IMPLIED VOLATILITY AND INTEREST RATES IN EARLY PRICING OF PUT OPTIONS ON AGRICULTURAL COMMODITIES

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Trading in options on futures contracts on agricultural commodities grown in the United States began in late 1984. While trading activity in these options has not been overwhelming, it is believed that trading has been sufficient to provide some substantial insight into the pricing of options when market conditions provide opportunities for more widespread use by farmers and those who buy from farmers. The focus in this study is on put options since it is expected that these options will be used more extensively than call options by farmers and those who buy from farmers. This paper reports on research on volatility and interest rates implied in the premiums of put options on futures contracts on cotton, corn, live hogs and live cattle in the first few months of trading in these options.

As correlation studies reported here show, the market does price the put options by at least a close approximation of the Black model of option pricing. The variables included in the Black model are the following:

f is the current price of the futures contract

s is the selected strike price

s-f is the difference between the strike and futures prices

d is days to maturity of the option

r is interest rate

v is volatility of the futures contract.

S-f is not generally regarded as a variable in the Black model, however, research has shown that more insight can be gained to option pricing by focusing on s-f than s as a separate variable. In estimating the premium for an option with the Black model the appropriate values of the variables other than r and v are unambiguously specified by the user. While individual users of the Black model may have specific ideas of what constitutes appropriate values for r and v, the values of these variables that are really appropriate to market pricing of options logically must be determined from option premiums determined by the market.

The research progressed by initially computing the time-value and s-f values from daily closing options premiums. The

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intrinsic-value is subtracted from the premiums since it is always exactly equal to the difference between s and f when s exceeds f on put options, and intrinsic value contains no information that is useful to the research.

Plots of time-value as a function of s-f show a characteristic function in which the time-value increases at an increasing rate as s-f progresses from large negative values to zero, and then the function decreases at a decreasing rate as s-f progresses from zero to large positive values. This functional relationship seen in the empirical data is inherent in the Black model.

The estimation of this function of time-value as function of s-f required the use of a unique regression equation with dummy variables and squared terms of s-f. The constant in the regression equation is an excellent estimate of the time-value of the put option when s=f, which is an option at-the-money. The regression function also fits the out-of-the-money option portion of the s-f range independent of the in-the-money portion of the s-f range except that both parts of the function are forced to have the same value at-the-money. This allows the two parts of the s-f range to have different curvature and thus different values on the variables that determine the time value. If the in-the-money and out-of-the-money portions of the s-f range were fitted separately it is not only possible but it is likely that the time-value at-the-money would have two different values for the same basic set of data.

Each regression equation was fitted to data computed from one calendar month of closing put option premiums. Tables 1, 2, 3 and 4 show the adjusted R squares for each of the regression equations estimated on time-value as a function of s-f. It can be seen that the R square values are generally very high and range from .74 to .99. About three-fourths of the R square values are greater than .90. In addition the slope coefficients on the s-f and s-f-squared values are, with a very few exceptions, significantly different from zero at very impressive levels. In every regression equation the intercept value (value of the option at-the-money) was significantly different from zero at very high levels of significance. These results from fitting the regression equations suggest that the form of the regression equations allow an excellent measurement of the relationship between time-value and s-f.

The regression equations were used to estimate the time-value for options at-the-money and out-of-the-money and in-the-money by s-f values of 1, 3 and 5 cents. The final step in the research process was to experimentally find the values of r and v applied to the Black model that produced the smallest sum of the absolute differences between the regression values of time-value and the Black model values of time-value with the solution constrained by the requirement that the values of r and v in the Black model must produce at-the-money time-values of exactly the same magnitude as the time-values in the regression equations.

