and Schlesinger (1993); Lapan and Moschini (1994); Netz (1996); and Lence (1995, 1996), and Frechette (2000) examined the role of basis risk within a price risk management strategy. Hirshleifer (1988) showed that transaction costs drive hedgers from the market; Simaan (1993) and Lence (1995, 1996) found that opportunity costs reduce optimal hedge ratios; Frechette (2000) showed how hedging costs affect the demand for futures, but none considered options. Wolf (1987) and Lapan, Moschini, and Hanson (1991) looked at the tradeoff between futures and options, but not their costs.

A synthesis of these approaches leads to a view of futures and options as substitute goods. Risk managers are demanders of risk management goods and are consumers of hedging products. Other market participants are suppliers of risk management goods and producers of hedging products. Each good has a price, which is determined by equilibrium between consumers and producers.

The demander of a risk management good chooses a hedge ratio or quantity for each good to maximize expected utility. Futures and options are substitute goods, from the demander's perspective, and the price of each will affect the demand of both types of goods. The situation is therefore analogous to the consumer choice problem with multiple goods and a budget constraint, except that a budget constraint is not necessarily binding for the hedger.

Producers may be competitive and may drive prices down to expected marginal cost, but expected marginal cost is unlikely to be zero. Broker's fees, transaction costs, opportunity costs, learning costs, and every other cost associated with futures and options hedging must be included. For example, initiating a futures position in corn may involve a \$25 per contract

broker's fee and \$1000 set aside in a margin account. If the hedger's discount rate is 12% annually, then the margin account costs \$120 per year per contract or \$10 per month per contract. Over a six-month horizon, the total cost from both these sources will be \$85 per contract. Additional costs of transacting and administering the hedge could easily increase the marginal hedging cost to more than \$100 per contract or 2 cents per bushel.

Within a demand systems framework it is easy to see that, holding everything else fixed, fewer futures contracts will be purchased by the hedger if futures cost 2 cents per bushel than would be purchased if they were free. What is not clear is the effect on options.

The tradeoff between futures and options was studied by Lapan, Moschini, and Hanson (1991), hereafter abbreviated LMH, but they did not consider costs. In this context, the results of LMH can be reinvestigated and reinterpreted. They start with a base case of unbiased pricing and then consider the effect of alternative expectations (on the part of the hedger). The same framework can be used here, starting with a base case of no marginal hedging costs and then considering the effect of positive marginal costs.

A reinterpretation of their model in terms of hedging costs leads to a reinterpretation of their primary result. Their primary result was that options add nothing to the optimal hedging strategy in unbiased markets. When futures and options are costless, their result is still correct, but when futures and options are not costless, the optimal (equilibrium) hedging strategy will almost always be a mixed strategy using both futures and options in some combination. This result is driven entirely by the costs faced by the hedger, including broker's fees, transaction costs, opportunity costs, learning costs, and every other cost associated with futures and options hedging. The result is not driven by nonlinearities or non-orthogonalities in the basis function or any other changes to the LMH model.

The importance of this result goes beyond academic circles. Agricultural hedging has grown in importance with the implementation of the FAIR Act in the United States, and agricultural futures and options have moved from relative obscurity to become part of the national consciousness. This research implies that hedgers ought to consider carefully the costs and benefits of hedging and to compare the costs of different risk management products before choosing a hedging strategy. In most cases a role exists for both futures and options, depending on the relative price of each and the marginal expected utility gained from hedging.

The demand for hedging products is similar to the demand for any other system of goods. Understanding the whole means understanding each of the parts and how they work together. Here the parts are hedging demand and supply, and the whole is equilibrium behavior. This approach to hedging demand is useful for understanding equilibrium hedging behavior when hedging demand and hedging supply conditions change because of government policy or actions by hedgers, brokerage houses, or exchanges.

The Model

The model used here is based on that of Lapan, Moschini, and Hanson (LMH). The LMH notation has been adopted to emphasize the differences and similarities of the two models. The purpose is to make it clear how the LMH results can be reinterpreted in a demand systems framework.

The variant of LMH to be used here is the one where hedging decisions are conditional on the output decision. This condition simplifies the presentation, and no new insights are gained by including output as a choice variable, beyond what LMH have already demonstrated. Another simplification will be that profits here will be considered per unit profits, which is different from but not inconsistent with LMH's model. The choice variables must be interpreted then as hedge ratios rather than quantities hedged.

LHM select an objective function using von Neumann and Morgenstern's (1944) expected utility paradigm. Kahl (1983); Berck (1981); Bond and Thompson (1985); Bond, Thompson, and Geldard (1985); Hirshleifer (1988); and Viaene and De Vries (1992) used Markowitz's (1952, 1959) mean-variance utility, but evidence indicates that neither Markowitz's (1952, 1959) mean-variance utility nor von Neumann and Morgenstern's (1944) expected utility can explain all the nuances of choice under uncertainty. Both remain in common use and differ little in prescriptive value for a wide range of models, as shown by Meyer (1987); Tew, Reid, and Witt (1991); Hanson and Ladd (1991); and Simaan (1993). Expected utility is used by Lapan and Moschini (1994), Lence (1995, 1996), and LMH. In particular, LMH explain why mean-variance utility is inappropriate for choices involving options.

The hedger is assumed to maximize expected utility of profits by choosing a pair of hedge ratios, x and z , where x is the portion of output hedged in the futures market and z is the portion of output hedged in the options market. LMH normalize by choosing x to be accomplished using short futures, so an output hedger will typically have a positive x , but they choose z to be accomplished using short puts (for the sake of mathematical elegance perhaps?) so an output hedger will typically have a negative z . As in LMH, only one option strike price, k , is considered. There are two periods in LMH's model, with the current futures price denoted f and the terminal futures price denoted p when realized, or p^- , when treated as a random variable. The options premium is denoted r , and the terminal option value is denoted v or v^- : $v = 0$ if $p \geq k$, and $v = k - p$ if $p < k$. The utility function $u(\cdot)$ is assumed to be continuous, monotonic increasing, and strictly concave: $u' > 0$ and $u'' < 0$. Utility is a function of profits, π , treated as a random variable:

$$(1) \quad \pi^- = b^- + (f-p^-)x + (r-v^-)z - c,$$

where b^- represents the local commodity price with spatial and temporal basis included, and c represents costs per unit. All money values have been adjusted by appropriate discount rates, suitably defined.

The hedger's optimization problem is

$$(2) \quad \text{Max}_{\{x,z\}} E[u(\pi^-)] ,$$

with E representing the expectations operator over all sources of uncertainty. The first order conditions are

$$(3a) \quad E[u'(\pi^-)(f-p^-)] = 0 , \text{ and}$$

$$(3b) \quad E[u'(\pi^-)(r-v^-)] = 0 .$$

The second order conditions are satisfied because of the conditions on $u(\cdot)$.

LMH take the model further by specifying the nature of basis risk:

$$(4) \quad b^- = \alpha + \beta p^- + \theta^- ,$$

where α and β are fixed parameters and θ^- is a random variable with mean zero and orthogonal to p^- (and therefore v^- , though they do not say so). Profits can then be rewritten as

$$(5) \quad \pi^- = \pi_0 + \theta^- + p^- (\beta-x) - v^- z , \text{ with}$$

$$(6) \quad \pi_0 = \alpha - c + fx + rz$$

as the deterministic part. Their insight is that setting $x = \beta$ and $z = 0$ satisfies the first order conditions (3a)-(3b) because the only remaining stochastic part of π^- is θ^- , which is orthogonal to $(f-p^-)$ and $(r-v^-)$, so that (3a)-(3b) are satisfied if $E(f-p^-) = 0$ and $E(r-v^-) = 0$. These last two conditions they note are simply the market unbiasedness conditions for efficient futures and options pricing. They conclude that unbiased markets with a linear basis rule like (4) yield no role for options in an optimal hedging portfolio.

Reinterpreting the model in a demand systems context means treating marginal hedging costs as prices faced by the hedger. The prices include such things as broker's fees, opportunity costs, and learning costs associated with futures and options hedging. They may also include perceived costs associated with uncertainty but not modeled by the expected utility framework, such as distributional ambiguity and nonlinear responses to distributional properties. In truth, there are many costs faced by an atomistic hedger that make the net hedged price less than $E p^-$ for futures and the net option premium greater than $E v^-$ for options, even if the markets themselves are perfectly unbiased. The major innovation here is the treatment of hedging demand separately from the market pricing mechanism (supply of hedging).

To proceed, consider a hedger who faces additional costs beyond f and r , the unbiased expectations of p^- and v^- . Call these extra costs t_x per unit hedged with futures and t_z per unit

hedged with options. Profits now become

$$(7) \quad \pi^- = b^- + (f-p^-)x + (r-v^-)z - t_x|x| - t_z|z| - c ,$$

where $|\cdot|$ is the absolute value operator. The absolute value operator is necessary because $z < 0$ in most cases for output hedgers, as discussed previously. In a parallel manner to equations (3a)-(3b), the first order conditions can be derived using the sign operator, defined by $\text{sgn}(x) = |x|/x$ for nonzero x , and $\text{sgn}(0) = 0$.

$$(8a) \quad E[u'(\pi^-)(f-p^- - t_x \text{sgn}(x))] = 0 , \text{ and}$$

$$(8b) \quad E[u'(\pi^-)(r-v^- - t_z \text{sgn}(z))] = 0 .$$

Profits can be expressed as in (5)-(6).

$$(9) \quad \pi^- = \pi_0 + \theta^- + p^- (\beta-x) - v^- z , \text{ with}$$

$$(10) \quad \pi_0 = \alpha - c + [f-t_x \text{sgn}(x)]x + [r-t_z \text{sgn}(z)]z .$$

Now, the solutions to (8a)-(8b) are different from the solutions to (3a)-(3b) because

$$(11a) \quad E[u'(\pi^-)(f-p^-)] = t_x E u'(\pi^-) > 0 , \text{ for positive } x, \text{ and}$$

$$(11b) \quad E[u'(\pi^-)(r-v^-)] = t_z \text{sgn}(z) E u'(\pi^-) < 0 , \text{ for negative } z.$$

Following the logic through, the solution requires $x < \beta$ and $z < 0$ whenever $t_x, t_z > 0$. The markets themselves may be unbiased, yet the hedger may choose to mix futures and options simply because hedging is not free.

The implication is that futures and options are substitute goods within most typical hedging portfolios. Totally differentiating the first order conditions, as in LMH's equations (11)-(12) under negative exponential utility, yields the signs of the demand slopes and cross-price slopes (partial derivatives) $-d|z|/dt_z > dx/dt_z = d|z|/dt_x > -dx/dt_x > 0$. The direct effects are negative, meaning the amount hedged with futures will decline as the marginal cost of hedging with futures increases, and the amount hedged with options will decline as the marginal cost of hedging with options increases. The cross effects are positive, meaning the amount hedged with one product will increase when the price of the other product rises. The cross-effects are also equal, proving the demand system symmetry condition. A different version of these results was derived by LMH but not interpreted in this way.