Table 1. Implied Interest Rates and Volatility of Cotton Put Options

	months to option maturity										
	15	14	13	12	11	10	9	8	7	6	5
December, 1985, option											
adjR^2				.76	.90	.92	.97	.98	.95	.96	.97
out-of-money											
r				13.9	-5.1	3.5	-5.0	10.0	19.5	16.2	22.0
v				8.2	7.7	7.6	6.5	7.6	8.0	8.3	10.2
in-the-money											
r				17.0	21.0	9.7	7.3	7.8	8.2	4.0	5.6
v				8.8	10.4	8.6	7.6	8.0	8.0	8.5	10.6
March, 1986, option											
adjR^2	.74	.87	.94	.96	.92	.95	.90	.94			
out-of-money											
r	20.0	-29.5	20.0	12.0	6.5	9.5	17.5	25.5			
v	10.4	6.3	10.4	8.9	8.2	7.9	8.5	8.7			
in-the-money											
r	29.3	51.3	8.0	11.7	15.0	16.8	7.8	6.9			
v	11.8	16.9	9.3	9.2	9.2	8.7	8.2	8.0			

Source: Computed as explained in text.

Table 2. Implied Interest Rates and Volatility of Corn Put Options

	months to option maturity									
	11	10	9	8	7	6	5	4	3	2
December, 1985, option										
adjR^2				.88	.95	.96	.95	.95	.88	.86
out-of-money										
r				7.0	5.5	2.5	7.8	45.0	228	475
v				12.6	13.8	14.3	17.0	17.2	22.7	25.1
in-the-money										
r				7.0	13.0	8.0	4.0	-1.0	17.0	-30
v				12.6	14.3	14.8	16.8	14.8	12.9	12.1
March, 1986, option										
adjR^2	.80	.90	.96	.93	.97	.94	.94			
out-of-money										
r		21.0	-5.0	11.0	39.5	78.0	278			
v		18.5	14.8	18.2	18.2	19.1	23.3			
in-the-money										
r	25.0	10.0	16.5	13.0	4.5	1.0	-8.0			
v	17.6	17.5	16.4	18.4	16.4	16.2	16.4			

Source: Computed as explained in text.

Table 3. Implied Interest Rates and Volatility of Hog Put Options

	months to option maturity				
	5	4	3	2	1
June, 1985, option					
adjR^2			.97	.84	.88
out-of-money					
r			41.5	60.5	279.0
v			14.0	19.7	26.2
in-the-money					
r			-3.5	-6.0	3.0
v			12.4	17.2	18.9
July, 1985, option					
adjR^2		.90	.89	.93	.91
out-of-money					
r		35.0	-2.0	110.0	149.0
v		14.6	18.4	25.0	26.0
in-the-money					
r		1.0	6.0	13.0	5.0
v		13.0	18.8	20.7	22.2
August, 1985, option					
adjR^2	.88	.85	.95	.96	.87
out-of-money					
r	17.0	23.0	-2.0	95.0	81.0
v	13.8	19.4	20.6	27.6	26.8
in-the-money					
r	-11.0	-1.0	4.0	10.0	3.0
v	12.4	17.8	21.0	23.4	24.6

Source: Computed as explained in text.

The results of this experimentation on the r and v values that minimized the sum of absolute differences between the Black model and the regression equation values are reported in the four tables in this report. The implied r and v values for March, 1986, out-of-the-money corn put options 11 months to maturity could not be estimated because there were very few premiums reported for out-of-the-money put options that month. The options for each of the four commodities began trading in either very late October or early November of 1984. With the exception of the out-of-money corn put option, the left-most values of r and v reported in the tables is the estimate for the month of November, 1984, which was the first full or nearly full month of trading in the options. The r and v values to the right of the left-most values in the tables are for the trading in the month of December. In the case of cotton and corn, the December and March contracts were selected for study since these are the contracts that are most often used by farmers and those who buy from farmers to hedge. Options mature approximately one month before the trading in the associated futures contracts concludes.

Table 4. Implied Interest Rates and Volatility of Cattle Put Options

	months to option maturity				
	5	4	3	2	1
February, 1985, option					
adjR^2				.95	.89
out-of-money					
r				182	370
v				16.5	16.0
in-the-money					
r				-10	-20
v				12.1	9.6
April, 1985, option					
adjR^2		.94	.97	.93	.90
out-of-money					
r		1	77	70	300
v		14.6	15.6	13.4	14.4
in-the-money					
r		-10	-20	-7	-7
v		14.2	12.4	11.5	9.6
June, 1985, option					
adjR^2	.98	.97	.97	.99	.98
out-of-money					
r	25	45	80	78	100
v	15.5	14.6	14.8	16.6	15.8
in-the-money					
r	13	-1	-2	2	-1
v	14.9	12.9	12.4	14.6	14.3

Source: Computed as explained in text.