A useful result from the arguments above is that $d|z|/dt_z < dx/dt_x < 0$. Options demand is more sensitive to price changes than is futures demand. This result is consistent with the idea of options as a luxury good and futures as a necessary good.¹ The implication is that policy makers and market professionals wishing to encourage hedging ought to focus on options, not futures. Reducing the marginal cost of options (including opportunity costs, fees,

and learning costs) could stimulate hedging demand and result in a larger aggregate hedge ratio.

To explore this issue further, we can derive the effect of t_x and t_z on the aggregate hedge ratio, $h = x + |z|$. It can be shown that $dh/dt_x > 0$ and $dh/dt_z < 0$, which means that an increase in the marginal cost of hedging in the futures market (an upward shift in the hedging supply curve for futures) will lead to an increase in the overall hedge ratio. Similarly, an increase in the marginal cost of hedging in the options market (an upward shift in the hedging supply curve for options) will lead in a decrease in the overall hedge ratio. Conversely, a decrease in the marginal cost of hedging with options will stimulate hedging demand, while a decrease in the marginal cost of hedging with futures will reduce total hedging demand.

This result seems counter-intuitive at first, until h is considered to be a composite good made up of futures and options. An increase in the price of one or the other component goods will always lead to a reduction in the purchase of that good and an increase in the purchase of its substitute; however, the demand for the composite good is not constrained to decrease. Here h is behaving as a Giffen good when t_x increases and as a normal good when t_z increases.

The confusion is allayed when the aggregate price is homogenized: restrict both to move together one-for-one, so that $dt_x = dt_z = dt$. Thus, $dh/dt = dh/dt_x + dh/dt_z = dx/dt_x + d|z|/dt_x + dx/dt_z + d|z|/dt_z$, the sign of which can be shown to be negative. Hedging shifts from options to futures as t increases because $dx/dt > 0$ and $d|z|/dt < 0$, but the total effect is to reduce the options position by more than the futures position as marginal costs increase. In aggregate, then, $dh/dt < 0$, which means the aggregate hedging demand curve slopes downward.

Beyond this, it is possible to characterize the share of hedging demand comprised by futures and the share comprised by options. The share comprised by futures is x/h , and $d(x/h)/dt_x = (dx/dt_x)/h - (x/h^2)(dx/dt_x + d|z|/dt_x) = [(h-x)dx/dt_x - xd|z|/dt_x]/h^2 = (|z|dx/dt_x - xd|z|/dt_x)/h^2 < 0$. The share comprised by options is $|z|/h$, and $d(|z|/h)/dt_z = (xd|z|/dt_z - |z|dx/dt_z)/h^2 < 0$. Each demand share curve (or equivalently each demand curve conditional on h) slopes downward.

The problem is compounded by the inadequacy of a simple marginal rate of substitution to characterize demand properties. The usual marginal rate of substitution measures the variation in one good's quantity that keeps utility fixed as another good's quantity is changed exogenously. Here, though, expected utility has already been maximized, and no change in quantities can keep it fixed without a corresponding change in prices. Indifference curves can be constructed to illustrate iso-expected utility contours. They are ellipses with a downward sloping major axis in $x-|z|$ space. The center of the ellipse is the optimum point, which depends on t_x , t_z , and the other parameters.

Another way to characterize the demand for hedging is by plotting expansion paths in $x-|z|$ space as t_x and t_z change. When $t_x = t_z = 0$, the optimum point is $(\beta, 0)$, as discussed

above. No options are purchased unless t_x is positive. As t_x increases, the expansion path moves northwest in the coordinate space until it meets the $|z|$ -axis. Similarly, if $t_z = 0$ and t_x is positive, then the expansion path moves from the $|z|$ -axis southeast to the x -axis as t_z rises. The slope of the expansion path depends on t_x , t_z , and the other parameters.

For most realistic parameter values the optimum is on neither the x -axis nor the $|z|$ -axis, but somewhere between. In realistic situations, then, futures and options ought to be mixed. This result is true even without any nonlinearities or nonorthogonalities in the basis relationship. The futures contract cannot be said to be a “better” hedging product without reference to hedging costs. To say so would be akin to saying meat is “better” than milk. Both are goods, and most people will select positive quantities of each in an optimal consumption portfolio, depending on personal preferences and market prices. Similarly, futures and options are both goods, and most hedgers ought to select positive quantities of each in an optimal hedging portfolio, depending on personal preferences and market prices. This line of research provides a foundation for choosing how to allocate hedge ratios between the two.

It also yields a useful way to compute the value of a market’s existence. In consumer theory, the consumer’s surplus is the total benefit accruing to the consumer because the opportunity exists to purchase the good. Here, the hedger’s surplus may be defined analogously [Frechette (2000)]. If $u^* = \max_{\{x,z\}} Eu(\pi^-)$, then the hedger’s surplus in the options market is e_z , where $u^* = \max_{\{x|z=0\}} Eu(\pi^- + e_z)$. Similarly, the hedger’s surplus in the futures market is e_x , where $u^* = \max_{\{z|x=0\}} Eu(\pi^- + e_x)$. The aggregate hedger’s surplus for both markets is e_{xz} , where $u^* = Eu(\pi^- + e_{xz})$. The value of e_{xz} will necessarily be greater than or equal to $e_x + e_z$ because futures and options are substitutes. The values will depend on t_x , t_z , and the other parameters.

Empirical Application

This section applies the concepts from the previous section to the hedging decision for dairy producers in five local regions of Pennsylvania. The dairy producers hedge their purchases of corn using long futures and long call options, exactly the reverse of the LMH case. The analysis is essentially identical in every other way. The negative exponential (constant absolute risk aversion) utility function is assumed, which results in a convenient way to compare results for different levels of risk aversion. Basis is specified as local price minus Chicago price. Expected utility is then computed as a numerical double-integral over price risk and basis risk and maximized (again numerically) with respect to x and z . The integrals were calculated by the trapezoidal method, and optimization was achieved by the simplex method.

Data

The data set is the same one used in Frechette (2000) and consists of (i) weekly corn cash prices collected by the Pennsylvania Department of Agriculture (PDA); and (ii) the

nearby corn futures price in Chicago. Local cash prices were collected through surveys and phone calls for five regions: Southeastern, Central, South Central, Western, and the Lehigh Valley. The prices were collected and reported by PDA on Monday mornings before the market opened and the futures price that corresponds most closely is the previous Friday's settlement price for the nearby futures contract. If the Chicago Board of Trade was closed due to a Monday holiday, then the closest day was used, matching the information sets as closely as possible in each case. All prices are reported in cents per bushel, for the years 1997-1998.

Table I displays summary statistics and the covariance structure used in this analysis. The table shows that the covariances are negative and relatively large in magnitude between the Chicago price and each regional basis, indicating that the hedge ratios may be quite low in these regions. These statistics represent actual results for the sample period, and the results represent optimal *ex post* behavior in the sense that hedgers are assumed to have known the covariance matrix before the sample period began. Individual hedgers' expectations will depend on the sample period and available information.

For the purpose of exposition, rational expectations and unbiased markets are assumed. Thus, f is the futures price and r is the expected value of $\max\{p - k, 0\}$.² However, $E p = f$ and $E v = r$ are true only if a complete set of week-ahead contracts exists. Week-ahead contracts do not exist in general, and hedgers roll over into the next nearest contract before the current contract expires. The distributions of the random variables depend on the hedging horizon. This problem was addressed as in Frechette (2000), by incorporating a constant discount rate of 0.2 percent per week and constant marginal storage costs of 0.5 cents per week. After the adjustments, the conditional covariance matrix was assumed to be constant, and a bivariate normal distribution was assumed. Ignoring the adjustments would cause an error that grows with the length of the horizon. Sensitivity analysis indicated that the variances changed by less than 4% when the discount rate and marginal storage costs were set to zero. The covariances changed by 3.6% to 9.1% depending on the region. The character of the results depends much more on the coefficient of risk aversion than on the choice of these parameters.

The coefficient of absolute risk aversion was set to span a range of possible farmer risk preferences. Lapan and Moschini (1994) and Lence (1995) were used as a guide to select values for the coefficient of absolute risk aversion after converting the units as in Raskin and Cochran. The relative risk aversion parameter in these studies ranged from 0 to 20 per year for a soybean farm. Adjusting to a weekly value (multiply by 52) in cents (divide by 100), adjusting from an output-based quantity to a much smaller input quantity (divide by roughly 5) requires a final scaling factor of roughly 0.1. The range from 0 to 20 corresponds to an approximate range for the coefficient of absolute risk aversion of 0 to 2 per cent. Reasonable values to span this range were chosen as 2.00 for high risk aversion, 0.20 for moderate risk aversion, and 0.02 for low risk aversion.

Hedging Demands

The computational hedging demand curves cannot be expressed analytically. Table II

displays the quantities demanded at regular grid points in (t_x, t_z) space for the Southeastern Pennsylvania region, with a coefficient of absolute risk aversion of 0.2 per cent. Most hedgers face price combinations in the lower left corner of Table II because of margin accounts. Margin accounts account for a large portion of the cost of hedging with futures, but are not required for options hedging. For example, if an options position costs \$25 plus the premium, then t_z is 0.5 cents per bushel, which corresponds to column 2 in the Table. If a futures position also costs \$25 but requires a margin account, then t_x could be as high as 2.5 cents per bushel, depending on the hedger's discount rate. If $t_x = 1.5$, corresponding to the fourth row of Table II, then the optimal hedge ratios are $x = 0.3298$ and $z = 0.3417$. 33 percent of a hedger's corn purchases will be hedged with futures and 34 percent will be hedged with options.

The four other regions of Pennsylvania display similar hedging demands. Figure 1 shows the futures hedging demand curves for each region, assuming the marginal cost of trading options is 0.5 cents per bushel. Figure 2 shows the corresponding options hedging demand curves, assuming the marginal cost of trading futures is 1.5 cents per bushel. The Southeastern region displays the highest demand for futures hedging with a hedge ratio of 0.66 when the price is zero and a cutoff price of 2.01 cents per bushel, at which the demand curve intersects the price axis. The Western region displays the lowest demand for futures hedging with a hedge ratio of 0.20 when the price is zero and a cutoff price of 1.23. The Southeastern region displays the highest demand for options with a hedge ratio of 0.90 when the price is zero and a cutoff price of 0.825. The Western region displays the lowest demand for options with a hedge ratio of 0.29 when the price is zero and a cutoff price of 0.825. The options cutoff prices are the same for every region.