The implied interest rates are found to be extremely variable ranging from minus 30 to plus 475 percent. This extreme variability in implied interest rates certainly suggests that not much credibility can be attached to their specific values. At representative values of the variables in the Black model, the estimated premiums are about twenty times more sensitive to proportionate changes in volatility than they are to changes in the interest rates. This relative insensitivity to interest rate changes is probably the principal explanation for the great variability in estimates of implied interest rates in the four tables. There does seem to be some tendency for the implied interest rates to be higher for out-of-the-money options than for in-the-money put options. It seems logical that if there was a difference in r between in or out-of-money put options that the higher r would be associated with the in-the-money option since these options include in the premium some intrinsic value in addition to the time value. In the Black model there is an inverse relationship between changes in r and changes in the premium. One possible conclusion might be that interest rate is simply not a very important variable in the pricing of options, and possibly it could either be ignored or replaced with another variable that would be found to explain the great variability attributed to implied interest rates.

In the data of Table 1 there seems to be lower adjusted R square values in the early months of trading the put options on cotton than in later months. This may indicate that there was some learning going on as traders learned how to appropriately price the options. This "learning" pattern is less pronounced in the put options on corn futures contracts. No such "learning" process seems to have existed in the pricing of put options on cattle and hog futures contracts.

The implied volatility on cotton futures contracts is lower than on the other three commodities. Also the values of v are generally lower for out-of-money options than for in-the-money options. Both of these findings are consistent with the fact that during the period represented in the option premium data the market price of cotton was only somewhat above the CCC loan price. The most obvious effect of the CCC loan price would be on limiting downward price moves and thus affecting the variability for out-of-money put option premiums. But when the CCC is holding large stocks of cotton the CCC loan price will also limit the upward price moves but to a smaller extent than the down moves.

While the market price of corn was also in the vicinity of its CCC loan price the CCC loan price on corn does not set an effective floor on the market price as the CCC loan price has historically done on the market price of cotton. The values of v for corn generally exceed those of not only cotton but also those for cattle. Only the values of v for hogs exceed those for corn. There is a strong tendency for the implied values of v on the out-of-money options to exceed those on in-the-money options on the commodities other than cotton. This suggests even more that the CCC loan price had a very great influence on the implied v for cotton so that the more general tendency for out-of-money options v to exceed in-the-money options v is reversed. The research techniques used in this research are somewhat primitive, the coverage of commodities and months of trading is limited and the trading activity in the options has not been extensive. For these reasons it is difficult to reach irrefutable conclusions based upon the evidence presented in this paper. Certain patterns do seem to exist in the results reported in the tables that provide a basis the following hypotheses that may be worthy of future research:

1. Users of options on futures contracts should use values of volatility implied from past option premiums rather than assuming that the appropriate volatility is a number computed using a particular formula on past futures prices.

2. Interest rates used on option pricing should be higher for out-of-money options than for in-the-money options.

3. Since implied interest rates are so erratic and the premiums insensitive to interest rate changes, the interest rate

should be assumed to be either zero or some small value.

4. The market prices options with a higher volatility for an out-of-money option than for the same option if it is in-the-money.

References

Al-Sakkaf, Ghazi. Put Options on Cotton Futures Contracts and Alternative Marketing Strategies, M.S. Thesis, Agricultural Economics Department, University of Arizona, 1986.

Belongia, Michael T. "Commodity Options: A New Risk Management Tool for Agricultural Markets," Federal Reserve Bank of St. Louis Review, Vol. 65, no. 6, June/July, 1983, pp. 5-15.

Black, Fischer. "The Pricing of Commodity Contracts," Journal of Financial Economics, Vol. 3, 1976, pp. 167-79.

Catlett, Lowell B. and Michael Boehlje. "Commodity Options, Hedging and Risk Premiums," North Central Journal of Agricultural Economics, Vol. 4, no. 2, July, 1982, pp. 95-101.

Firch, Robert S. and Ghazi Al-Sakkaf. "Cotton Put Options: Low Price Insurance?," 1986 Proceedings Beltwide Cotton Production Conferences," National Cotton Council, Memphis, Tenn., pp. 319-322.