Little difference is observed when the coefficient of absolute risk aversion is changed. Tables III and IV display the demand for futures and options for the Southeastern Pennsylvania region when the coefficient of absolute risk aversion is 2.0 and 0.02. One interesting result to notice from these tables is that for some combinations of t_x and t_z the demand for hedging, overall, decreases as the level of risk aversion increases. This phenomenon occurs when options comprise a large part of the hedging portfolio and the optimal decision is to over-hedge. As the level of risk aversion rises, the way hedgers reduce risk is to reduce the hedge ratio, rather than increase it, because they are already over-hedged. In summary, higher levels of risk aversion cause the overall hedge ratio to move toward the optimal no-cost hedge ratio, sometimes resulting in an increase and sometimes a decrease.

The analyses above are based on the availability of an at-the-money option contract. When other strike prices are considered a curious pattern emerges. Low strike prices are always preferred to high ones, leading to the conclusion that in-the-money options are preferred to out-of-the-money options. This conclusion requires further analysis, as common practice indicates out-of-the-money options to be superior. The mean Chicago price was 255.22 cents per bushel, and Table II shows the results for an at-the-money strike price of 255 cents per bushel for the base case with moderate risk aversion. Table V displays the hedging demands for Southeastern Pennsylvania using an in-the-money strike price of 253 cents per

bushel. Options demand is higher for all combinations of t_x and t_z below the diagonal.

This result depends on how fast the cost of options changes as the strike price deviates from the current futures price. The marginal cost of supplying options away from the money may be substantial due to low liquidity. Deep in-the-money options will have a high marginal cost and shallow in-the-money options will be preferred by input hedgers. The supply curve for options at different strike prices is left for future research.

The hedging demands discussed in the last section generate hedger's surplus estimates that depend on t_x , t_z , and the other parameters of the model. Table VI displays the hedger's surplus estimates under moderate and high levels of risk aversion for each of the five Pennsylvania regions. Hedger's surplus is displayed for futures, options, and total. All surplus estimates were zero under low risk aversion, indicating that the opportunity to hedge is valueless or nearly valueless for hedgers with low risk aversion. Under moderate risk aversion, the Southeastern region had the highest hedger's surplus at 1.7 cents per bushel, and the Western, Central, and South Central regions had a hedger's surplus of zero.

The covariance structure between the Chicago price and each regional basis is the main determinant of the level of hedger's surplus estimated. Lower strike prices generated higher estimates of hedger's surplus, but changing the strike price did not affect the character of the hedger's surplus estimates. Quantitative comparisons are difficult to make because all surplus estimates are conditioned on the same fixed values for $t_x = 1.5$ cents per bushel and $t_z = 0.5$ cents per bushel, which may vary with the strike price as noted above.

An important qualitative comparison can be made between e_x and e_z . The value of e_x is the value of the opportunity to trade futures, given that options are available to trade. The value of e_z is the value of the opportunity to trade options, given that futures are available to trade. In most cases e_x and e_z were very small. For example, e_x is 0.091 cents per bushel and e_z is 0.054 cents per bushel for the Southeastern region at a moderate level of risk aversion. The surplus values are very small, indicating that futures and options are close substitutes.

Another qualitative comparison can be drawn between e_x and e_z on one hand and e_{xz} on the other. The value of e_{xz} is the value of the opportunity to trade futures or options, compared to the alternative of neither. At the moderate level of risk aversion, the Southeastern region had an e_{xz} value of 1.737 cents per bushel. Compared to 0.091 for e_x and 0.054 for e_z , the 1.737 seems very large. This result indicates that hedgers place a relatively high value on the opportunity to hedge, but they aren't particular about whether they hedge with futures or with options.

At a high level of risk aversion hedging becomes more valuable to hedgers. The value of e_{xz} rises dramatically to 19.295 cents per bushel for the Southeastern region. The value of e_x rises to 0.952, and the value of e_z falls to 0.005. The expected utility model tends to equate risk with variance, leading to a preference against both downside and upside variance. The difference between e_x and e_z is due to the hedger's assumed preference against variance. This result indicates that the expected utility model may need to be reconsidered for studies of this

sort in the future. It might be replaced by a model that more adequately represents hedgers' preference against downside risk without penalizing upside risk.

Implications & Conclusions

There are several practical implications of these results. First, LMH's conclusion that options play no role in the optimal hedging portfolio needs to be conditioned and clarified. LMH make it clear that they assume markets are unbiased, and they explain how biased expectations may lead to different results. However, they do not consider that hedging is costly. In most real situations faced by hedgers the optimal mix of futures and options will not be one-sided.

Second, the overall hedge ratio falls when the marginal cost of trading options increases, but in many cases it actually rises when the marginal cost of trading futures increases. The optimal position in those cases is an over-hedged position that declines as the level of risk aversion increases. These counter-intuitive conclusions indicate that over-hedging may not always be speculative.

Third, options hedging is more sensitive to price than is futures hedging. Hedging demand can be stimulated most easily by reducing the marginal cost of options trading. Such a reduction can be accomplished by reducing learning costs and management costs by educating hedgers about options. It can also be stimulated directly by subsidizing options and allowing hedgers to learn by experience, as with the Dairy Options Pilot Program. Targeting options is likely to have more effect on the overall hedge ratio than targeting futures.

Fourth, futures and options are highly substitutable, though not one-for-one. The hedger's surplus forgone by closing one market or the other is small in most cases, though the hedger's surplus forgone by closing both markets is much higher. Low levels of risk aversion make hedging unimportant and reduce the hedger's surplus to zero. For high and moderate levels of risk aversion, however, hedgers are nearly indifferent between the optimal futures-only strategy and the optimal options-only strategy. The optimal futures-only strategy involved a lower hedge ratio than the optimal options-only hedge ratio.

Overall, the relationship between futures and options still needs to be explored. The supply curve for hedging products needs to be developed in more detail. The relationship among the marginal costs of hedging options with different strike prices must be understood more fully. Alternative decision rules to expected utility should be considered to account properly for asymmetric attitudes toward upside and downside risk. Future research along these lines may contribute to a better understanding of hedging behavior and the development of more effective risk management strategies for hedgers.

References

- Berck, P. "Portfolio Theory and the Demand for Futures: the Case of California Cotton." *American Journal of Agricultural Economics* 63(3)(August 1981): 466-74.
- Bond, G. E., S. R. Thompson, and J. M. Geldard. "Basis Risk and Hedging Strategies for Australian Wheat Exports." *Australian Journal of Agricultural Economics* 29(3)(December 1985): 199-209.
- Bond, G. E. and S. R. Thompson. "Risk Aversion and the Recommended Hedging Ratio." *American Journal of Agricultural Economics* 67(4)(November 1985): 870-2.
- Briys, E., M. Crouhy, and H. Schlesinger. "Optimal Hedging in a Futures Market with Background Noise and Basis Risk." *European Economic Review* 37(June 1993): 949-60.
- Castelino, M. G. "Basis Volatility: Implications for Hedging." *Journal of Financial Research* 12(2)(Summer 1989): 157-72.
- Castelino, M. G. "Hedge Effectiveness: Basis Risk and Minimum-Variance Hedging." *Journal of Futures Markets* 12(April 1992): 187-201.
- Castelino, M. G., J. C. Francis, and A. Wolf. "Cross-Hedging: Basis Risk and Choice of the Optimal Hedging Vehicle." *The Financial Review* 26(2)(May 1991): 179-210.
- Frechette, D. L. "The Demand for Hedging and the Value of Hedging Opportunities." Working Paper (February 2000).
- Hanson, S. D. and G. W. Ladd. "Robustness of the Mean-Variance Model with Truncated Probability Distributions." *American Journal of Agricultural Economics* 73(2)(May 1991): 436-45.
- Hirshleifer, D. "Risk, Futures Pricing, and the Organization of Production in Commodity Markets." *J. Political Economy* 96(6)(December 1988): 1206-20.
- Kahl, K. H. "Determination of the Recommended Hedging Ratio." *American Journal of Agricultural Economics* 65(August 1983): 603-5.
- Lapan, H. E., G. Moschini, and S. D. Hanson. "Production, Hedging, and Speculative Decisions with Options and Futures Markets." *American Journal of Agricultural Economics* 73(1)(February 1991): 66-74.
- Lapan, H. E. and G. Moschini. "Futures Hedging under Price, Basis, and Production Risk." *American Journal of Agricultural Economics* 76(August 1994): 465-77.
- Lence, S. H. "The Economic Value of Minimum-Variance Hedges." *American Journal of Agricultural Economics* 77(May 1995): 353-64.
- Lence, S. H. "Relaxing the Assumptions of Minimum-Variance Hedging." *Journal of Agricultural and Resource Economics* 21(1)(July 1996): 39-55.
- Markowitz, H. M. "Portfolio Selection." *Journal of Finance* 7(1)(March 1952): 77-91.

- Markowitz, H. M. *Portfolio Selection: Efficient Diversification of Investments*. New York: Wiley, 1959.
- Meyer, J. "Two-Moment Decision Models and Expected Utility Maximization." *American Economic Review* 77(June 1987): 421-30.
- Moser, J. T. and B. Helms. "An Examination of Basis Risk Due to Estimation." *Journal of Futures Markets* 10(5)(October 1990): 457-67.
- Netz, J. "An Empirical Test of the Effect of Basis Risk on Cash Market Positions." *Journal of Futures Markets* 16(3)(May 1996): 289-311.
- Park, T. and F. Antonovitz. "Basis Risk and Optimal Decision Making for California Feedlots." *Journal of Futures Markets* 10(3)(June 1990): 259-71.
- Paroush, J. and A. Wolf. "Production and Hedging Decisions in the Presence of Basis Risk." *Journal of Futures Markets* 9(6)(December 1989): 547-63.
- Raskin, R. and M. J. Cochran. "Interpretations and Transformations of Scale for the Pratt-Arrow Absolute Risk Aversion Coefficient: Implications for Generalized Stochastic Dominance." *Journal of Agricultural and Resource Economics* 11(2)(December 1986): 204-10.
- Simaan, Y. "What is the Opportunity Cost of Mean-Variance Investment Strategies?" *Management Science* 39(5)(May 1993): 578-87.
- Tew, B. V., D. W. Reid, and C. A. Witt. "The Opportunity Cost of a Mean-Variance Efficient Choice." *Financial Review* 26(1)(February 1991): 31-43.
- Turnovsky, S. J. "The Determination of Spot and Futures Prices with Storable Commodities." *Econometrica* 51(September 1983): 1363-87.
- Viaene, J.-M. and C. G. de Vries. "International Trade and Exchange Rate Volatility." *European Economic Review* 36(6)(August 1992): 1311-21.
- von Neumann, J. and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton, NJ: Princeton University Press, 1944.
- Wolf, A. "Optimal Hedging with Futures Options." *J. Econ. and Bus.* 39(1987): 141-58.

Table I
Regional Price Covariance Structure
Nominal Cents/Bushel, 1997-98

Region	N*	Price Mean	Price Variance	Basis Variance	Covariance with Chicago
Southeastern PA	104	291.40	33.57	14.60	-20.07
Central PA	104	287.40	14.51	37.51	-41.05
South Central PA	104	284.80	18.54	42.73	-41.65
Western PA	104	279.50	32.05	67.68	-47.37
Lehigh Valley	85	274.70	34.81	17.93	-21.98
Chicago	104	255.22	59.11		

* Data collection for Lehigh Valley began May 1997. All other data are for Jan. 1997 – Dec. 1998.

Table II
Hedging Demand
Southeastern Pennsylvania
Moderate Risk Aversion

		t_z					
		0.0	0.5	1.0	1.5	2.0	2.5
0.0	0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0
0.5	0.4772 0.2611	0.6182 0	0.6182 0	0.6182 0	0.6182 0	0.6182 0	0.6182 0
1.0	0.2467 0.5849	0.5558 0.0356	0.5758 0	0.5758 0	0.5758 0	0.5758 0	0.5758 0
t_x 1.5	0 0.9016	0.3298 0.3417	0.5359 0	0.5359 0	0.5359 0	0.5359 0	0.5359 0
2.0	0 0.9016	0.0153 0.7485	0.4118 0.1287	0.4912 0	0.4912 0	0.4912 0	0.4912 0
2.5	0 0.9016	0 0.7674	0.1118 0.5140	0.4490 0	0.4490 0	0.4490 0	0.4490 0

Prices in cents per bushel. Futures hedge ratios listed above options hedge ratios.

Table III
Hedging Demand
Southeastern Pennsylvania
High Risk Aversion

		t_z					
		0.0	0.5	1.0	1.5	2.0	2.5
0.0		0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0
0.5		0.6421 0.0262	0.6562 0	0.6562 0	0.6562 0	0.6562 0	0.6562 0
1.0		0.6185 0.0585	0.6500 0.0036	0.6520 0	0.6520 0	0.6520 0	0.5758 0
t_x							
1.5		0.5851 0.1022	0.6274 0.0342	0.6478 0	0.6478 0	0.6478 0	0.6478 0
2.0		0.5318 0.1679	0.5959 0.0749	0.6356 0.0129	0.6435 0	0.6435 0	0.6435 0
2.5		0.4118 0.3060	0.5455 0.1366	0.6056 0.0514	0.6393 0	0.6393 0	0.6393 0

Prices in cents per bushel. Futures hedge ratios listed above options hedge ratios.

Table IV
Hedging Demand
Southeastern Pennsylvania
Low Risk Aversion

	t_z					
	0.0	0.5	1.0	1.5	2.0	2.5
0.0	0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0
0.5	0 0.9522	0.2375 0	0.2375 0	0.2375 0	0.2375 0	0.2375 0
1.0	0 0.9522	0.0005 0	0.0002 0	0.0002 0	0.0002 0	0.0002 0
t_x 1.5	0 0.9522	0 0	0 0	0 0	0 0	0 0
2.0	0 0.9522	0 0	0 0	0 0	0 0	0 0
2.5	0 0.9522	0 0	0 0	0 0	0 0	0 0

Prices in cents per bushel. Futures hedge ratios listed above options hedge ratios.

Table V
Hedging Demand
Southeastern Pennsylvania
In-the-Money Options

		t_z					
		0.0	0.5	1.0	1.5	2.0	2.5
t_x	0.0	0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0	0.6605 0
	0.5	0.3896 0.3515	0.6182 0	0.6182 0	0.6182 0	0.6182 0	0.6182 0
t_x	1.0	0 0.8325	0.4735 0.1532	0.5758 0	0.5758 0	0.5758 0	0.5758 0
	1.5	0 0.8325	0.0809 0.6350	0.5359 0	0.5359 0	0.5359 0	0.5359 0
t_x	2.0	0 0.8325	0 0.7313	0.1768 0.4319	0.4912 0	0.4912 0	0.4912 0
	2.5	0 0.8325	0 0.7313	0 0.6405	0.2667 0.2456	0.4490 0	0.4490 0

Prices in cents per bushel. Futures hedge ratios listed above options hedge ratios.

Table VI
Hedger's Surplus
Southeastern Pennsylvania

	Moderate Risk Aversion			High Risk Aversion		
	e_x	e_z	e_{zx}	e_x	e_z	e_{zx}
Southeastern PA	0.091	0.054	1.737	0.952	0.005	19.30
Central PA	0	0.053	0.242	0.317	0.005	5.02
South Central PA	0	0.053	0.221	0.300	0.005	4.69
Western PA	0	0.036	0.067	0.151	0.005	2.05
Lehigh Valley	0.075	0.054	1.539	0.892	0.005	17.91

$t_x = 1.5$ cents/bushel, $t_z = 0.5$ cents/bushel
Surplus measured in cents per bushel.

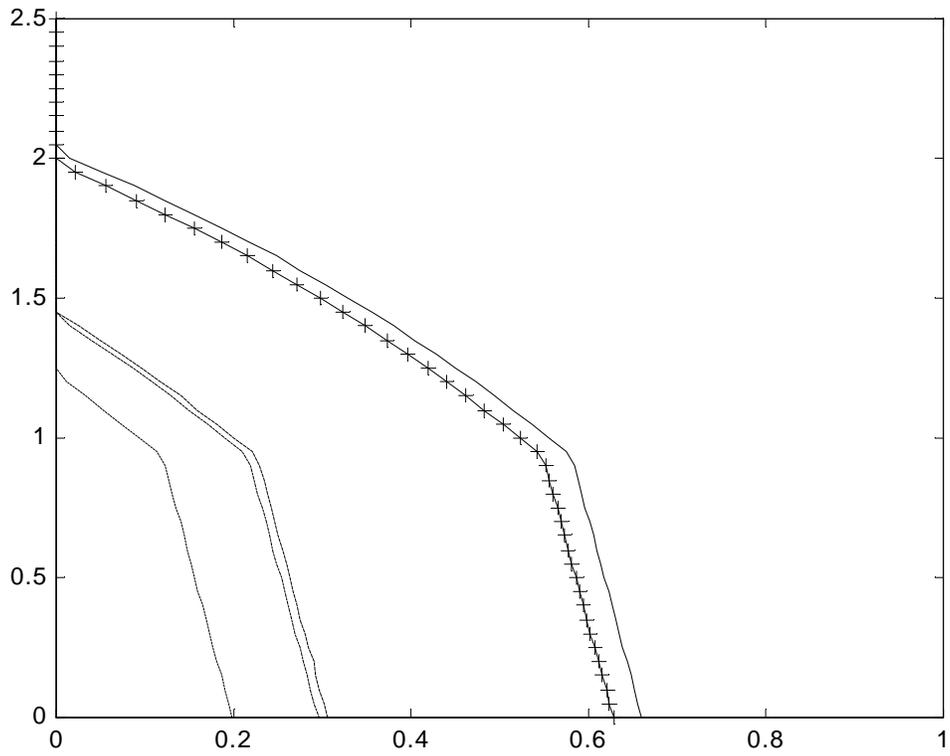


Figure 1

Futures Hedging Demands

Moderate risk aversion. Futures cost in cents/bushel on vertical axis. Hedge ratio (futures) on horizontal axis. $t_z = 0.5$. SE solid, C dotted, SC dot-dash, W dash, LV cross hair.

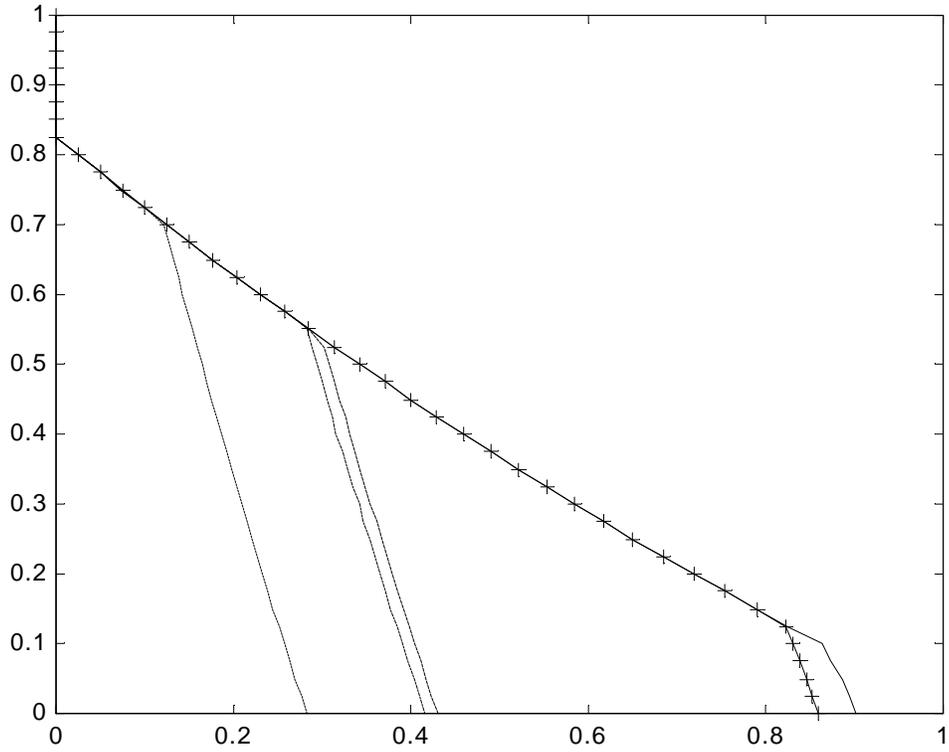


Figure 2

Options Hedging Demands

Moderate risk aversion. Option cost in cents per bushel on vertical axis. Hedge ratio (options) on horizontal axis. $t_x = 1.5$. SE solid, C dotted, SC dot-dash, W dash, LV cross hair.

Endnotes

¹ To see why, interpret an increase in t_x as an equivalent decrease in t_z and decrease in hedging expenditures, denoted y . Similarly, an increase in t_z is equivalent to a decrease in t_x and a decrease in hedging expenditures. Consider that $dx/dt_z = d|z|/dt_x$, and dx/dt_x is smaller in magnitude than $d|z|/dt_z$, so dx/dy must be smaller in magnitude than $d|z|/dy$. Thus, the synthetic expenditure elasticity for futures is smaller than the expenditure elasticity for options, meaning futures are more of a necessary good and options are more of a luxury good.

² The base case for the strike price is at-the-money, but other strike prices will be considered.